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Angular Diameters (*bimba*) of the Sun, Moon and Earth's Shadow-cone in Indian Astronomy: A Comparative Study

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Abstract

In the procedures for computing solar and lunar eclipses the angular diameters (*bimba*) of the eclipsing and the eclipsed bodies play a very important role. The possibility of an eclipse as also its duration depend on these parameters. In classical Indian astronomical texts procedures for the *bimbas* are given in various forms in different texts. Most texts give the *bimbas* in terms of the true daily motions of the Moon and the Sun (*MDM* and *SDM*). But, some others – including *sāriṇīs* (astronomical tables) determine the *bimbas* as functions of other parameters viz., the duration of the running *nakṣatra* or the *manda* anomalies of the Sun and the Moon. In the present paper, a comparative study of the various procedures given in different texts is attempted.

Key words: Angular diameters, *Bimba, Grahalāghava Karaṇakutūhala, Makaranda sāriṇī, Sāriṇī, Shadow-cone, Sūryasiddhānta,*

Abbreviations: In what follows we use the following multi-lettered notations for conveniences of easy identification and computer programming.

SDIA: the Sun's diameter, *MDIA* : the Moon's diameter, *SHDIA* : Diameter of the earth's shadow-cone at moon's orbital plane, *STMD* : the Sun's true daily motion, *MTDM* : the Moon's true daily motion.

1. Introduction

It is well-known that in the computations of solar and lunar eclipses the angular diameters (*bimba*) play a crucial role. In fact these parameters along with the celestial latitude (*śara*) of the Moon determine the possibility of an eclipse as also its duration. Of course the Sun and the Moon need to be in conjunction for a solar eclipse and in opposition for a lunar eclipse.

In the Indian context a systematic development in the study and use of the angular diameters of the participating bodies is perceptible right from the pre-Āryabhaṭan time upto the period of the Kerala contribution. In the following sections we present the results of a few classical Indian astronomical texts one by one and compare them.

2. Diameters of the Sun, Moon and Earth's Shadow-cone according to *Sūryasiddhānta* (*SS*)

sārdhāni ṣaṭ sahasrāṇi yojanāni vivasvataḥ|

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viṣkaṃbho maṇḍalasyendoḥ sāśītiṣtu catuśśataṃ

sphuṭasvahuktyā guṇitau madhybhuktyoddhṛtau sphuṭau ||

SS, ch-IV, 1-2

The (mean) diameters of the Sun and the Moon are respectively 6500 and 480 *yojanas*. These are multiplied by their individual true daily motions and divided by the mean daily motions to get the true diameters (in *yojanas*).

The popular classical text *Sūryasiddhānta* gives expressions for the angular diameters of the eclipsing (*chādaka*) and the eclipsed (*chādya*) bodies in terms of *yojanas* (linear units) as follows.

(i) Sun's diameter (*Ravi bimba*):

$$SDIA = \left(\frac{STDM \times 6500}{SMDM}\right) \times \left(\frac{S.Revns}{M.Revns}\right) \quad .....(2.1)$$

where *S.Revns* and *M.Revns* are respectively the revolutions of the Sun and the Moon in a *mahāyuga* of 432×10^4 years. The remaining variables are explained earlier in **Abbreviations**. Here the text takes the Sun's linear diameter as 6500 *yojanas*.

(ii) Moon's diameter (*Candra bimba*):

$$MDIA = \left(\frac{MTDM \times 480}{MMDM}\right) yojanas \quad .....(2.2)$$

where *MTDM* = the Moon's true daily motion, *MMDM* = the Moon's mean daily motion. Moon's linear diameter is taken as 480 *yojanas*.

(iii) Diameter of Earth's shadow-cone *(Bhūcchāyā bimba)*:

$$SHDIA = \left(\frac{MTDM \times 1600}{MMDM}\right)$$

$$= \left[(SDIA - 1600) \times \frac{480}{6500}\right] yojanas \quad .....(2.3)$$

where 1600 is the earth's linear diameter in *yojanas*.

The true daily motions of the Sun and the Moon, *STDM* and *MTDM* vary with their respective *manda* anomalies (*mandakendra, MK*). The variation of the Sun's true daily motion with its *MK* is given by

$$STDM = 59.1388 - (1.98109)\cos(MK) - (0.041285)\cos(2MK) \quad .....(2.4)$$

in *kalās* (arc min.)

Here, *manda* anomaly
MK=Sun's *Mandocca* (Apogee) – Mean Sun

Similarly, the Moon's true daily motion, in terms of its *manda* anomaly *MK* is given by

$$MTDM = 790.5666 - \left(\frac{PRD}{360}\right) \times 783.9 \times \cos(MK) \text{ in } kalās \text{ (arc min.)} \quad ....(2.5)$$

where *PRD* is the Moon's *manda paridhi* (periphery). Here also, *MK* = Moon's *Mandocca* (Apogee) – Mean Moon

From (2.4) and (2.5), as *MK* varies from 0^0 to 360^0, the minimum values of the true daily motions (*TDM*) of the Sun and the Moon are as shown in Table 2.1.

Table 2.1: Minimum and maximum true daily motions.

Body	Minimum *TDM*	Maximum *TDM*
Sun	56′.83	61′.43
Moon	720′.8866	860′.2466

The variations of the Sun's true daily motion (*STDM*) and its diameter (*Ravi bimba*) *SDIA* with *MK* are shown in Table 2.2.

Table 2.2: Variations of *STDM* and *SDIA* with Sun's *MK*.

MK (Deg)	*STDM* (Min)	*SDIA* (Min)
0	56.83	31.24
30	57.19	31.44
60	58.03	31.90
90	59.13	32.51
120	60.24	33.11
150	60.84	33.44
180	61.43	33.77

Similarly the variations of the Moon's true daily motion (*MTDM*) and its diameter

(*MDIA*, *Candra bimba*) are shown in Table 2.3.

Table 2.3: Variations of *MTDM* and *MDIA* with Moon's *MK*.

MK (Deg)	*MTDM* (Min)	*MDIA* (Min)
0	720.89	28.89
30	730.85	29.28
60	756.35	30.31
90	790.57	32.68
120	824.78	33.05
150	850.28	34.07
180	860.25	34.47

The angular diameter of the earth's shadow-cone (*bhūcchāyā bimba, SHDIA*) varies with the true daily motions of both the Sun and the Moon (*STDM* and *MTDM*). While *STDM* varies from the minimum of 56′.83 to the maximum of 61′.43, the corresponding extrenal values of *MTDM* in the case of the Moon are respectively 720′.8866 and 860′.2466 (see Table 2.1).

Table 2.4: *SHDIA* according to *Karaṇakutūhala*

STDM → *MTDM* ↓	57′	58′	59′	60′	61′	62′
720	72.3	71.9	71.4	71	70.6	70.1
730	73.6	73.2	72.8	72.3	71.9	71.5
740	75	74.5	74.1	73.7	73.3	72.8
750	76.3	79.5	75.5	75	74.6	74.2
760	77.7	77.2	76.8	76.4	75.9	75.5
770	79	78.6	78.1	77.7	77.3	76.9
780	80.3	79.9	79.5	79.1	78.6	78.2
790	81.7	81.3	80.8	80.4	80	79.5
800	83	82.6	82.2	81.7	81.3	80.9
810	84.4	83.9	83.5	83.1	82.7	82.2
820	85.7	85.3	84.9	84.4	84	83.6
830	87.1	86.6	86.2	85.8	85.3	84.9
840	88.4	88	87.6	87.1	86.3	86.3
850	89.8	89.3	88.9	88.5	88	87.6
860	91.1	90.7	90.2	89.8	89.4	89
870	92.4	92	91.6	91.2	90.7	90.3

We have listed in Table 2.4 the values of the diameter of the earth's shadow-cone (*SHDIA*) for *MTDM* from 720′ to 860′ and *STDM* from 57′ to 62′ respectively. According to the *Karaṇakutūhala* of Bhāskara II (epoch 1183 C.E.)

We notice from Table 2.4 that the diameter of the earth's shadow-cone *SHDIA* is minimum at 70′.1 for the minimum *MTDM* = 720′ and the maximum *STDM* = 62′. Similarly, *SHDIA* is maximum at 92′.4 for the maximum *MTDM* = 870′ and minimum *STDM* = 57′.

3. Sun's Diameter according to other Texts

It is interesting to compare the expressions and the values of the Sun's angular diameter according to a few other Indian astronomical texts.

Gaṇeśa Daivajña (1520 CE) in his ***Grahalāghava*** mentions:

vyasuśaragatīṣvaṃśo digyugbhavedvapuruṣṇagor

atha sitaruco bimbaṃ bhuktiryugācalabhājitā |

tadapi himagorbimbaṃ trighnaṃ nijeśalavānvitaṃ

vivasu bhvati kṣmābhābimbaṃ kilāṅgulapūrvakam ||

- *GL*, ch-V, 3

From the Sun's daily motion (in *kalās*) 55 is subtracted, the difference is divided by 5 and the result is added to 10, giving the Sun's diameter in *aṅgulas*. The Moon's daily motion (in *kalās*) divided by 74 gives the Moon's diameter in *aṅgulas*. 3 times the Moon's diameter (in *aṅgulas*) divided by 11 is add to 3 times the Moon's diameter; from the result subtract 8 to get the diameter of the earth's shadow-cone (*bhū-bhā*).

The above procedures are used in what follows under separate heads.

(i) From the above we have the following expression for the Sun's diameter:

$$SDIA = \left(\frac{STDM - 55}{5}\right) + 10 \text{ } a\dot{n}gulas \qquad \text{.....(3.1)}$$

where *STDM* is Sun's true daily motion in *kalās*. An *aṅgula* = 3 *kalās* (arc minutes).

(ii) Brahmagupta in his *karaṇa* text (handbook), *Khaṇḍakhādyaka* (*KDK*) provides

bhavadaśaguṇite ravi śaśigatī nakhaiḥ svarajinairhṛte māne |

ṣaṣṭyā bhaktaṃ tatvā'ṣṭaguṇitayōrantaraṃ tamasaḥ ||

- KDK, ch-IV, 2

The true daily motions of the Sun and the Moon multiplied respectively by 11 and 10, and divided by 20 and 247, give their angular diameters in minutes. The difference between 8 times the true motion of the moon and 25 times that of the Sun, when divided by 60 gives the angular diameter of the earth's shadow in minutes.

i.e. $SDIA = STDM \times \left(\frac{11}{20}\right)$ *kalās* …..(3.2)

(iii) In the southern part of India a popular system of astronomical computations prevalent is the *Vākya Paddhati*. In this system the true positions of planets, using their synodic and anomalistic periods, are given in the form of simple meaningful Sanskrit sentences (*vākyas*). Here each letter of a given sentence represents a number following the *kaṭapayādi* system of letter numerals. The popular text in this genre, *Vākya karaṇa* obtains Sun's diameter from the expression[1]

$SDIA = STDM \times \left(\frac{5}{9}\right)$ *kalās* …..(3.3)

(iv) The famous commentator Viśvanātha Daivajña (c.1620 C.E.) in his *Udāharaṇa* gloss on *GL* computes the Sun's diameter from the expression

$SDIA = STDM \times (2/11)$ *aṅgulas* …..(3.4)

For the purpose of comparison the values of the Sun's angular diameter according to the above different texts forthe increasing anomaly (from the apogee) arecomputed and presented in Table 3.1 at intervals of 20^0 of Sun's anomaly. These are compared with the values obtained from modern computations.

From Table 3.1 we observe that the values of Bhāskara II and Viśvanātha differ the least from the corresponding modern values while those of the *Vākya* system differ the maximum.

Table 3.1: Sun's Diameter according to different Texts.

MK (Deg)	*Grahalāghava*	*Khaṇḍakhādyaka*	*Vākya*	Bhāskara II and Viśvanātha	Modern
0	31.27	31.41	31.73	31.15	31.50
20	31.35	31.49	31.8	31.22	31.54
40	31.57	31.69	32.0	31.43	31.63
60	31.90	31.99	32.32	31.73	31.77
80	32.30	32.36	32.69	32.09	31.95
100	32.71	32.74	33.07	32.47	32.13
120	33.09	33.08	33.43	32.81	32.31
140	33.39	33.36	33.69	33.08	32.45
160	33.58	33.53	33.87	33.26	32.54
180	33.65	33.59	33.93	33.32	32.58

1. *Vākya pañcādhyāyī,* Cr.Ed.by T.S. Kuppanna Śastri and K.V. Śarma, KSR Inst., Madras, 1962. Chaper IV, page number 95 and 96

4. Moon's Diameter According to Different Texts

(i) Brahmagupta's *Khaṇḍakhādyaka* (epoch March 23, 665 C.E.) gives the following expression for *MDIA* [sec 3 ii)

$$MDIA = \frac{10}{247} \times MTDM$$

The rationale for the above expression is provided by taking

$$MDIA = \text{Mean diameter} \times \left(\frac{\text{MTDM}}{\text{MMDM}}\right)$$

$$= \left(\frac{\mathbf{32'}}{\mathbf{790'31''}}\right) \times MTDM$$

$$= \left(\frac{\mathbf{1}}{\mathbf{24.7036}}\right) \times MTDM$$

$$\approx \left(\frac{10}{247}\right) \times MTDM \qquad \text{.....(4.1)}$$

(ii) According to *Grahalāghava* of Gaṇeśa Daivajña, the Moon's *bimba*

$$MDIA = \left(\frac{MTDM}{74}\right) \textit{aṅgulas}$$

$$= \frac{3}{74} \times MTDM \textit{ kalās} \qquad \text{.....(4.2)}$$

(iii) According to the *Vākya* system

$$MDIA = \left(\frac{MTDM}{25}\right) \textit{kalās} \qquad \text{.....(4.3)}$$

5. Diameter of the Earth's Shadow-cone according to other Texts

(i) *Khaṇḍakhādyaka* provides the following expression. [sec 3 ii)

$$SHDIA = \left(\frac{8 \times MTDM - 25 \times STDM}{60}\right) \textit{kalās} \text{.....(5.1)}$$

(ii) Lalla's in his *Śiśyadhī-vṛddhi-da* calculates *SHDIA* using the formula:

$$SHDIA = \left(\frac{2 \times MTDM}{15}\right) - \left(\frac{11}{20} - \frac{2}{15}\right) \times STDM \textit{ kalās} \text{ .....(5.2)}$$

(iii) In the *Vākya* system the expression for *SHDIA* is given as follows:

$$SHDIA = \left(\frac{5}{2}\right) \times MDIA \textit{ kalās} \qquad \text{......(5.3)}$$

(iv) Gaṇeśa Daivajña's *Grahalāghava* (1520 CE)

$$SHDIA = \left(\frac{3}{11}\right) \times MDIA + 3 \times MDIA)\text{-}8 \textit{ aṅgulas} \text{ ......(5.4)}$$

(v) Viśvanātha in his commentary on *Grahalāghava* modifies the formula for *SHDIA* as

$$SHDIA = \left(\frac{MTDM}{22}\right) - \left(\frac{STDM}{7}\right) - \left(\frac{6}{11}\right) \textit{aṅgulas} \text{ .....(5.5)}$$

Table 5.1: Minimum and maximum SHDIA

Text	Min. *SHDIA* (*kalās*)	Max. *SHDIA* (*kalās*)
Khaṇḍakhādyaka	70.17	92.67
Śiśyadhī-vṛddhi-da	70.2	92.7
Vākya	73.5	86
Grahalāghava	71.53	90.1
Udāharaṇa	70.0	93

In the *Vākya* system as well as in the *Grahalāghava* the diameter of the shadow-cone is given only in terms of *MDIA*. From Table 5.1 we observe that the range of the variation of the shadow's diameter is widest from 70′ to 93′ in the case of Viśvanātha's procedure. On the other hand, in the *Vākya* system, the range is narrowest from 73′.5 to 86′.

Actually the shadow-cone depends directly on the motions and the relative distances of both the Sun and the Moon. This fact has been taken into account by Brahmagupta, Lalla and Viśvanātha. The values of *SHDIA* according to modern procedure varies from 76′.67 to 93′.02. The minimum angular diameter 76′.67 is attained when the Sun's anomaly is 0^0 and the Moon's anomaly is 180^0. Similarly the maximum angular diameter 93′.02 when the Sun's anomaly is 180^0 and the Moon's anomaly is 0^0. Here the anomalies are measured from the respective perigees.

Interestingly Copernicus (1473- 1543) in his *De Revelutionibus* gives the ratio of the diameter of the earth's shadow-cone to that of the Moon as $k = \frac{403}{150} = 2.6866$. Thus, the text gives the greatest and the least diameters as

$$d_{min} = k \times 0^0 30' \qquad = 1^0 20' 36'' = 80' 36''$$

$$d_{max} = k \times 0^0 35' 38'' = 1^0 35' 44'' = 95' 44''$$

6. According to Bhāskara's *Karaṇakutūhala* Diameters of Sun, Moon and Earth's Shadow-cone

Bhāskara II gives the expression for these angular diameters in the following *śloka* in his *Karaṇakutūhala* (*KRK*)

bimbaṃ vidhoḥ syāt svagatir yugādri bhaktā

raverdasrahatā śivāptā||

trighnīndubhuktis turagāṅgabhaktā

bhūbhārka bhuktyādri lavenahīnā|

-*KRK*, ch-IV,7,8

The Moon's (*angular*) diameter (*bimbam*) is its (*daily*) motion divided by 74, (the diameter) of the Sun is that (its daily motion) multiplied by 2 and divided by 11. The shadow's diameter (*bhū-bhā*) is the Moon' daily motion multiplied by 3 and divided by 67 reduced by one-seventh of the Sun's motion.

This means

(i) $MDIA = \frac{MTDM}{74}$ *aṅgulas*

(ii) $SDIA = \frac{STDM \times 2}{11}$ *aṅgulas*

(iii) $SHDIA = (MTDM \times \frac{3}{67}) - (STDM \times \frac{1}{7})$ *aṅgulas*

Example: Lunar eclipse dated *śaka* 1542, *Mārgaśīrṣa śukla* 15 (*paurṇimā*), Wednesday, corresponding to December 9, 1620 CE (G).

That day, $STDM = 61' \ 21''$ and $MTDM = 829' \ 35''$

Therefore,

(i) $SDIA = 61'21'' \times \frac{2}{11} = 11|09$ *aṅg*

(ii) $MDIA = \frac{829' \ 35''}{74} = 11|12$ *aṅg*

(iii) $SHDIA = 829'35'' \times \frac{3}{67} - 61'21'' \times \frac{1}{7} = 28|22$ *aṅg*

6.1 Rationale for the *bimbas*:

It is assumed that the angular diameter is proportional to the true daily motion of a body and given by

$$\text{True } bimba = \text{Mean } bimba \times \frac{\text{True daily motion}}{\text{Mean daily motion}}$$

(i) In the case of the Moon, the mean daily motion is 790', taking the Moon's mean angular diameter as close to 32' (see P.C Sengupta (1934)), we have the Moon's true diameter (*MDIA*) given by

$$MDIA = \frac{32' \times MTDM}{790'} kal\bar{a}s$$
$$= \frac{MTDM}{\left(\frac{790 \times 3}{32}\right)} = \frac{MTDM}{74.0625} \ a\dot{n}g$$

where *MTDM* is the Moon's true daily motion. Bhāskara II has taken the denominator as 74.

(ii) For the Sun, the mean daily motion is $59' \ 8''$ and taking the Sun's mean angular diameter as $32' \ 31''$ (P.C. Sengupta (1934)), we have the Sun's true angular diameter (*SDIA*) given by

$$SDIA = \frac{32'31'' \times STDM}{59'8''} \ kal\bar{a}s$$
$$= \frac{STDM}{\left(\frac{59'8'' \times 3}{32' \ 31''}\right)} = a\dot{n}g$$

Bhāskara II has taken the denominator as 5.5 i.e., 11/2 so that

$$SDIA = \frac{2}{11} \times STDM \ a\dot{n}g$$

where *STDM* is the Sun's true daily motion.

(iii) The angular diameter of the earth's shadow-cone at the Moon's orbit is given by $SHDIA = 2 \times (p + p' - s)$ .....(6.1)

where p and p' are the horizontal parallax of the Sun and the Moon and s is the Sun's angular semi-diameter (see fig 6.1).

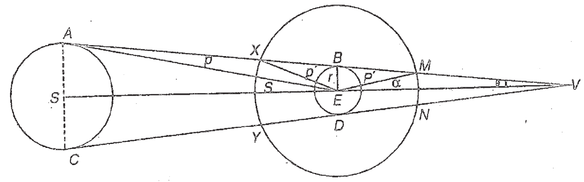

Fig. 6.1: Angular diameter of the Shadow-cone

In Fig 6.1, the angular diameter of the cross-section of the shadow cone is represented by arc *MN*. Let the semi-angle *MEV* subtended by *MN* at the centre of the earth be α.

We have

p = the Sun's horizontal parallax = $E\hat{A}X$

p' = the Moon's horizontal parallax = $E\hat{M}B = E\hat{X}B$

s = the Sun's angular semi-diameter = $S\hat{E}A$

θ = semi-vertical angle of the shadow-cone = $E\hat{V}B$

Now, in triangle *MEV*, we have

$$\alpha + \theta = p' \text{ so that } \alpha = p' - \theta \quad \ldots..(6.2)$$

Similarly, we have from the triangle *AEV*,

$$\theta = s - p' \quad \ldots..(6.3)$$

From (6.2) and (6.3) we get,

$$\alpha = p' - (s - p) \text{ or } \alpha = p + p' - s \quad \ldots..(6.4)$$

The assumption in *siddhāntic* texts is that the horizontal parallax of a body is $\left(\frac{1}{15}\right)^{th}$ of its true daily motion. Accordingly,

$p = \frac{MTDM}{15}$ and $p' = \frac{STDM}{15}$ in *kalās* and the Sun's angular diameter $2s = \frac{6}{11}$ *STDM*

Substituting these in (6.1), we have

$$\text{SHDIA} = 2\left[\frac{MTDM}{15} + \frac{STDM}{15} - \frac{3}{11}STDM\right] \textit{kalās}$$

$$= 2\left[\frac{MTDM}{15} + \left(\frac{1}{15} - \frac{3}{11}\right)STDM\right] \textit{kalās}$$

$$= 2\left[\frac{MTDM}{15} + \left(\frac{11-45}{165}\right)STDM\right] \textit{kalās}$$

$$= \left[\frac{MTDM}{\left(\frac{15\times3}{2}\right)} - \frac{STDM}{\left(\frac{165\times3}{34\times2}\right)}\right] \textit{aṅg}$$

$$= \frac{MTDM}{22.5} - \frac{STDM}{7.279} \quad \textit{aṅg}$$

$$= \frac{MTDM \times 3}{67.5} - \frac{STDM}{7.279} \textit{aṅg}$$

Bhāskara II has taken the two denominators respectively as 67 and 7.

7. Conclusion

In the preceding sections we have discussed the algorithms of diameters (*bimbas*) of the Sun, Moon and the earth's shadow-cone (*bhūcchāyā bimba*). According tosome classical Indian astronomical texts and the results are compared, where feasible, with those from the modern procedures.

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