Precise Determination of the Ascendant in the *Lagnaprakaraṇa* - III

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Abstract

Authored by the celebrated mathematician-astronomer Mādhava, the *Lagnaprakaraṇa* is an important astronomical text dedicated to the determination of the *udayalagna* or the ascendant, and is notable for its technical brilliancy and multiple approaches to a given problem. In continuation of our previous papers on this text, here we discuss additional methods for precisely determining the *udayalagna* as described in the third chapter of the *Lagnaprakaraṇa*. The procedure prescribed here makes use of two quantities knows as the *śaṅku* and the *dṛggati*.

Key words: Ascendant, *dṛkkṣepakoṭikā*, *dṛggati*, *lagna*, *Lagnaprakaraṇa*, Mādhava, *śaṅku*

1 Introduction

In our previous two papers,[1] we have discussed a variety of methods of determining the ascendant described in verses 31–52 of the *Lagnaprakaraṇa*, constituting its second chapter. In this paper, we discuss further methods for determining the ascendant described in verses 53–61 of the *Lagnaprakaraṇa*, constituting its third chapter. These verses describe the method to determine the gnomons (*śaṅku* and *dṛggati*) corresponding to the Sun and an ecliptic point whose longitude is ninety degrees less than that of the Sun's, and then show how to determine the ascendant therefrom.

It may be noted that this paper is to be read in conjunction with our earlier paper as various physical and mathematical quantities described therein are employed here as well. In this regards, it may be reiterated that we employ the symbols λ, α, δ, and z to respectively refer to the longitude, right ascension, declination, and zenith distance of a celestial body. The *kālalagna*, the latitude of the observer, and the obliquity of the ecliptic are denoted by the symbols α_e, ϕ, and ϵ respectively. It may also be mentioned that all the figures in this paper depict the celestial sphere for an observer having a northerly latitude ϕ. In these figures, N, S, E, and W denote the cardinal directions north, south, east, and west, while P and K denote the poles of the celestial equator and the ecliptic respectively.

2 Obtaining the *śaṅku*

कृतायनार्कस्य भुजां द्युजीवां[2]
चरं कलाप्राणभिदां च नीत्वा ।
कलासुभेदं पुनरत्र भानौ
संस्कृत्य तद्वर्जितकाललग्नात् ॥५३॥
नीतां भुजाज्यां समभिन्नदिक्त्वे

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[1] See [12] and [13]. Also see [11], [14], and [10] for our discussion on some of the foundational astronomical parameters described by Mādhava in the first chapter of the *Lagnaprakaraṇa*.

[2] भुजा द्युजीवां in the manuscripts. Corrected for grammar.

चरान्वितोनां द्युगुणेन[3] हत्वा ।
हृत्वा त्रिजीवाकृतिलम्बकांशेन
अवाप्तमर्कस्य[4] वदन्ति शङ्कुम् ॥५४॥

kṛtāyanārkasya bhujāṃ dyujīvāṃ
caraṃ kalāprāṇabhidāṃ ca nītvā |
kalāsubhedaṃ punaratra bhānau
saṃskṛtya tadvarjitakālalagnāt ||53||
nītāṃ bhujājyāṃ samabhinnadiktve
carānvitonāṃ dyuguṇena hatvā |
hṛtvā trijīvākṛtilambakāṃśena
avāptamarkasya vadanti śaṅkum ||54||

Having computed the Rsine of the precession corrected longitude of the Sun, the day-radius (*dyujīvā*), the ascensional difference (*cara*), and the *prāṇakalāntara*, and having applied the *prāṇakalāntara* to [the longitude of] the Sun, the *kālalagna* is decreased by that [quantity]. The Rsine computed from that [modified *kālalagna*], increased or decreased by the [Rsine of] ascensional difference (*cara*) depending on the sameness or difference in the directions [of the declination and the latitude], is multiplied by the day-radius (*dyuguṇa*) and divided by one-*lamba*th part of the square of the radius (*trijīvā*). [Scholars] state the quotient [thus obtained] to be the gnomon (*śaṅku*) of the Sun.

These two verses, in the *upajāti* metre, gives the following relation for the measure of the gnomon corresponding to the height of the Sun at any instant:

$$\begin{aligned}\textit{śaṅku} = \Big[&\textit{bhujājyā}\ \big(\textit{kālalagna} - [\textit{kṛtāyanārka} \pm \\ &\textit{kalāsubheda}]\big) \pm \textit{carajyā}\Big] \times \textit{dyuguṇa} \div \\ &\textit{trijīvākṛtilambakāṃśa}\end{aligned}$$

or,

$$R\cos ZS = \frac{[R\sin(\alpha_e - \alpha) \pm R\sin\Delta\alpha_s] \times R\cos\delta}{\frac{R^2}{R\cos\phi}}, \quad (1)$$

where α_e is the *kālalagna*, α and δ are the Sun's right ascension[5] and declination, and $\Delta\alpha_s$ refers to the 'instantaneous' ascensional difference of the Sun, calculated using δ.

The validity of the above relation can be understood with the help of Figure 1a, which depicts the Sun (S) having declination δ at a given instant. The altitude of the Sun is given by the arc $SF = 90 - ZS$. Therefore, the measure of a gnomon corresponding to the height of the Sun is given by

$$\textit{śaṅku} = SX = R\sin SF = R\cos ZS.$$

Now, considering the spherical triangle PZS in the figure, we have $PS = 90 + \delta$, and $PZ = 90 - \phi$. We note that the angle $H = BT = 90 - BE$.[6] However, $BE = \alpha_e - \alpha$.[7] Therefore, $H = 90 - (\alpha_e - \alpha)$. Now, applying the cosine rule in the spherical triangle PZS, we have

$$\begin{aligned}\cos ZS &= \cos(90-\phi)\cos(90+\delta) + \\ &\qquad \sin(90-\phi)\sin(90+\delta)\cos H \\ &= [\sin(\alpha_e - \alpha) - \tan\phi\tan\delta]\cos\phi\cos\delta,\end{aligned}$$

which can be written as

$$\cos ZS = [\sin(\alpha_e - \alpha) - \sin\Delta\alpha_s]\cos\phi\cos\delta, \quad (2)$$

where $\sin\Delta\alpha_s$ is the *cara* calculated using (16) from [14], however using the declination of the Sun at the given instant, rather than at sunrise. In the given figure, the declination of the Sun is negative. That is, the Sun and the latitude—represented by the arc ZT—are on opposite sides of the equator. Therefore, the ascensional difference is negative in the above relation.

When the Sun's declination is positive, it will lie on the same side of the equator as the arc representing the latitude. Then, in the spherical triangle PZS, we will have $PS = 90 - \delta$, while the other parameters stay the same. Upon applying the cosine rule in this spherical triangle again, we get

$$\begin{aligned}\cos ZS &= \cos(90-\phi)\cos(90-\delta) + \\ &\qquad \sin(90-\phi)\sin(90-\delta)\cos H \\ &= \sin\phi\sin\delta + \cos\phi\cos\delta\sin(\alpha_e - \alpha) \\ &= [\sin(\alpha_e - \alpha) + \sin\Delta\alpha_s]\cos\phi\cos\delta. \qquad (3)\end{aligned}$$

[3] चरान्वितोना द्युगुणेन in [17] and चरान्वितोनाद्ध्युगुणेन in [18]. Both appear to be transcribing errors.

[4] त्रिजीवाहृतिलम्बकांशेनावाप्तमर्कस्य in the manuscripts. This appears to be a transcribing error.

[5] The expression *kṛtāyanārka* ± *kalāsubheda* is equal to $\lambda \pm |\lambda - \alpha| = \alpha$.

[6] From (5) in [12] we know that the difference between the *madhyakāla* and the *kālalagna*, that is the arc TE, is 90 degrees.

[7] This is the difference in the right ascensions of the east cardinal point (α_e) and the Sun (α).

Converting to the Indian cosine, and taken together, (2) and (3) yield

$$R\cos ZS = [R\sin(\alpha_e - \alpha) \pm R\sin\Delta\alpha_s]\cos\phi\cos\delta,$$

which is the same as (1).

3 Obtaining the *dṛggati*

अथ त्रिभोनिताद्भानोः अपमं तस्य कोटिकाम् ।।
चरं प्राणकलाभेदं अपि प्राग्वदिहानयेत् ॥५५॥
कृत्वा भास्वति वित्रिभे निजकलाप्राणान्तरं तत्पुनः
त्यक्त्वा कालविलग्नतो भुजगुणस्तच्छेषजातः क्रमात् ।
दिग्भेदैक्यवशाच्चरोनसहितः कोट्यापमस्याहतः
त्रिज्याकृत्यवलम्बकांशविहृतो भानोर्भवेद्दृग्गतिः ॥५६॥

atha tribhonitādbhānoḥ
apamaṃ tasya koṭikām |
caraṃ prāṇakalābhedaṃ
api prāgvadihānayet ||55||
kṛtvā bhāsvati vitribhe nijakalā-
-prāṇāntaraṃ tatpunaḥ
tyaktvā kālavilagnato bhujaguṇas-
-tacheṣajātaḥ kramāt |
digbhedaikyavaśāccaronasahitaḥ
koṭyāpamasyāhataḥ
trijyākṛtyavalambakāṃśavihṛto
bhānorbhaveddṛggatiḥ ||56||

Now, one should compute [the Rsine of] the declination (*apama*) and its Rcosine, the ascensional difference (*cara*), and also the *prāṇakalāntara* from the longitude of Sun minus three signs (*tribhonitādbhānu*) as earlier.
Having applied the *prāṇakalāntara* to the Sun's longitude minus three signs (*vitribha bhāsvat*), and again subtracting that [result] from the *kālalagna*, [compute] the Rsine of the residue obtained. That increased or decreased by the ascensional difference (*cara*) depending on the directions [of the declination and the latitude] being same or different, multiplied by the Rcosine of the declination (*koṭyāpama*) and divided by one-*lamba*th part of square of radius (*trijyā*) would become the *dṛggati* of the Sun.

These two verses, in the *anusṭubh* and *śārdūlavikrīḍitam* metres respectively, give the following relation for the *dṛggati* of the Sun, which is defined as the height of a point on the ecliptic whose longitude is ninety degrees less than that of the Sun:

$$\begin{aligned} d\d{r}ggati = \Big[&bhujaguṇa\ (kālalagna - [vitribha\ bhāsvat\ \pm \\ &nija\text{-}kalāprāṇāntara]) \pm carajyā\Big] \times \\ &koṭyapama \div trijyākṛtyavalambakāṃśa \end{aligned}$$

or,

$$R\cos ZS' = \frac{[R\sin(\alpha_e - \alpha') \pm R\sin\Delta\alpha'_s] \times R\cos\delta'}{\frac{R^2}{R\cos\phi}}, \quad (4)$$

where α_e is the *kālalagna*, α' and δ' are the right ascension[8] and declination of an ecliptic point (S') ninety degrees behind the Sun, and $\Delta\alpha'_s$ is the 'instantaneous' *cara* of this point, computed using δ'.

The validity of the above relation can be understood with the help of Figure 1b, which depicts the point S' on the ecliptic, having declination δ', and which is ninety degrees behind the position of the Sun depicted in Figure 1a. The altitude of the point S' is given by the arc $S'F' = 90 - ZS'$. Therefore, the measure of a gnomon corresponding to the height of S' is given by

$$d\d{r}ggati = S'Y = R\sin S'F' = R\cos ZS'.$$

Now, considering the triangle PZS' in the figure, we have $PS' = 90 + \delta'$, and $PZ = 90 - \phi$. We note that the angle $H' = TB' = 90 - B'W$.[9] However, $B'W = \alpha' - (\alpha_e - 180)$.[10] Thus, $H' = (\alpha_e - \alpha') - 90$. Now, applying the cosine rule in the spherical triangle PZS', we have

$$\begin{aligned} \cos ZS' &= \cos(90-\phi)\cos(90+\delta') + \\ &\qquad \sin(90-\phi)\sin(90+\delta')\cos H' \\ &= [\sin(\alpha_e - \alpha') - \tan\phi\tan\delta']\cos\phi\cos\delta', \end{aligned}$$

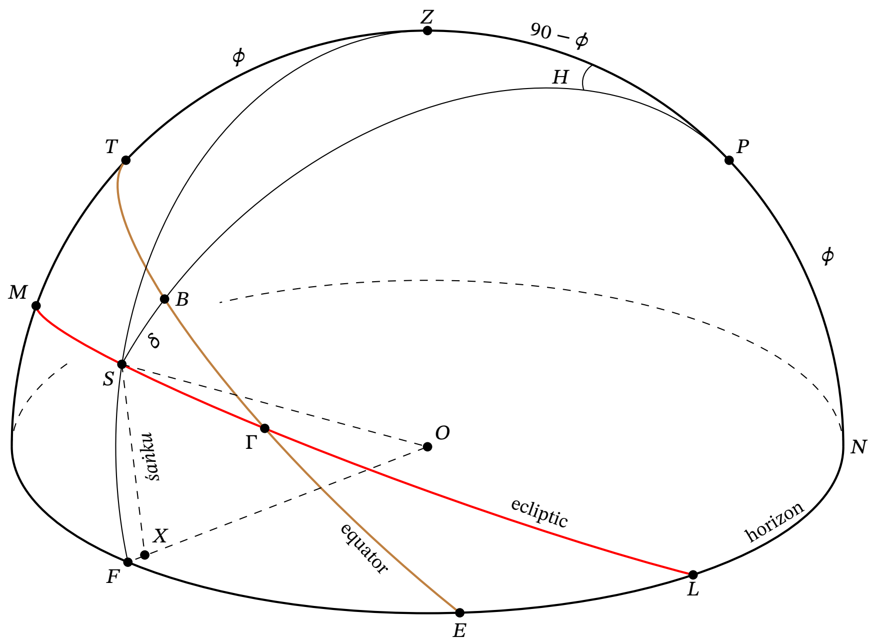
which can be written as

$$\cos ZS' = [\sin(\alpha_e - \alpha') - \sin\Delta\alpha'_s]\cos\phi\cos\delta', \quad (5)$$

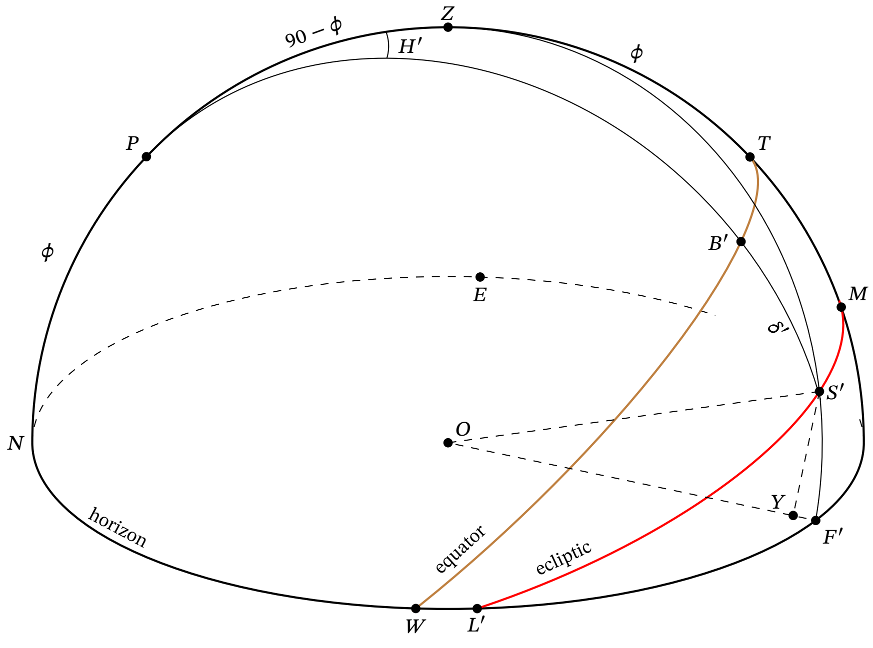
[8]Taking λ as the longitude of the Sun, the expression *vitribha bhāsvat* ± *nija-kalāprāṇāntara* is nothing but $\lambda' \pm |\lambda' - \alpha'|$, where $\lambda' = \lambda - 90$. The *nija-kalāprāṇāntara* (see verse 31), which is the same as the *nija-prāṇakalāntara* discussed in verse 31, has no special meaning here as it is being applied to the longitude.

[9]From (5) in [12] we know that the difference between the *madhyakāla* and the *kālalagna*, that is the arc TE, is 90 degrees. As $EW = 180$, we have $TW = 90$.

[10]This is the difference in the right ascensions of S' (α') and the west cardinal point ($180 - \alpha_e$).



(a) *Śaṅku*



(b) *Dṛggati*

Figure 1 Determining the *śaṅku* and the *dṛggati*.

where $\sin\Delta\alpha'_s$ is the *cara* calculated using (16) in [14], however using the declination (δ') of the point S'. In the given figure, the declination of S' is negative. That is, S' and the latitude—represented by the arc ZT—are on opposite sides of the equator. Therefore, the ascensional difference is negative in the above relation.

When the declination of S' is positive, it will lie on the same side of the equator as the arc representing the latitude. Then, in the spherical triangle PZS', we will have $PS' = 90 - \delta'$, while the other parameters will stay the same. Upon applying the cosine rule again in this spherical triangle, we get

$$\begin{aligned}\cos ZS' &= \cos(90-\phi)\cos(90-\delta') + \\ &\qquad \sin(90-\phi)\sin(90-\delta')\cos H' \\ &= \sin\phi\sin\delta' + \cos\phi\cos\delta'\sin(\alpha_e-\alpha') \\ &= [\sin(\alpha_e-\alpha') + \sin\Delta\alpha'_s]\cos\phi\cos\delta'. \qquad (6)\end{aligned}$$

Converting to the Indian cosine, and taken together, (5) and (6) yield

$$R\cos ZS' = [R\sin(\alpha_e-\alpha') \pm R\sin\Delta\alpha'_s]\cos\phi\cos\delta',$$

which is the same as (4).

4 Relationship between the *śaṅku*, the *dṛggati*, and the *dṛkkṣepakoṭikā*

शङ्कुदृग्गतिवर्गैक्यमूलं दृक्क्षेपकोटिका ।
सैवापक्रमवृत्तस्य क्षितिजात्परमोन्नतिः ॥५७॥

śaṅkudṛggativargaikyamūlaṃ dṛkkṣepakoṭikā |
saivāpakramavṛttasya kṣitijātparamonnatiḥ ||57||

The square root of the sum of squares of *śaṅku* and *dṛggati* is the Rcosine of *dṛkkṣepa* (*dṛkkṣepakoṭikā*). That itself is the maximum height of the ecliptic from horizon.

This verse, in the *anusṭubh* metre, gives the following relation between *dṛkkṣepakoṭikā*, *śaṅku*, and *dṛggati*:

$$\begin{aligned} &\textit{dṛkkṣepakoṭikā} = \sqrt{\textit{śaṅku}^2 + \textit{dṛggati}^2} \\ \text{or,}\quad &R\cos z_d = \sqrt{(R\cos ZS)^2 + (R\cos ZS')^2}, \qquad (7)\end{aligned}$$

where, z_d is the zenith distance of the *dṛkkṣepalagna* or the nonagesimal.

Deriving the relation between *dṛkkṣepakoṭikā*, *śaṅku*, and *dṛggati*

The validity of this relation can be understood with the help of Figure 2a, which depicts the celestial sphere from the point of view of the ecliptic. Here, S and S' refer to the positions of the Sun, and a point ninety degrees behind it, while D represents the *dṛkkṣepalagna*, and Z the zenith of the observer. Figure 2b depicts the perpendiculars ZX, ZD', and ZY dropped from Z on to the radii OS', OD, and OS of the celestial sphere respectively.[11] We therefore have

$$\begin{aligned} ZX &= R\sin ZS', & ZD' &= R\sin z_d, & ZY &= R\sin ZS, \\ OX &= R\cos ZS', & OD' &= R\cos z_d, & OY &= R\cos ZS,\end{aligned}$$

where z_d is the zenith distance of the *dṛkkṣepalagna*. We have previously referred to the terms $R\sin z_d$ and $R\cos z_d$ to be the *dṛkkṣepajyā* and the *paraśaṅku* or *dṛkkṣepakoṭikā* respectively.[12]

Now, in right-angled triangle $ZD'X$, we have

$$XD'^2 = (R\sin ZS')^2 - (R\sin z_d)^2,$$

which can be rewritten as

$$XD'^2 = (R\cos z_d)^2 - (R\cos ZS')^2 = OD'^2 - OX^2,$$

which proves that the triangle $D'OX$ is right-angled at X. Similarly, we can show that the triangle $D'OY$ is right-angled at Y. Therefore, $D'XOY$ is a rectangle, whose diagonal OD' is equal to $R\cos z_d$ (shown above). Now, from either of the triangles $D'OX$ or $D'OY$, it can be easily seen that

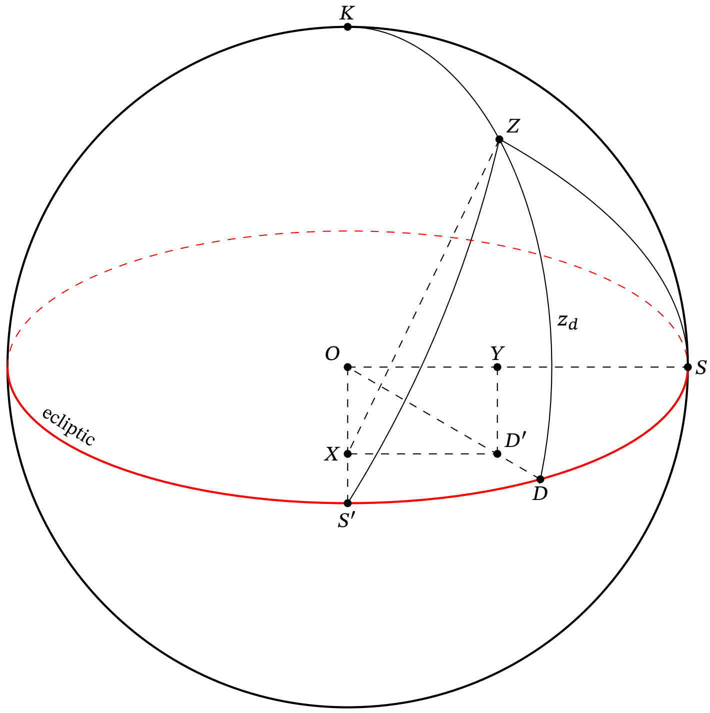
$$R\cos z_d = \sqrt{(R\cos ZS)^2 + (R\cos ZS')^2},$$

which is the relation given in the verse. It may be noted that the above relation will be valid for any two points separated by ninety degrees on the ecliptic.[13] It may also be noted that when the Sun is at the *dṛkkṣepalagna*, the *dṛggati* will have to be measured from the setting ecliptic point, which is the point ninety degrees behind the Sun. As this point lies on the horizon, the *dṛggati* is zero. In this case, the Sun's *śaṅku* is equal to the *dṛkkṣepakoṭikā*.

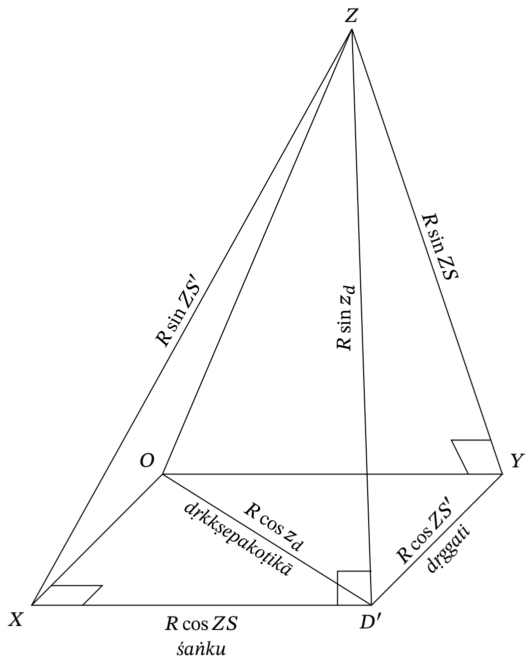
[11] It may be noted that the base $D'XOY$ of this figure is a rectangle as we shall show below. It only appears to be a parallelogram as we are trying to depict a three dimensional figure here.

[12] For the *dṛkkṣepajyā* see (9) in [12], or (2) and (3) in [13]. For *paraśaṅku* or *dṛkkṣepakoṭikā* see (12) in [12] and (10) in [13].

[13] Indeed, this property is later made use of in Chapter 5 of the *Lagnaprakaraṇa*, with respect to the equinoctial and solstitial points of the ecliptic.



(a) Visualising the *śaṅku*, *dṛggati*, and the *dṛkkṣepakoṭikā* on the plane of the ecliptic.



(b) The relation between the *śaṅku*, *dṛggati*, and the *dṛkkṣepakoṭikā*.

Figure 2 Determining the *dṛkkṣepakoṭikā*.

Showing that the *dṛkkṣepakoṭikā* is the maximum altitude of the ecliptic

From the discussion for (12) in [12] we know that the *paraśaṅku* or the *dṛkkṣepakoṭikā* is the altitude of the *dṛkkṣepalagna*. It can be shown that this is the maximum altitude of the ecliptic by considering Figure 3. In this figure, the points D, L, and Z indicate the *dṛkkṣepalagna*, the *udayalagna*, and the zenith respectively. The point B is a general point on the ecliptic. Now, in the spherical triangle ZBL, we have $ZL = 90$. Applying the cosine rule in this triangle, we have

$$\cos ZB = \sin BL \cos(B\hat{L}Z).$$

The point B will correspond to the highest point of the ecliptic when ZB is the least, or equivalently, when $\cos ZB$ is maximum. Inspecting the above relation, we can see that at any given instant, $\cos ZB$ is maximum when $\sin BL$ is maximum,[14] or $BL = 90$. However, from (14) in [12], we know this is only possible when B coincides with the *dṛkkṣepalagna* at D. Therefore, one can conclude that the altitude of the ecliptic is maximum at the *dṛkkṣepalagna*. Thus, the gnomon corresponding to this point, which is the *paraśaṅku* or the *dṛkkṣepakoṭikā*, is the maximum height of the ecliptic. Indeed, the name *paraśaṅku* itself indicates that this is the tallest *śaṅku*.

A note on the *dṛggati*

From (7), it is clear that

$$\begin{aligned} d\d{r}ggati &= \sqrt{(R\cos z_d)^2 - (R\cos ZS)^2} \\ &= \sqrt{(R\sin ZS)^2 - (R\sin z_d)^2}. \end{aligned}$$

The *Mahābhāskarīya*,[15] the *Śiṣyadhīvṛddhidatantra*,[16] and the *Siddhāntaśiromaṇi*[17] also define the *dṛggati* as above. However, the *Sūryasiddhānta*,[18] and the *Tantrasaṅgraha*[19] define it as

$$d\d{r}ggati = \sqrt{R^2 - (R\sin z_d)^2} = R\cos z_d,$$

which however is called the *dṛkkṣepakoṭikā* in this text. From (7) we can see that $R\cos z_d$ can be equal to the *dṛggati* only when the Sun is on the horizon and the *śaṅku* is zero. In this case, the *dṛggati* would be the the gnomon corresponding to the *dṛkkṣepalagna*, which would be the point ninety degrees away from the Sun. This is obviously the *dṛkkṣepakoṭikā* itself. As we have already shown, this is the maximum height of the ecliptic, and therefore, the maximum possible value of the *dṛggati*. Nīlakaṇṭha seems to have recognised this and clarifies in his *Āryabhaṭīya-bhāṣya* that the value given in the *Sūryasiddhānta* is the maximum *dṛggati*.[20] Perhaps, this is how the *Tantrasaṅgraha* definition is to be understood as well.

5 Determining the *udayalagna* using the *śaṅku* and the *dṛggati*

त्रिज्याहते दृग्गतिशङ्कुजीवे
दृक्क्षेपकोट्या विहृते यथोक्तम्।
चापीकृते वित्रिभभानुभान्वोः
कुर्यात्क्रमेणोदयलग्नसिद्ध्यै ॥५८॥
स्वर्णं दृग्गतिजं शङ्कुदृग्गत्योर्दिग्भिदैक्यतः[21] ।
तत्र शङ्कोरजादित्वे चक्रार्धमपि योजयेत्॥५९॥
शङ्कुजं शङ्कुदृग्गत्योर्दिक्साम्येऽन्तरतः क्रमात् ।
स्वर्णं क्षेप्यं च चक्रार्धं तुलादित्वे तु दृग्गतेः ॥६०॥

trijyāhate dṛggatiśaṅkujīve
dṛkkṣepakoṭyā vihṛte yathoktam |
cāpīkṛte vitribhabhānubhānvoḥ
kuryātkrameṇodayalagnasiddhyai ||58||
svarṇaṃ dṛggatijaṃ śaṅku-
-dṛggatyordigbhidaikyataḥ |
tatra śaṅkorajāditve
cakrārdhamapi yojayet ||59||
śaṅkujaṃ śaṅkudṛggatyor-
-diksāmye'ntarataḥ kramāt |
svarṇaṃ kṣepyaṃ ca cakrārdhaṃ
tulāditve tu dṛggateḥ ||60||

The Rsines corresponding to the *dṛggati* and the *śaṅku*, which are multiplied by the radius (*trijyā*) and divided by the Rcosine of the *dṛkkṣepa* (*dṛkkṣepakoṭikā*), are converted to arc. One should

[14] At any given instant, the angle $B\hat{L}Z$ is fixed irrespective of the position of B on the ecliptic.
[15] [3], p. 275.
[16] [15], p. 135.
[17] [4], p. 381.
[18] [7], p. 166
[19] [20], pp. 309-312

[20] [1], p. 81.
[21] शङ्कुं दृग्गत्योर्दिग्भिदैक्यतः in [17]. Appears to be a transcribing error.

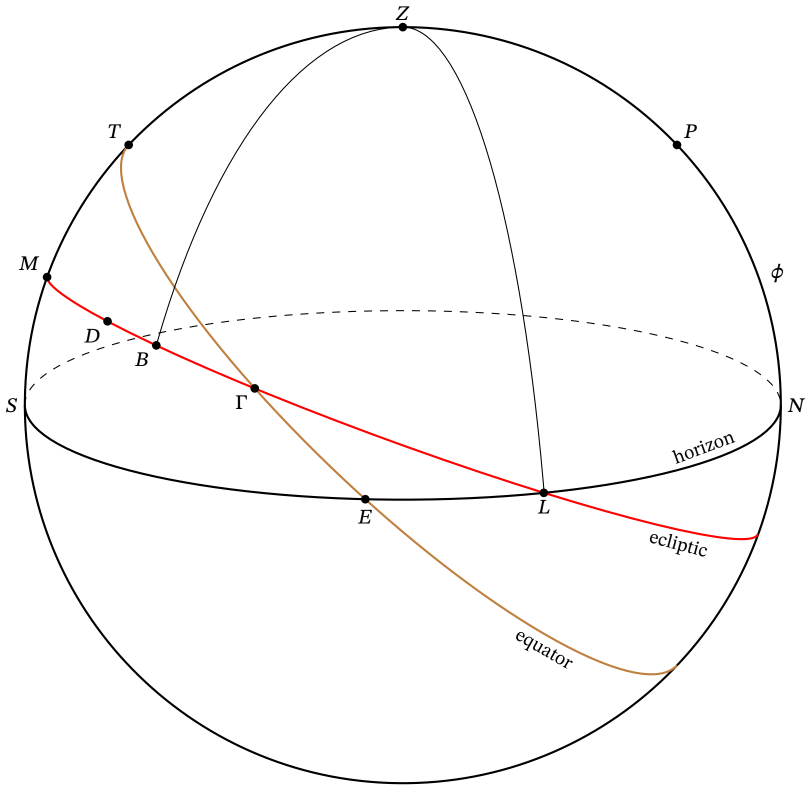


Figure 3 Proving that the *dṛkkṣepakoṭikā* is the maximum altitude of the ecliptic.

apply those (arcs) to [the longitudes of] the Sun minus three signs and the Sun respectively, as stated [now], in order to obtain the rising ecliptic point (*udayalagna*).

That [arc] obtained from *dṛggati* becomes additive or subtractive [to the longitude of the Sun minus three signs] depending on the difference or similarity in the directions of the *śaṅku* and the *dṛggati* [with respect to the horizon]. There, if the *śaṅku* is in [the six signs] Aries etc. (*ajādi*) [from the setting ecliptic point], then one should add a semi-circle (*cakrārdha*) also.

That [arc] computed from the *śaṅku* is additive or subtractive [to the longitude of the Sun] depending on if the directions of the *śaṅku* and the *dṛggati* are the same or different [with respect to the horizon] respectively. And, indeed a semi-circle (*cakrārdha*) needs to be added if the *dṛggati* is in [the six signs] Libra etc. (*tulādi*) [from the setting ecliptic point].

These three verses, the first in the *indravajrā* metre and the latter two in the *anusṭubh* metre respectively, prescribe a method to determine the *udayalagna* using the *dṛggati* or the *śaṅku*. The relation to determine the *udayalagna* from the *dṛggati* is stated to be as follows:

$$udayalagna = vitribhabhānu \pm cāpa\left(\frac{dṛggati \times trijyā}{dṛkkṣepakoṭikā}\right)$$

... [*śaṅku* in *tulādi*]

$$udayalagna = vitribhabhānu \pm cāpa\left(\frac{dṛggati \times trijyā}{dṛkkṣepakoṭikā}\right) + cakrārdha.$$

... [*śaṅku* in *ajādi*]

Taking λ_l as the longitude of the *udayalagna*, λ as the longitude of the Sun (S), $\lambda' = \lambda - 90$ as the longitude of a point S' ninety degrees behind the Sun, and accounting for the sameness or difference in the directions of the *śaṅku* and the *dṛggati*, the above relations can be written in mathematical notation as follows:

$$\lambda_l = \lambda' - R\sin^{-1}\left(\frac{dṛggati \times R}{R\cos z_d}\right) + 180 \qquad (8)$$

... [S and S' above horizon]

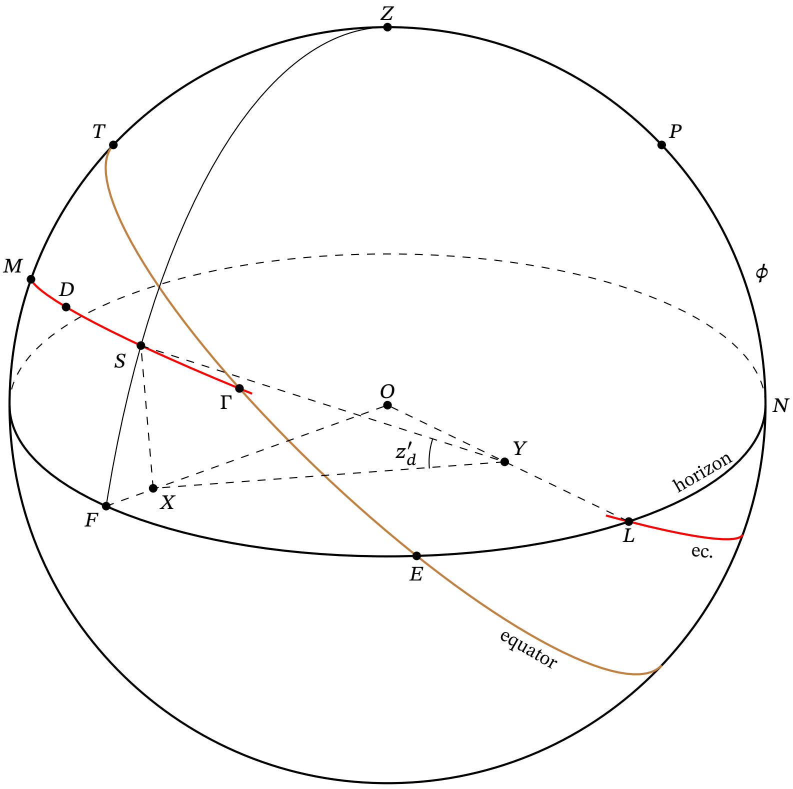


Figure 4 Determining the arc SL using a planar triangle.

$$\lambda_l = \lambda' + R\sin^{-1}\left(\frac{dṛggati \times R}{R\cos z_d}\right) \quad (9)$$

... [S below, S' above horizon]

$$\lambda_l = \lambda' - R\sin^{-1}\left(\frac{dṛggati \times R}{R\cos z_d}\right) \quad (10)$$

... [S and S' below horizon]

$$\lambda_l = \lambda' + R\sin^{-1}\left(\frac{dṛggati \times R}{R\cos z_d}\right) + 180. \quad (11)$$

... [S above, S' below horizon]

It may be noted that the mathematical relations for the *dṛggati* and the *dṛkkṣepakoṭikā* are given by (4) and (7) respectively.

The verse also gives the following relations to determine the *udayalagna* from the *śaṅku*:

$$udayalagna = bhānu \pm cāpa\left(\frac{śaṅku \times trijyā}{dṛkkṣepakoṭikā}\right)$$

... [*dṛggati* in *ajādi*]

$$udayalagna = bhānu \pm cāpa\left(\frac{śaṅku \times trijyā}{dṛkkṣepakoṭikā}\right) + cakrārdha.$$

... [*dṛggati* in *tulādi*]

Accounting for the sameness or difference in the directions of the *śaṅku* and the *dṛggati*, the above relations can be expressed in mathematical notation as

$$\lambda_l = \lambda - R\sin^{-1}\left(\frac{śaṅku \times R}{R\cos z_d}\right) + 180 \quad (12)$$

... [S above, S' below horizon]

$$\lambda_l = \lambda + R\sin^{-1}\left(\frac{śaṅku \times R}{R\cos z_d}\right) \quad (13)$$

... [S and S' above horizon]

$$\lambda_l = \lambda - R\sin^{-1}\left(\frac{śaṅku \times R}{R\cos z_d}\right) \quad (14)$$

... [S below, S' above horizon]

$$\lambda_l = \lambda + R\sin^{-1}\left(\frac{śaṅku \times R}{R\cos z_d}\right) + 180. \quad (15)$$

... [S and S' below horizon]

Here, the mathematical relation for the *śaṅku* is given by (1).

In what follows, we show the validity of the above expressions. For convenience, we first show the derivation of the *udayalagna* from the *śaṅku* below. However, first, a brief note on how the quadrants of the ecliptic are to be understood for the purpose of this verse.

The quadrants of the ecliptic

In standard practice, the four quadrants of the ecliptic are considered to be the four ninety degree arcs confined between adjacent equinoctial and solstitial points, with the arc between the vernal equinox and the summer solstitial point being considered the first quadrant. In contrast to this standard practice, for the purpose of this verse, the author appears to have conceived of the ecliptic as consisting of four quadrants, where the first quadrant corresponds to the ninety degree arc from the setting ecliptic point (L') to the nonagesimal (D). The second quadrant in this case would be the arc between the nonagesimal and the rising ecliptic point (L). The third and fourth quadrants are to be understood accordingly. The Sun is depicted in the first to the fourth quadrants thus defined in Figures 5a–5d respectively.

With respect to this convention, the terms *ajādi* (0 to 180 degrees) and *tulādi* (180 to 360 degrees) in the verse clearly correspond to the halves of the ecliptic which lie above and below the horizon respectively. The purpose of this definition will become clear in our discussion below.

Deriving the *udayalagna* from the *śaṅku*

The derivation of the *udayalagna* from the *śaṅku* can be understood by considering Figure 1a. Here, the *udayalagna* (L) can be obtained by adding the arc SL to the Sun's (S) longitude. In spherical triangle SFL, we have $SF = 90 - ZS$, $S\hat{F}L = 90$,[22] and from (17) in [12] we know $S\hat{L}F = 90 - z_d$. Then, applying the sine rule in this spherical triangle, we have

$$\sin SL = \frac{\cos ZS \times \sin 90}{\cos z_d},$$

which gives the measure of the arc from the Sun to the *udayalagna*:

$$SL = R\sin^{-1}\left(\frac{R\cos ZS \times R}{R\cos z_d}\right) \tag{16}$$

$$= R\sin^{-1}\left(\frac{\textit{śaṅku} \times \textit{trijyā}}{\textit{dṛkkṣepakoṭikā}}\right). \tag{17}$$

[22]The great circle arc ZFS, which is the vertical passing through S, is perpendicular to the horizon.

It may be noted that we have considered the Sun to be in the eastern hemisphere here. If the Sun were in the western hemisphere instead, the above relation would give the measure of the arc $L'S$ from the setting ecliptic point (L') to the Sun.

Alternatively, the same relation can be derived using the planar triangle SXY shown in Figure 4. In this figure, S once again gives the position of the Sun, and ZSF is the vertical passing through S. The point L is the *udayalagna*, and O is the centre of the celestial sphere. The line segment SX is the perpendicular dropped from S on to the horizon, which we know as the *śaṅku*. The line segment SY is the perpendicular dropped from S on to the radius OL in the plane of the ecliptic, and is therefore equivalent to $R\sin SL$. XY is a line segment on the horizon. The angle between SY and XY represents the angle between the horizon and the ecliptic. Thus, from (17) in [12], we have $S\hat{Y}X = 90 - z_d$. Now, in the planar right-angled triangle SXY, which is perpendicular to the horizon, we have

$$\sin(90 - z_d) = \frac{\textit{śaṅku}}{R\sin SL}.$$

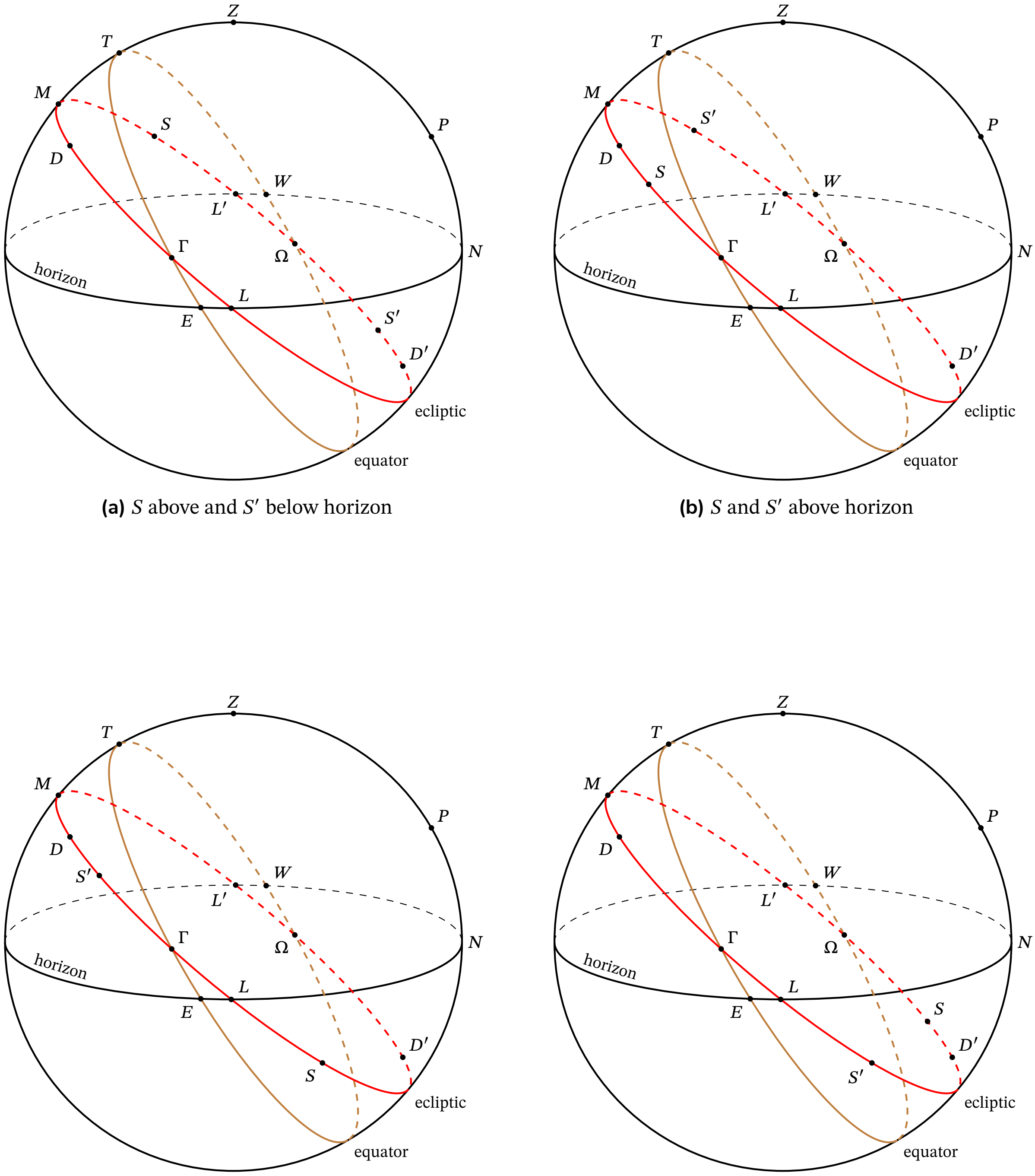
Upon rearranging, we obtain the length of the arc

$$SL = R\sin^{-1}\left(\frac{\textit{śaṅku} \times R}{R\cos z_d}\right),$$

which is equivalent to the second expression in the RHS of the relations (12)–(15). In Figure 4, adding this arc to the longitude of Sun gives the *udayalagna* . It may be noted that, with respect to the special convention adopted for counting the quadrants of the ecliptic in this verse, the Sun can be considered to be in the second quadrant in this case. This is clearly seen in Figure 5b, where the Sun lies between the nonagesimal (D) and the rising ecliptic point (L). It may also be noted that both S and S' are above the horizon in this case, and that S' is *ajādi* (first quadrant). Thus, we obtain (13).

When the Sun is in the third quadrant, as shown in Figure 5c, it can be seen that the arc LS needs to be subtracted from the Sun's longitude to obtain the *udayalagna*. This gives (14). In this case, S and S' are on opposite sides of the horizon, and S' is *ajādi* (second quadrant).

When the Sun is in the fourth quadrant, as shown in Figure 5d, the arc determined by the above relation will actually be the arc SL'. Therefore, to obtain the *udayalagna*, we need to add this arc to the Sun's longitude to obtain L', and then add a further 180 degrees to obtain



(a) S above and S' below horizon

(b) S and S' above horizon

(c) S below and S' above horizon

(d) S and S' below horizon

Figure 5 Determining the *udayalagna* from the *śaṅku* and the *dṛggati*.

the longitude of point L. This is the procedure laid out in (15). In this case, S and S' both lie below the horizon, and S' is *tulādi* (third quadrant).

Finally, when the Sun is in the first quadrant of the ecliptic, as shown in Figure 5a, we need to subtract the arc $L'S$ (whose magnitude is the same as SL above) from the Sun's longitude (to obtain L'), and then add 180 degrees to obtain the longitude of point L. We thus obtain (12). Here, S and S' are on opposite sides of the horizon, and S' is *tulādi* (fourth quadrant).

It may be noted that a similar procedure for obtaining the *udayalagna* is also discussed by Nīlakaṇṭha in his *Tantrasaṅgraha*.[23]

Deriving the *udayalagna* from the *dṛggati*

The derivation of the *udayalagna* from the *dṛggati* can be understood by considering Figure 1b. Here, the *udayalagna* can be obtained from the longitude of S' by first subtracting the arc $L'S'$ to obtain the setting ecliptic point (L'), and then adding 180 degrees to obtain the rising ecliptic point. The measure of the arc $L'S'$ can be obtained from the spherical triangle $S'F'L'$. In this triangle, we have $S'F' = 90 - ZS'$, $S'\hat{F}'L' = 90$,[24] and from (17) in [12] we know $S'\hat{L}'F' = 90 - z_d$. Applying the sine rule in this spherical triangle, we have[25]

$$\sin L'S' = \frac{\cos ZS' \times \sin 90}{\cos z_d},$$

which gives the measure of the arc

$$\begin{aligned} L'S' &= R\sin^{-1}\left(\frac{R\cos ZS' \times R}{R\cos z_d}\right) \\ &= R\sin^{-1}\left(\frac{dṛggati \times trijyā}{dṛkkṣepakoṭikā}\right), \end{aligned}$$

which is the same as the second expression in the RHS of the relations (8)–(11).[26] Subtracting this arc from S' yields the point L', whereupon, adding 180 degrees gives the *udayalagna*. Thus, we obtain the expression (8). With respect to the special convention adopted for measuring the quadrants of the ecliptic in this verse, S' can be considered to be in the first quadrant in this case. This is clearly seen in Figure 5b, where S' lies between the setting ecliptic point (L') and the nonagesimal. Here, both S and S' are above the horizon, and S is *ajādi* (second quadrant).

When S' lies in the second quadrant, as shown in Figure 5c, the *udayalagna* can be obtained by simply adding the arc $S'L$ (whose magnitude is the same as $L'S'$ above) to it. This yields (9). In this case, S and S' are on opposite sides of the horizon, and S is *tulādi*.

When S' lies in the third quadrant, as shown in Figure 5d, the *udayalagna* can be obtained by subtracting the arc LS' (whose magnitude is the same as $L'S'$ above) from it. This gives (10). Here, S and S' lie below the horizon, and S is *tulādi*.

Finally, when S' lies in the fourth quadrant, as shown in Figure 5a, the *udayalagna* can be obtained by adding the arc $S'L'$ to obtain L', and then adding a further 180 degrees to obtain L. Thus, we get (11). In this case, S and S' again lie on opposite sides of the horizon, and S is *ajādi*.

6 Determining the *udayalagna* from any two ecliptic points which are ninety degrees apart

एवमेवेष्टलग्नाच्च ततो वित्रिभतोऽपि च ।
प्राग्विलग्नमिहानेयं भुक्तिज्ञैर्न्यायदर्शिभिः ॥६१॥

evameveṣṭalagnācca
tato vitribhato'pi ca |
prāgvilagnamihāneyaṃ
bhuktijñairnyāyadarśibhiḥ ||61||

Similarly, from any desired point on the ecliptic (*iṣṭalagna*) and from the ecliptic point three signs less from it (*vitribha*) also, the orient ecliptic point should be computed by the scholars of planetary motions who understand the principles [of astronomy].

A careful inspection of the relations given in this chapter reveals that the only necessary condition for determining the *udayalagna* as prescribed here is that we consider two points on the ecliptic which are ninety degrees apart. Therefore, this verse, in the *anusṭubh* metre, states that the *udayalagna* can be determined from any two desired

[23] See [20], pp. 245–248.

[24] This is the angle made by the vertical passing through S' with the horizon.

[25] This relation can also be determined using planar triangles as shown in the case of the *śaṅku*.

[26] It may be noted that we have considered S' to be in the western hemisphere here. If S' were in the eastern hemisphere instead, the above relation would give the measure of the arc $S'L$ from S' to the rising ecliptic point (L).

ecliptic points (say σ and σ') which are ninety degrees apart, using the procedure described in verses 58–60. For this, the *śaṅku* of σ, and the *dṛggati* from σ', should be first determined using (1) and (4), however substituting the right ascension, ascensional difference, and declination of the S and S' with the corresponding quantities of σ and σ'. Then, as in the case of the S and S', the *udayalagna* can be determined from σ and σ' using (12)–(15) and (8)–(11) respectively.

7 Conclusion

In this paper, we have discussed two techniques for determining the ascendant by respectively employing the two gnomons (*śaṅku* and *dṛggati*) corresponding to the Sun and a point on the ecliptic ninety degrees behind the Sun. The methods discussed in our previous papers first determine positions on the ecliptic such as *dṛkkṣepalagna*, *rāśikūṭalagna*, *unmaṇḍalalagna*, etc., and determine the ascendant therefrom. However, the third chapter of the *Lagnaprakaraṇa*—discussed in this paper—describes the procedure of determining the exact *lagna* directly from the position of the Sun at any given instance, by determining its corresponding gnomon at that instance.

The procedures for determining the *śaṅku* and *dṛggati* makes use of the innovative concept of *kālalagna*,[27] as well as the 'instantaneous' ascensional difference. The author gives different relations for determining the ascendant depending upon the directions and positions of the *śaṅku* and the *dṛggati*. He states this complex relationship in two short verses. This once again highlights the ability of the author to not only superbly visualise the motions of the celestial bodies in the celestial sphere, but also to effectively communicate the resulting relationships in a precise and succinct manner.

Verses 57 and 61 in this chapter are also quite interesting. The former verse states the relationship between the *śaṅku*, the *dṛggati*, and the *dṛkkṣepakoṭikā*, which is of the form of the sides of a right-angled triangle. Thus, it is evident that the author was able to visualise the relevant right-angled triangle clearly in his mind. The latter verse generalises the procedure given in this chapter by noting that the ascendant can be determined from any two ecliptic points which are ninety degrees apart, using the stated procedure. This implies that the author was well aware of the rationale behind the procedure, and understood that one did not always have to start with the position of the Sun and determination of its gnomon. Indeed, the author employs a similar procedure for determining the ascendant by considering the gnomons dropped from the equinoctial and solstitial ecliptic points (which are ninety degrees apart) in the fifth chapter of the *Lagnaprakaraṇa*. These and other procedures of determining the ascendant, as given in the *Lagnaprakaraṇa*, will be discussed in future papers.

[27] For a detailed discussion on *kālalagna*, see [10].

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