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Project Report

Sūryaprakāṣa of Sūryadāsa — Critical Edition, Eng. Tr. and Explanatory Notes for *Vargaprakṛti* and *Granthasamāpti**

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Introduction

Bījagaṇita of Bhāskaracārya II holds the distinction of being the first text to be entirely devoted to Algebra which accounts for its immense popularity. In Indian tradition, it is customary to find commentaries growing along with original texts and the field of mathematics is no exception. Thus we find many commentaries on the *Bījagaṇita* of Bhāskaracārya II and so far six commentaries have been identified. Of these the printed edition of *Bījapallava* or *Navāṅkura* of Kṛṣṇa Daivajña is available. A critical study of the *Bījapallava* was taken up for Doctoral thesis by the present Project Investigator. It was later published as a book "*Bījapallava of Kṛṣṇa Daivajña, Algebra of the sixteenth century–A critical study*" by the Kuppuswami Sastri Research Institute in 2012. It has had good reviews, both in national and international journals and since then three editions have been published.

Sūryadāsa (1507–1585) was a versatile genius who wrote on a wide variety of topics. Three sections of his commentary *Sūryaprakāṣa* namely the *upodghāta*, *ṣaḍviḍhaprakaraṇa* and *kuṭṭakādhikāra* have already been edited. The remaining sections deal with *vargaprakṛti*–Indeterminate equation of the second degree, *cakravāla*–the cyclic method, *ekavarṇa samīkaraṇa*–linear equations, *madhyamāharaṇa*–quadratic equations, *anekavarṇasamīkaraṇa*–linear equations with more than one unknown, *madhyamā-haraṇabheda*–quadratic equations with more than one unknown, *bhāvita*–operations of products with several unknowns and *granthasamāpti*–concluding portion of the text.

Sūryaprakāṣa of Sūryadāsa being the earliest Commentary on *Bījagaṇita* of Bhāskarahas helped greatly in understanding the original text better. *Sūryaprakāṣa* seems to have been a popular text as is evinced by the fact that H T Colebrooke and many modern authors refer to him often. It was therefore rewarding to critically edit the rest of the chapters (mentioned above), which are still available in Manuscript form. It explains every verse and solves almost every example of Bhāskara's text in order to teach the students the rules of mathematics. He follows a logical and consistent program of exposition and explanation throughout his *Sūryaprakāṣa*.

The five manuscripts of *Sūryaprakāṣa* of Sūryadāsa selected for study were (i) British Library, San I.O. 1533a; (ii) Prājnapāthaśalā Mandala, Wai 9777/ 11–2/551; (iii) British Library, San I.O.1533a; (iv) British Museum, London 447, Ff 46, 19th century and (v) British Museum, London 448, Ff 40, 19th century. The first two Mss. were already available. The other three were procured for collation.

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The problems identified in these Mss. are: (i) Legibility was very low in three of the manuscripts. (ii) Two manuscripts had a number of, mathematical errors—for instance the numbers were wrongly given; the denominators were missing in the fractions.(iii) Portions of the text were deranged in one manuscript.(iv)The *sūtras* giving the rules and examples were found only in one manuscript. The manuscripts had to be diligently studied and compared to avoid mathematical errors and to arrive at an error-free text.

The Project Report has the following format: (i) The Sanskrit text contains the rules and examples (given in bold) of Bhāskarācārya as well as the commentary of Sūryadāsa. (ii) For easy reference, these and other *sūtras* (from previous chapters) quoted by Sūryadāsa are also given in a separate Appendix. (iii) English translation has been given for the verses and commentary of Sūryadāsa. (iv) Mathematical explanation has been given wherever required using modern notation; some technical terms have also been explained. (v) Emendations in the text are marked by the box bracket [ ]. (vi) Definition for a few words has been given and (v) A select bibliography has been added.

The chapters taken up for the Project are divided into three sections:

In the **first section**, the Rules related to the solution of the *Vargaprakṛti* equation $Nx^2 + 1 = y^2$ are discussed. The *bhāvanā* method of Brahmagupta was explained by Bhāskara and elaborated by Sūryadāsa. The famous *cakravāla* method propounded by Bhāskara was also explained in detail. The equation $61x^2 + 1 = y^2$ dealt with European mathematicians with much awe and solved by Bhāskara 500 years earlier has been illustrated in detail in the report with modern mathematical notation.

At least from the time of Brahmagupta, mathematicians in India were attempting the harder problem of solving equations of the second degree. In his *Brāhmasphuṭa siddhānta*, Brahmagupta gave a partial solution to the problem of solving $Nx^2 + 1 = y^2$.

The fundamental step in Brahmagupta's method for the general solution in positive integers of the equation $Nx^2 + 1 = y^2$ where N is any non-square integer, is to consider two auxiliary equations $Nx^2 + k_i = y^2$, $i = 1, 2$ with k_i being chosen from $k_i = \pm 1, \pm 2, \pm 4$. A procedure known as *bhāvanā*, applied repeatedly, wherever necessary, helps us in deriving at least one possible solution of the original *vargaprakṛti viz.* $Nx^2 + 1 = y^2$. Using the *bhāvanā*, an infinite number of solutions can be obtained. Brahmagupta could find this auxiliary equation only by trial and error.

Remarkable success was achieved by Jayadeva and Bhāskara when he introduced a simple method to derive the auxiliary equation. This equation would have the required *kṣepas* ±1, ±2, ±4, simultaneously with two integral solutions from any auxiliary equation empirically formed with any simple value of the *kṣepa*. This method is the famous *cakravāla* or cyclic method, so named for its iterative character.

Bhāskara's *cakravāla* is based on the lemma: If $Na^2 + x = b^2$ is an auxiliary equation where a, b, k are integers, k being negative or positive then,

$$N\left\{\frac{am+b}{k}\right\}^2 + \frac{m^2-N}{k} = \left\{\frac{bm+Na}{k}\right\}^2$$

where m is any arbitrary whole number.

The rationale is simply the following.

Consider the equations:

$Na^2 + x = b^2$

$N1^2 + m^2 - N = m^2$

Using Brahmagupta's *bhāvanā* we have:

$N(am + b)^2 + k(m^2 - N) = (Na + mb)^2$

Dividing by k^2

$$N\left\{\frac{am+b}{k}\right\}^2+\frac{m^2-N}{k}=\left\{\frac{bm+Na}{k}\right\}^2$$

m can be so chosen such that $\frac{am+b}{k}$ is an integer since its value can be determined by means of *kuṭṭaka*, *viz.*, by solving the equation $ax + b = ky$ for integer solution and taking the solution for x as m. Obviously there can be infinite number of values for m. But Bhāskara says m should be so chosen as to make $|m^2 - N|$ minimum. If $\frac{m^2-N}{k}$ is equal to ±1, ±2, ±4, then Brahmagupta's *bhāvanā* can be applied immediately. If $\frac{m^2-N}{k}$ is not one of the above values, then the *kuṭṭaka* is performed again and again till $\frac{m^2-N}{k}$ is equal to ±1, ±2, ±4. It is possible that Bhāskara was aware that this process will end after a finite number of steps.

An Interesting Anecdote (Stedall, p. 318)

There is an interesting story behind the equation of the type $Nx^2 + 1 = y^2$. In 1657, the famous French mathematician Fermat sent a public challenge to his friend, Bernard Freniclede Bessy and then on to Brouncker and Wallis in England to solve the equation $61x^2 + 1 = y^2$ in integers. "We await", he challenged, "the solutions which, if England or Belgian and Celtic Gauls cannot give them, Narbonian Gaul will…"(meaning himself). None of them succeeded in solving the equation. It was only in 1732 that the renowned mathematician Euler gave a complete solution. But remarkably, the very same equation had been dealt with and solved in a few steps by Bhāskara II by the famous *cakravāla* method more than five centuries earlier. Bhāskara gave the least solution as $x = 226153980$ and $y = 1766319049$. No wonder Andre Weil (p. 81) exclaims "what would have been Fermat's astonishment if some missionary back from India had told him that his problem had been successfully tackled there by native mathematicians almost six centuries earlier".

Later, in the 18th century CE, Euler gave Brahmagupta's Lemma and its proof. He was aware of Brouncker's work on the above equation as presented by Wallis, but he was totally unaware of the contribution of the Indian mathematicians. He gave the basis for the continued fraction approach to solving the above equation which was put into a polished form by Lagrange in 1766. Lagrange published his *Additions to Euler's Elements of Algebra* in 1771, and this contains his rigorous version of Euler's continued fraction approach to Pell's equation.

In the **second section**, the application of *bīja* in relation to single variable is discussed. Equations are divided according to their degrees i) equations in one unknown *ekavarṇasamīkaraṇa* and ii) equations in several unknowns *anekavarṇa-samīkaraṇa. Ekavarṇasamīkaraṇa* is further divided into i) linear equations with one unknown and ii) *madhyamāharaṇa* quadratic equations. This section deals with the above two types of equations.

In linear equations, Bhāskara provides examples from various branches of mathematics such arithmetic progression, simple interest and so on. *Saṅkramaṇa*, a special case of solving simultaneous equations is also explained. In *madhyamāharaṇa,* Śridharācārya's formula for solving quadratic equation is discussed. Though every quadratic equation has two roots, the cases where they cannot be accepted are also discussed. Sūryadāsa gives his own formula which is very innovative.

The **third section** deals with *anekavarṇa-samīkaraṇa, bhāvita* and *granthasamāpti. Anekavarṇasamīkaraṇa* is again divided into *anekavarṇasamīkaraṇa*–linear equations with more than one unknown, and *madhyamāharaṇa-bheda*–quadratic equations with more than one unknown. *Bhāvita*–operations of products with several unknowns are explained both algebraically and with figures. In *granthasamāpti*–concluding portion of the text, Bhāskara has given some interesting information about his family. His father and teacher was one Maheśvara who was a well-known preceptor. Bhāskara wrote the text *Bījagaṇita* under his tutelage. He has imbibed the teachings of Brahmagupta, Śridharā, Padmanābha and others and given the essence in his text. In his final verse he exhorts all mathematicians to read his text for improving their intellect.

Definition of a few words

Vargaprakṛti	$Nx^2 + 1 = y^2$	Equation of the affected square
Prakṛti	N	Coefficient of x^2
hrasva, kaniṣṭha, laghu	x	smaller root
jyeṣṭha	y	greater root
kṣepa	k	additive
abhyāsa		multiplication
vajrābhyāsa		cross multiplication
nyāsa		statement
bhāvanā		composition
kuṭṭaka		pulverizer
upapatti		demonstration of proof

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