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Determination of *Kālalagna* in the *Lagnaprakaraṇa*

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Abstract

The concept of the *kālalagna* is an important and innovative contribution of the Kerala school of astronomy, and is employed for a variety of astronomical computations in texts such as the *Tantrasaṅgraha*, the *Candracchāyāgaṇita*, the *Karaṇapaddhati*, and the *Gaṇita-yukti-bhāṣā*. This concept appears to have been first introduced by Mādhava (c. 14th century), the pioneer of the Kerala school, in his *Lagnaprakaraṇa*. In this text, Mādhava makes innovative use of the *kālalagna* to determine the exact value of the *udayalagna*, or the ascendant, for the first time in the annals of Indian astronomy. This paper discusses the various techniques of determining the *kālalagna* described in the *Lagnaprakaraṇa*.

Key words: *Asu*, *Kālalagna*, *Lagnaprakaraṇa*, *Mādhava*, *Nāḍī*, *Prāṇa*.

1. INTRODUCTION

The Sanskrit term *lagna* literally means 'that which is touching or intersecting'. In the astronomical context, the *lagna* usually refers to the longitude of the ecliptic point at the intersection of the ecliptic and the eastern horizon, and is more properly known as the *udayalagna*. More often than not, the term *lagna* is used to refer to the *lagnamāna*, which is the time interval taken by a *rāśi* (a thirty degree segment of the ecliptic) to rise at the observer's location.[1]

In the Indian tradition, the *lagnamāna* plays a crucial role in fixing the time for conducting various socio-religious events. For instance, if a wedding invitation states that the event would occur in the *vṛṣabha-lagna*, this means that the auspicious event is to take place within the time taken by the corresponding thirty degree segment (30–60 degrees) of the ecliptic to rise above the horizon at the observer's location. On the other hand, among other applications, the computation of the *udayalagna* is important for determining the occurrence and visibility of a solar eclipse at a given location.

Astronomical works such as the *Sūrya-siddhānta*, the *Brāhmasphuṭasiddhānta*, and the *Śiṣyadhīvṛddhidatantra*, describe a standard procedure[2] for first determining the rising times of the *rāśis*, and therefrom the *udayalagna*. This procedure involves a certain approximation because

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[1] For instance, Amarasiṃha in the *Digvargaprakaraṇa* of the *Amarakoṣa* (1.3.230) defines the *lagna* as: राशीनामुदयो लग्नं ते तु मेषवृषादयः । (The rising [time] of the *rāśis* is *lagna*. They are *meṣa*, *vṛṣa* etc.).

[2] This standard procedure is perhaps best described in the *Tripraśnādhikāra* of the *Śiṣyadhīvṛddhidatantra*. For a detailed discussion, see Lalla (*Śiṣyadhīvṛddhidatantra*, pp. 61–69).

[3] Given the rising time of a *rāśi*, the rule of three is employed to determine the portion of a *rāśi* which rises in a desired amount of time. This introduces an error as the *rāśi* does not rise linearly with time.

of the use of the rule of three.[3] Perhaps due to the importance of determining the *udayalagna* accurately, Āryabhaṭa II suggests an improvement in this procedure.[4] The result thus obtained, though more accurate than the earlier procedure, is nevertheless still approximate.

Kerala astronomers starting with Mādhava found a way to circumvent the approximations involved in the computation of the *udayalagna* by introducing the ingenious concept of the *kālalagna*,[5] or the time interval between the rise of the vernal equinox and a desired later instant. While the application of the *kālalagna* is encountered in texts such as the *Tantrasaṅgraha*, the *Candracchāyā-gaṇita*, the *Karaṇapaddhati*, and the *Gaṇita-yukti-bhāṣā*, this concept appears to have been first introduced by Mādhava in his hitherto unpublished *Lagnaprakaraṇa*,[6] a text dedicated to the computation of the *udayalagna*.

The *Lagnaprakaraṇa* brings about a revolutionary change in Indian astronomy by describing several precise relations for the *udayalagna*. Over 139 verses across eight chapters, the *Lagnaprakaraṇa* makes myriad uses of the *kālalagna* to determine various intermediate quantities such as *madhyakāla*, *madhyalagna*, *dṛkkṣepajyā*, *dṛkkṣepakoṭikā*, *unmaṇḍalalagna*, *śaṅku*, *dṛggati*, *viṣuvannara*, *ayanāntaśaṅku* etc., all in the service of determining the *udayalagna*. Given the importance of the *kālalagna*, Mādhava devotes six verses of the *Lagnaprakaraṇa* to discuss various methods of obtaining this quantity. In this paper, we explain in detail the technical content of these verses, along with their rationale.

2. DETERMINING THE *KĀLALAGNA*

The *Lagnaprakaraṇa* discuss five methods of calculating the *kālalagna* (verses 25–29) and also makes a brief remark (verse 30) regarding the invariability of this quantity for observers located on a given meridian of longitude. In the following discussion, α_e represents the *kālalagna*, while λ, α, and δ correspond to the precession-corrected longitude, right ascension, and declination of the Sun respectively. The following discussion also makes use of the concepts of *prāṇakalāntara* and *cara*. The *prāṇakalāntara*, or the difference in the longitude and corresponding right ascension, is an important concept used frequently in the *Lagnaprakaraṇa*, and is represented as $|\lambda - \alpha|$. The various techniques described for determining this quantity in the *Lagnaprakaraṇa* have been discussed in an earlier paper.[7] The *cara* or the ascensional difference of an entity at a given latitude is represented by the symbol $\Delta\alpha$. The various techniques given for determining the *cara* in the *Lagnaprakaraṇa* have also been discussed in a separate paper.[8] It may also be noted that the longitude of a celestial body is generally measured in

[4] In verses 38–45 of the *Tripraśnādhikāra* of his *Mahāsiddhānta*, Āryabhaṭa II (*Mahāsiddhānta*, pp. 81–83) suggests determining the rising times of ten degree segments of the ecliptic, and gives the corresponding table of rising times. Due to the smaller range, applying the rule of three in ten degree segments of the ecliptic helps improve the accuracy of the *udayalagna* calculations. Later, in the *Spaṣṭādhikāra* of his *Siddhāntaśiromaṇi*, Bhāskara (*Siddhāntaśiromaṇi*, p. 193) also suggests determining the rising times of ten or fifteen degree segments of the ecliptic to improve the accuracy of calculations. However, Bhāskara (*Siddhāntaśiromaṇi*, p. 211) once again uses thirty degree segments of the ecliptic to determine the *udayalagna* in the *Tripraśnādhikāra* of the same text.

[5] In his *Candracchāyāgaṇita* (see Sarma, "Contributions to the Study of the Kerala School of Hindu Astronomy and Mathematics", p. 1486), Nīlakaṇṭha explains the term *kālalagna* as: कालात्मकं लग्नं, घटिकामण्डलगतं लग्नमिति यावत् । ([*Kālalagna* is] the *lagna* [expressed] in the form [i.e., units] of time, which means *lagna* [measured] on the equator.)

[6] The authors obtained 2 manuscripts of the *Lagnaprakaraṇa* from the Prof. K. V. Sarma Research Foundation, Chennai, and once copy from the Kerala University Oriental Research Institute and Manuscripts Library, Thiruvananthapuram. See Bibliography for details.

[7] See Kolachana, Mahesh, and Ramasubramanian, "Mādhava's multi-pronged approach for obtaining the *prāṇakalāntara*".

[8] See Kolachana, Mahesh, Montelle, et al., "Determination of ascensional difference in the *Lagnaprakaraṇa*".

kalās or *liptās*, both of which correspond to one-sixtieth of a degree or one arc-minute of the ecliptic. Longitude may also be measured in degrees (*aṃśa*). The right ascension of a celestial body is generally measured in *prāṇas* or *asus*, a unit of time close to four seconds, which is the time taken by one arc-minute of the celestial equator to cross the prime meridian.

2.1. *KĀLALAGNA* IN DEGREES

कृतायनांशे पुनरत्र भानौ
चरं कलाप्राणभिदां च कृत्वा ।
तदंशकेषु द्युगताश्च नाडीः
षड्घ्नीः क्षिपेत् कालविलग्नसिद्ध्यै ॥२५॥

kṛtāyanāṃśe punaratra bhānau
caraṃ kalāprāṇabhidāṃ ca kṛtvā |
tadaṃśakeṣu dyugatāśca nāḍīḥ
ṣaḍghnīḥ kṣipet kālavilagnasiddhyai ||25||

Here, having applied the ascensional difference (*cara*) and also the *kalāprāṇabhidā* (i.e. *prāṇa-kalāntara*) to the precession-corrected longitude of the Sun, one should add six times the *nāḍīs* elapsed on the day to those degrees (*aṃśa*) [of longitude] to obtain the *kālalagna* (*kālavilagna*).

This verse (in the *upajāti* metre) gives the following relation to determine the *kālalagna* in degrees:

$$\begin{aligned} \textit{kālalagna} &= \textit{kṛtāyanāṃśa bhānu} \pm \\ &\quad \textit{prāṇakalāntara} \pm \textit{cara} + \\ &\quad 6 \times \textit{nāḍīs} \text{ elapsed} \\ \text{or,} \quad \alpha_e &= \lambda \pm |\lambda - \alpha| \pm |\Delta\alpha| + 6n, \end{aligned} \qquad (1)$$

where α_e is the *kālalagna*, and n is the number of *nāḍīs* elapsed during the day.[9]

As mentioned earlier, the *kālalagna* is the time interval between the rise of the vernal equinox and any desired later instant. If t_γ and t_d denote the rising time of the vernal equinox and the time at a desired later instant respectively, then we have the *kālalagna*

$$\alpha_e = t_d - t_\gamma.$$

By introducing the rising time of the true Sun (t_s), the above expression may be written as

$$\alpha_e = (t_d - t_s) + (t_s - t_\gamma). \qquad (2)$$

Here, the expression $t_d - t_s$ gives the time elapsed since sunrise up to any desired instant, and the expression $t_s - t_\gamma$ gives the time interval between the rise of the vernal equinox and sunrise. Clearly, the sum of these two expressions gives the *kālalagna*.

Now, we will validate the expression given in the verse by showing that

1. $t_d - t_s = 6n$, and
2. $t_s - t_\gamma = \lambda \pm |\lambda - \alpha| \pm |\Delta\alpha|$.

The first expression is easily proven, as by definition, a *nāḍī* is a measure of time (measured from sunrise) equivalent to six degrees of the equator. Therefore, multiplying the number of *nāḍīs* elapsed since sunrise with six gives the time elapsed since sunrise in degrees.

As applying the *prāṇakalāntara* ($|\lambda - \alpha|$) to the Sun's longitude (λ) gives its right ascension (α), the second expression above can be written as

$$t_s - t_\gamma = \alpha \pm |\Delta\alpha|.$$

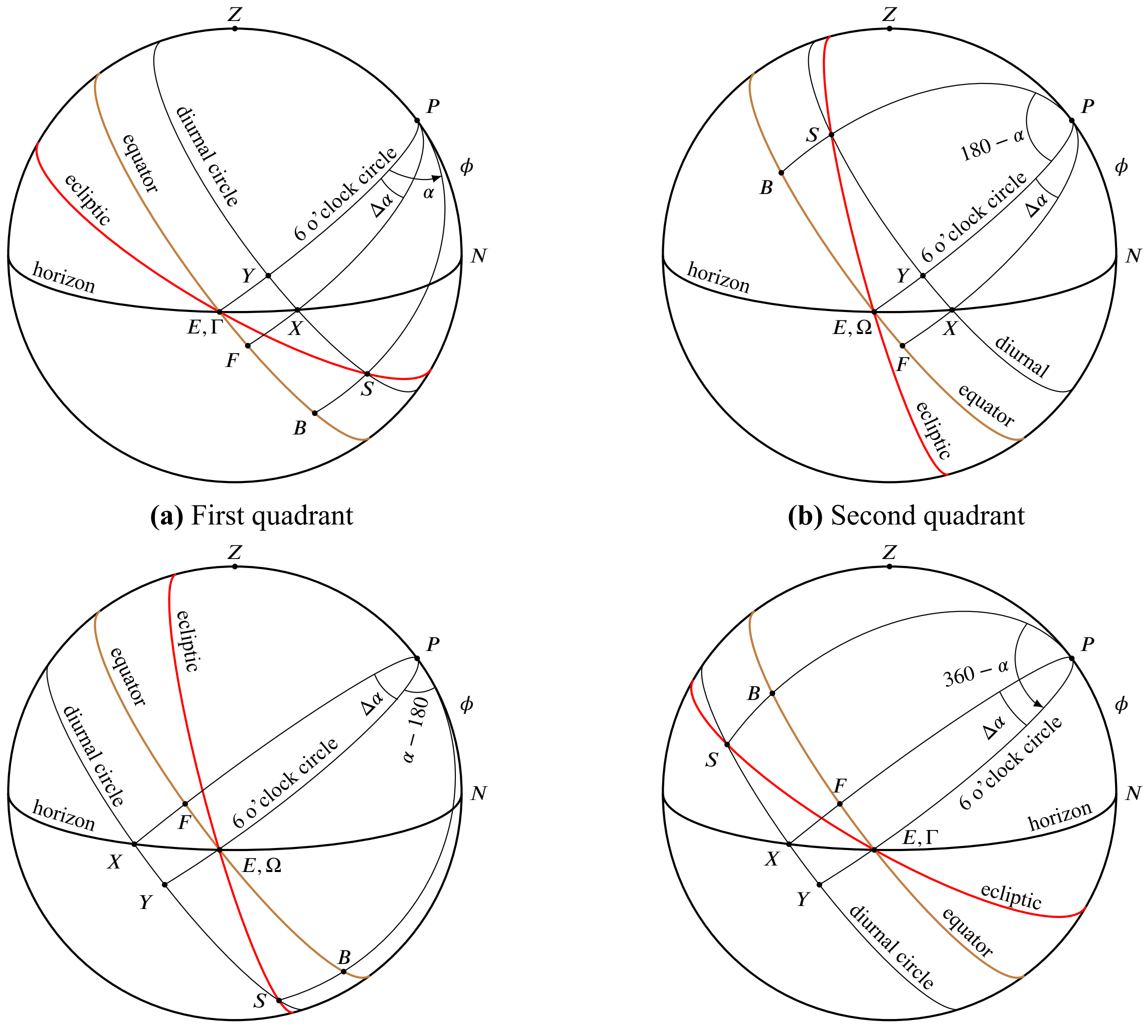
The validity of this restated expression can be understood through Fig. 1.[10] This figure shows the Sun in different quadrants of the ecliptic, and highlights the time interval between the rise of the vernal equinox (Γ) and the Sun.

When the Sun is in the first quadrant (i.e. $0^\circ \leq \lambda < 90^\circ$, Fig. 1a), it can be seen that it yet needs to cover the path SX to rise after the rise of Γ. Therefore, as time is measured on the equator, the time interval between the rise of the vernal equinox and the Sun is given by the length of the arc FB:

$$t_s - t_\gamma = \alpha - \Delta\alpha.$$

[9] The *nāḍī* was a standard unit of time in India, equalling twenty-four minutes, or six degrees of the equator. In the present context, the *nāḍīs* are to be counted from sunrise.

[10] All figures in this paper depict the celestial sphere for a general latitude, as seen from the outside.



(a) First quadrant **(b)** Second quadrant

(c) Third quadrant **(d)** Fourth quadrant

Fig. 1. Determining the *kālalagna* when the Sun is in different quadrants.

When the Sun is in the second quadrant (i.e. $90^\circ \leq \lambda < 180^\circ$, Fig. 1b), it has already traversed the distance XS on its diurnal circle post sunrise, before the rise of the autumnal equinox (Ω). Denoting t_ω as the rising time of Ω, measuring along the equator in the figure, we have

$$t_\omega - t_s = 180 - \alpha + \Delta\alpha.$$

However, as Γ is 180 degrees ahead of Ω, i.e. at the West point, and as the Sun has risen before Ω, we have

$$t_s - t_\gamma = 180 - (180 - \alpha + \Delta\alpha) = \alpha - \Delta\alpha.$$

When the Sun is in the third quadrant (i.e. $180^\circ \leq \lambda < 270^\circ$, Fig. 1c), it has yet to cover the distance SX to rise after the rise of Ω. Therefore, we have

$$t_s - t_\omega = \alpha - 180 + \Delta\alpha.$$

As Γ is 180 degrees ahead of Ω (at the West point), and as the Sun rises after Ω, we have

$$t_s - t_\gamma = 180 + (\alpha - 180 + \Delta\alpha) = \alpha + \Delta\alpha.$$

When the Sun is in the fourth quadrant (i.e. $270^\circ \leq \lambda < 360^\circ$, Fig. 1d), it has already traversed the distance XS post sunrise, before the rise of Γ. Therefore, the corresponding arc FB on the equator gives the difference in the rising times. We have

$$t_\gamma - t_s = 360 - \alpha - \Delta\alpha.$$

However, as we are interested in that instance of the rise of the vernal equinox which occurs before sunrise, we need to consider the previous instance of the rise of Γ here. Therefore, we have

$$t_s - t_\gamma = 360 - (360 - \alpha - \Delta\alpha) = \alpha + \Delta\alpha.$$

Hence, we prove that

$$t_s - t_\gamma = \alpha \pm |\Delta\alpha|,$$

and thereby show the validity of (1).

2.2. *KĀLALAGNA* IN ARC-MINUTES

दिनकृति कृतलिप्ताप्राणभेदे स्वदोर्ज्यां
चरमचरविनिघ्नीं त्रिज्ययाहृत्य लब्धम् ।
कृतधनुरपि कृत्वा वास्य लिप्तासु भूयः
क्षिपतु दिनगतासून् काललग्नस्य सिद्ध्यै ॥२६॥

dinakṛti kṛtaliptāprāṇabhede svadorjyāṃ
caramacaravinighnīṃ trijyayāhṛtya labdham |
kṛtadhanurapi kṛtvā vāsya liptāsu bhūyaḥ
kṣipatu dinagatāsūn kālalagnasya siddhyai ||26||

Or, in the *prāṇakalāntara* corrected [longitude of the] Sun, having applied the arc-converted quotient of the division of the product of its (the *prāṇakalāntara* corrected Sun's) *dorjyā* and [the Rsine of] the maximum *cara* by the radius, to its (the result's) arc-minutes again add the [time in] *asus* elapsed during the day to obtain *kālalagna*.

Whereas the previous verse gave the relation to determine the *kālalagna* in degrees, this verse (in the *mālinī* metre) shows how to calculate the same quantity in arc-minutes:

$$\begin{aligned} \textit{kālalagna} &= \textit{kṛtaliptāprāṇabhedadinakṛt} \pm \\ &\qquad \textit{dhanuṣ}\left(\frac{\textit{svadorjyā} \times \textit{caramacara}}{\textit{trijyā}}\right) \\ &\qquad + \textit{dinagatāsu} \\ \text{or,} \quad \alpha_e &= \alpha \pm R\sin^{-1}\left(\frac{R\sin\alpha \times R\sin\Delta\alpha_m}{R}\right) \\ &\qquad + \textit{asus}\text{ elapsed.} \qquad (3) \end{aligned}$$

The verse which is somewhat terse, is to be understood in the following manner. The quantity which is being operated upon is the *prāṇakalāntara* corrected longitude of the Sun, which is nothing but its right ascension. To this quantity is applied the inverse sine of the quotient of the division of the product of the sine of the right ascension and the maximum *cara* by the radius. Again, to this result, the number of *asus* elapsed during the day are added. Thus we obtain the relation stated in the verse.

The above relation is the same as (1) given in the previous verse, however taking the value of ascensional difference from verse 22 of the *Lagnaprakaraṇa*, which gives the following relation:[11]

$$\begin{aligned} \textit{carajyā} &= \frac{\textit{kālajīvā} \times \textit{paramacarajyā}}{\textit{tribhajīvā}} \\ \text{or,} \quad R\sin\Delta\alpha &= \frac{R\sin\alpha \times R\sin\Delta\alpha_m}{R}, \qquad (4) \end{aligned}$$

where $\Delta\alpha_m$ is the maximum ascensional difference at a given latitude. The relation given in the verse also employs *asus* as the time unit, instead of *nāḍīs*, which indicates that this relation is designed to give the *kālalagna* in arc-minutes. Therefore, though not explicitly stated in the verse, the other quantities need to be determined in arc-minutes too.

2.3. *KĀLALAGNA* AT NIGHT

रात्रावप्येवमेवेदं
काललग्नं समानयेत् ।

[11] See Kolachana, Mahesh, Montelle, et al., "Determination of ascensional difference in the *Lagnaprakaraṇa*".

किन्त्वत्र षड्भयुक्तोऽर्को
नाड्यो रात्रिगताः स्मृताः ॥२७॥

rātrāvapyevamevedaṃ
kālalagnaṃ samānayet |
kintvatra ṣaḍbhayukto'rko
nāḍyo rātrigatāḥ smṛtāḥ ||27||

One should compute this *kālalagna* in just the same way [as for the day] for the night too. But here [in the stated procedure], the [longitude of the] Sun is increased by six signs, and the *nāḍīs* should be considered as those elapsed at the night.

This verse (in the *anuṣṭubh* metre) states that the *kālalagna* at night is determined in the same manner as it is for the day, with the caveats that in the calculations (i) the longitude of the Sun is to be increased by six signs or 180 degrees, and (ii) instead of the *nāḍīs* elapsed since sunrise, one is to consider the *nāḍīs* elapsed after sunset.

We will show the validity of this statement by first calculating *kālalagna* for a desired instant at night using (1), and then showing that the same result is obtained by employing the procedure described in this verse.

We have shown in our discussion of verse 25 that (1) can be expressed as (2), that is

$$\alpha_e = (t_d - t_s) + (t_s - t_\gamma),$$

where

$$t_s - t_\gamma = \alpha \pm \Delta\alpha,$$

with the ascensional difference being applied negatively when the Sun is in the first two quadrants of the ecliptic, and positively otherwise. At anytime during the night, the other expression $(t_d - t_s)$ is equivalent to the time elapsed since sunrise to the desired instant at night. This would be the sum of the duration of the day in degrees and the degree measure of the *nāḍīs* elapsed at night (n_r). When the Sun is in the first two quadrants of the ecliptic, the duration of the day in degrees is equal to $180+2\Delta\alpha$. When the Sun is in the third and fourth quadrants of the ecliptic, the duration of the day is equal to $180 - 2\Delta\alpha$. Therefore,

$$t_d - t_s = 180 \mp 2\Delta\alpha + 6n_r.$$

Then, the general expression for the *kālalagna* at an instant at night is

$$\begin{aligned}\alpha_e &= \alpha \pm \Delta\alpha + 180 \mp 2\Delta\alpha + 6n_r \\ &= \alpha + 180 \mp \Delta\alpha + 6n_r,\end{aligned} \tag{5}$$

which can also be written as

$$\begin{aligned}\alpha_e = (\lambda + 180) &\pm |(\lambda + 180) - (\alpha + 180)| \\ &\mp \Delta\alpha + 6n_r.\end{aligned}$$

Comparing the above expression with (1), we notice that determining the *kālalagna* at night is equivalent to determining the *kālalagna* by considering the Sun to be 180 degrees further along its path, and considering only the *nāḍīs* elapsed at night.[12] This result has been stated in a simple manner in the verse.

For the purpose of illustration, we will discuss the case when the Sun is in the first quadrant. In this case,

$$t_s - t_\gamma = \alpha - \Delta\alpha,$$

and

$$t_d - t_s = 180 + 2\Delta\alpha + 6n_r.$$

Therefore, (5) reduces to

$$\alpha_e = \alpha + 180 + \Delta\alpha + 6n_r. \tag{6}$$

This expression can be visualised with the help of Fig. 2, which depicts an instant (at night) where both the vernal equinox (Γ) and the Sun (S) have set. The figure shows segments SF and $S'H$ of the diurnal circle of the Sun. The *kālalagna* at

[12]Note that neither the magnitude nor the sign of the *prāṇakalāntara* change when the longitude of the Sun is 180 degrees apart. Note also that the sign of the ascensional difference is reversed, as would be expected when the Sun is in the other hemisphere.

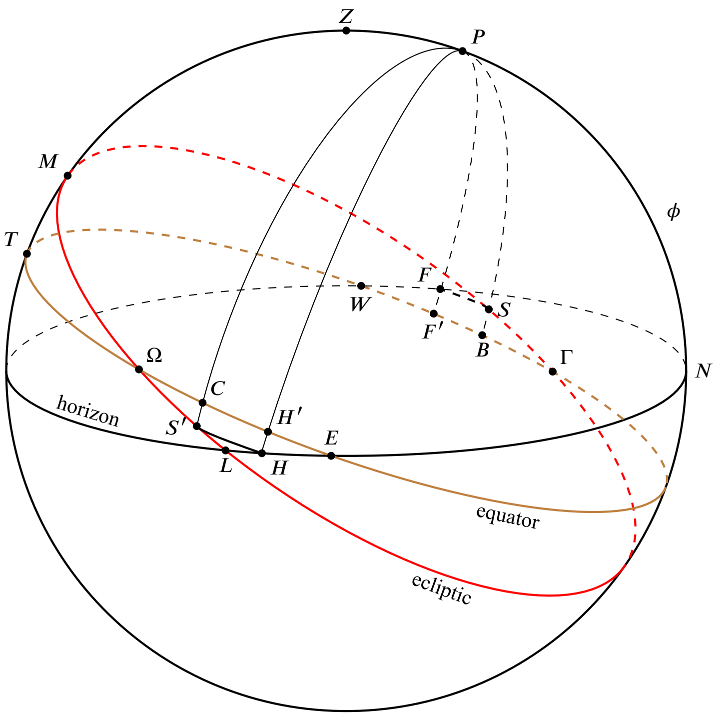


Fig. 2. Determining the *kālalagna* at night.

the depicted instant is equivalent to the arc ΓE,[13] where

$$\Gamma B = \alpha, \qquad F'W = \Delta\alpha,$$
$$BF' = 6n_r, \qquad WE = 180.$$

Therefore, each of the terms in (6) can be clearly visualised on the arc ΓE. That is, the *kālalagna* at night is given by

$$\begin{aligned} \alpha_e &= \Gamma E \\ &= \Gamma B + BF' + F'W + WE. \end{aligned}$$

However, we also have

$$\Gamma E = \Gamma C + CH' + H'E,$$

where ΓC corresponds to the right ascension of S', which is 180 degrees away from S. Since[14]

$$\Gamma\Omega = WE = BC = 180,$$

and

$$H'E = F'W = \Delta\alpha,$$

we can easily show that

$$\Omega E = \Gamma W, \qquad \Omega C = \Gamma B = \alpha,$$
$$\Gamma C = 180 + \alpha, \qquad CH' = BF' = 6n_r.$$

Therefore, the *kālalagna* at any instant during the night can be calculated by taking the Sun to be 180 degrees further along at S', and by considering only the *nāḍīs* elapsed at night.

2.4. CALCULATING *KĀLALAGNA* WHEN A DESIRED *RĀŚI* IS RISING

इष्टराश्युदयेऽप्येवं
तस्मिन्नयनसंस्कृते ।
उदयार्कोक्तवत् कुर्यात्
काललग्नस्य सिद्धये ॥२८॥

[13] By definition, the time interval between the rise of the vernal equinox and any desired instant will be the measure of the arc ΓE. See verse 30.

[14] Note that ecliptic points 180 degrees apart have equivalent ascensional difference.

iṣṭarāśyudaye'pyevaṃ
tasminnayanasaṃskṛte |
udayārko'ktavat kuryāt
kālalagnasya siddhaye ||28||

At the time of rising of a desired *rāśi*, when it has been corrected for precession, one should do the computation as stated for the rising Sun for obtaining the *kālalagna*.

This verse (in the *anuṣṭubh* metre) simply states that the procedure for determining the *kālalagna* at the instant of rise of a desired *rāśi*[15] is the same as that for the rising Sun. Putting $n = 0$ in (1), we can see that the *kālalagna* at sunrise is naturally given by

$$\alpha_e = \lambda \pm |\lambda - \alpha| \pm |\Delta\alpha| \, .$$

Following a procedure similar to that described in verse 25 (Section 2.1), and employing the longitude (λ_r), *prāṇakalāntara* ($|\lambda_r - \alpha_r|$) and ascensional difference ($\Delta\alpha_r$) corresponding to the desired *rāśi* instead that of the Sun, we can easily show that the *kālalagna* at the instant of rising of the *rāśi* is given by:

$$\alpha_e = \lambda_r \pm |\lambda_r - \alpha_r| \pm \Delta\alpha_r. \qquad (7)$$

It may be noted that the *kālalagna* at the time of rising of a particular *rāśi* can also be obtained by employing (1) directly. However, this would require the knowledge of the Sun's longitude, ascensional difference, *nāḍīs* elapsed in the day, etc. On the other hand, the longitudes and ascensional differences of the *rāśis* were well known to astronomers of yore. Thus, it would be simpler to determine the *kālalagna* at the time of rising of a particular *rāśi* by employing (7).

2.5. *KĀLALAGNA* FROM THE MEAN SUN

मध्यमसावनसिद्ध-
द्युगतप्राणैः समन्वितो वापि ।
सायनमध्यमभानुः
कालविलग्नं तदा भवति ॥२९॥

madhyamasāvanasiddha-
dyugataprāṇaiḥ samanvito vāpi |
sāyanamadhyamabhānuḥ
kālavilagnaṃ tadā bhavati ||29||

Or, even when the precession corrected [right ascension of the] mean Sun is combined with the elapsed *prāṇas* of the day computed with respect to this mean Sun, it produces *kālavilagna* (*kālalagna*).

This verse (in the *āryā* metre) states that the *kālalagna* can also be determined from the sum of the right ascension of the mean Sun and the elapsed *prāṇas* in the mean civil day. That is,

$$\begin{aligned} kālalagna &= sāyanamadhyamabhānu \\ &\quad + madhyamasāvanasiddhadyugataprāṇa \end{aligned}$$

or,

$$\alpha_e = \alpha_{ms} + prāṇas \text{ elapsed since mean sunrise,} \qquad (8)$$

where α_{ms} denotes the right ascension of the mean Sun.

To validate this relation, we need to understand the terms (i) mean Sun, and (ii) mean civil day, employed in the above verse. Ramasubramanian and Sriram (see *Tantrasaṅgraha*, p. 82) define these terms as follows:[16]

The 'mean Sun' is a fictitious body which is moving along the equator uniformly with the average angular velocity of the true Sun. In other words, the right ascension of the mean Sun (denoted by R.A.M.S) increases by 360 degrees in the same time period as the longitude of the true Sun increases by 360 degrees. As the R.A.M.S increases uniformly, the time interval between

[15] *Rāśi* here refers to a zodiac sign, which corresponds to one-twelfth of the ecliptic.
[16] It may be noted that the mean Sun referred to in this verse is not to be confused with the mean Sun used to determine the true Sun in the *mandasphuṭa* calculations described in various astronomical texts. That mean Sun is a fictitious body which moves uniformly along the ecliptic with the average angular velocity of the true Sun.

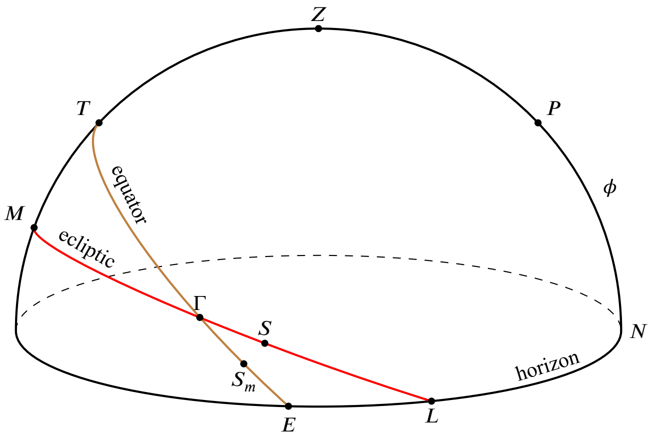


Fig. 3. Determining the *kālalagna* from the mean Sun.

the successive transits of the mean Sun across the meridian or the 6 o'clock circle is constant. This is the mean civil day. All the civil time measurements are with reference to the mean Sun.

As the right ascension of the mean Sun (α_{ms} in our notation) is determined from the vernal equinox and measured along the equator, this same quantity also gives the time difference in the rise of the vernal equinox and the mean Sun. If the vernal equinox rises at instant t_γ, and the mean Sun rises at t_{ms}, then

$$t_{ms} - t_\gamma = \alpha_{ms}.$$

This time difference is depicted by the arc ΓS_m in Fig. 3, where S_m indicates the position of the mean Sun on the equator.

The time elapsed in *prāṇas* since mean sunrise to a desired instant (t_d) is given by $t_d - t_{ms}$. This is equivalent to the arc $S_m E$ in Fig. 3. Therefore, the *kālalagna*, which is the time interval between the rise of the vernal equinox and a desired later instant, is given by

$$\begin{aligned}\alpha_e &= t_d - t_\gamma \\ &= (t_d - t_{ms}) + (t_{ms} - t_\gamma) \\ &= \alpha_{ms} + \textit{prāṇas} \text{ elapsed since mean sunrise,}\end{aligned}$$

which is the expression given in the verse. This quantity is also equal to the arc ΓE on the equator. The next verse states this very point.

2.6. EQUATORIAL ARC WHICH GIVES THE *KĀLALAGNA*

पूर्वस्वस्तिकघटिका-
सम्पातं काललग्नमाहुरतः ।
समयाम्योदक्स्थानां
कालविलग्नस्य नैव भेदः स्यात् ॥३०॥

pūrvasvastikaghaṭikā-
sampātaṃ kālalagnamāhurataḥ |
samayāmyodaksthānāṃ
kālavilagnasya naiva bhedaḥ syāt ||30||

[Scholars] state the intersection of the East cardinal point and the equator (*ghaṭikā* [*maṇḍala*]) to be *kālalagna*. Therefore, there would be no difference in the *kālalagna* for those [observers] who are [located] on a given meridian of longitude (*samayāmyodak*).

The verse (in the *gīti* metre) states that (i) the intersection of the east cardinal point and the equator, i.e. the time interval implied by the equatorial arc ΓE, gives the measure of the *kālalagna*,[17] and that (ii) the *kālalagna* is the same for all observers on a given meridian of longitude.

[17] The right ascension α_e of the east cardinal point is equal to the measure of the arc ΓE.

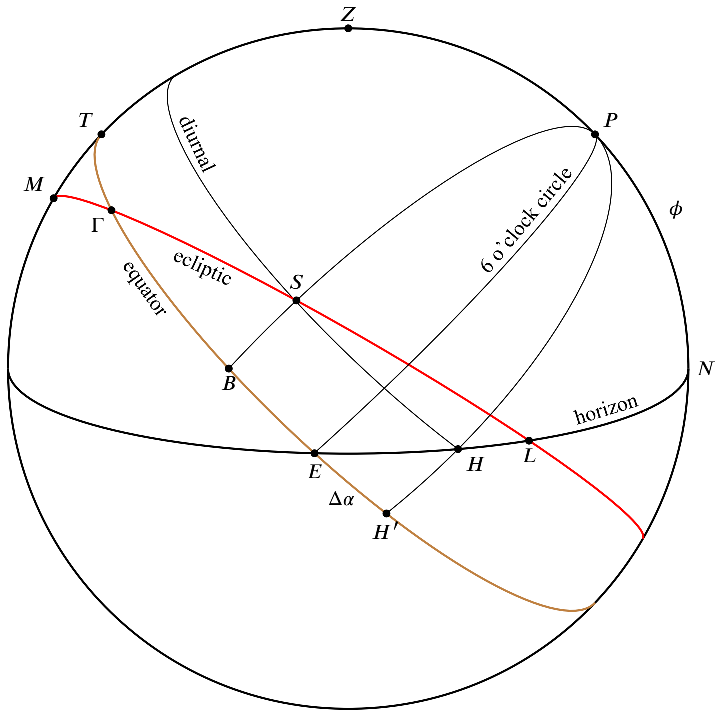


Fig. 4. Equatorial arc corresponding to the *kālalagna*.

The former assertion can be validated using Fig. 4, where the Sun (S) is in the first quadrant of the ecliptic and above the horizon. Here, we have $\Gamma B = \alpha$, $EH' = \Delta\alpha$, and $BH' = 6n$, where n is the *nāḍīs* elapsed since sunrise.[18] Therefore, we have

$$\Gamma E = \Gamma B + BH' - EH' = \alpha - \Delta\alpha + 6n,$$

which is equivalent to (1) when the Sun is in the first quadrant. Therefore, it is clear that the arc ΓE is equivalent to the *kālalagna* in this instance. Similarly, it can be shown that the arc ΓE gives the *kālalagna* when the Sun is in the other quadrants of the ecliptic as well.

The second assertion in the verse can be understood from the fact that the vernal equinox, being a point on the equator, rises at the same instant for all observers on a given meridian of longitude. Therefore, the time interval between the rise of the vernal equinox and a desired instant, which is the *kālalagna*, would be the same for all observers on that longitude.

3. CONCLUSION

The *Lagnaprakaraṇa* discusses the determination of the *udayalagna* through a multitude of approaches. The *kālalagna* is central to many of these calculations. Therefore, it is not surprising that the author has invested six verses to discuss this important topic in his text. Just as in the case of the *udayalagna*, the author discusses various means of obtaining the *kālalagna*. Nīlakaṇṭha Somayājī (*Tantrasaṅgraha*, pp. 240–242) and Jyeṣṭhadeva (*Gaṇita-yukti-bhāṣā*, pp. 579–581) discuss a technique of determining the *kālalagna* similar to that described in Section 2.1 (though they state the result in arc-minutes in-

[18] The equatorial arc BH' gives the time taken by the Sun to traverse the length HS of the diurnal circle since sunrise. As one *nāḍī* is a measure of time that equals six degrees of the equator, the time elapsed since sunrise when measured in degrees is equal to $6n$.

stead of degrees).[19] Jyeṣṭhadeva (*Gaṇita-yukti-bhāṣā*, pp. 579–581) and Putumana Somayājī (*Karaṇapaddhati*, pp. 283–285) also discuss a technique similar to that shown in Section 2.4. The other methods discussed in this paper appear to be unique. Therefore, it appears that the *Lagnaprakaraṇa* is the source of inspiration for the *Tantrasaṅgraha*, the *Candracchāyāgaṇita*, the *Karanapaddhati*, and the *Gaṇita-yukti-bhāṣā*.

Verse 25 describes the fundamental method of determining *kālalagna* in degrees using the longitude and ascensional difference of the Sun, and the time elapsed since sunrise. The procedure of determining the *kālalagna* in arc-minutes is described in Verse 26. The method of determining the *kālalagna* at night, discussed in verse 27, is especially impressive. In this technique, the author displays great imagination to simplify the calculation of the *kālalagna* at night based upon his deep understanding of the celestial sphere and the motion of various entities upon it. He thus correctly concludes that the *kālalagna* at night can be determined by (i) considering the Sun to be six signs (180 degrees) further along its path, and (ii) considering only the *nāḍīs* elapsed at night. Verse 28 describes how to determine the *kālalagna* at the time of rising of a *rāśi*, without utilising the position of the Sun. Verse 29 employs the abstract concept of the mean Sun to give yet another interesting method of calculating the *kālalagna*. Verse 30 describes the equatorial arc which corresponds to the measure of the *kālalagna* and states that this quantity will be the same for all observers on a given meridian of longitude.

The above variety of approaches provide a hint as to the passion of the author for the subject, and his mastery over mathematics as well as astronomy. This perhaps gives us a clue as to his reputation as the *Golavid*, or the knower of the [celestial] sphere, in the Kerala astronomical tradition.

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