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Historical Notes

Asutosh Mookerjee's Contribution on Nineteenth Century Modern Mathematics: A Bird's Eye View

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Abstract

This paper attempts to highlight the multifarious genius of Sir Asutosh Mookerjee. Though widely acclaimed as an eminent jurist and a famous educationist, he had a great passion for Mathematics and desired to be a mathematician which could not be fulfilled. He had interest in various branches of mathematics, both Pure and Applied. He was well versed with the works of mathematical giants of Europe. His mathematical contributions appearing in well-known journals of India and Europe earned him great recognition.

Keywords: Asutosh Mookerjee, A Cayley, G Mainardi, Calcutta High Court, Differential Equation, Geometry, G Monge, J J Sylvester, J L Lagrange, University of Calcutta

1. Introduction

Asutosh Mookerjee (1864 – 1924 CE) had great passion for Mathematics right from his early school days. At the age of seventeen, he published his first paper in *Messenger of Mathematics*. In a short span of seven years (1884 -1890 CE) he contributed sixteen mathematical papers of enduring relevance in the *Messenger of Mathematics*, *Quarterly Journal of Pure and Applied Mathematics* and *Journal of the Asiatic Society of Bengal*. He published papers dealing with Geometry, Elliptic Function, Differential Equation, Hydrokinetics etc. He was greatly influenced by European mathematicians viz. Euclid, J L Lagrange, A M Legendre, G Monge, G Mainardi, A Clebsch, J J Sylvester, AJC Cunningham and A Cayley.

Asutosh Mookerjee was born in Calcutta. His academic career was one of uniform brilliance. He stood first in the first class in M A in Mathematics in 1885 CE from the University of Calcutta. He was awarded the prestigious Premchand Roychand Scholarship in Mathematics and Science. Being unable to obtain a permanent post of lecturer in Mathematics in the University of Calcutta, he shifted to law. He was a Justice of Calcutta High Court from 1904 till the end of 1923 CE. For few months he officiated as the Chief Justice of Bengal. He held office of the Vice-Chancellor from 1906 -1914 CE. He was again appointed Vice-Chancellor in 1921 CE and held office for two years. All through his life, Asutosh Mookherjee made tireless efforts for the all round progress of the University of Calcutta.

Asutosh Mookerjee had interest in various branches of mathematics both Pure and Applied viz. geometry, calculus, differential equation, elliptic function, hydrokinetics etc.

He was obsessed with geometry. In this field his idol was the Greek Geometer Euclid [fl.c 300 BCE] often referred to as the 'father of

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Geometry'. Regarding study of geometry he was attracted to French mathematicians A M Legendre (1752-1833) and G Monge (1749-1818). Legendre's outstanding contribution '*Éléments de géométrie*' (1794) which deals with the important aspects of Euclidean Geometry fascinated him. In this book Legendre greatly rearranged and simplified many propositions of Euclid's 'Elements' and made the book more effective. Monge was also a geometer with a new outlook and authored outstanding books on geometry.

While in upper classes of school, he solved 'the Riders and Exercises' of Euclid's book and named it 'Exercises of Euclid' (Das Gupta, 2013 p.161). This is an unpublished book and has been kept in 'Asutosh Collection Section', National Library, Calcutta.

The study of the masters of Geometry resulted in the publication of his first paper, entitled 'Proof of Euclid 1, 25' (Mookherjee, 1880-81, pp.122-123), where he gave an elegant new proof of the 25^{th} proposition of the first book of Euclid. It is noteworthy that he published the above paper at the age of seventeen when he was a first year student of the Presidency College, Calcutta. During the period 1883-84, Asutosh, an undergraduate, published his second paper entitled 'Extension of a theorem of Salmon' (Mookherjee, 1883-84, pp. 157-160). In this paper, he stated a theorem of Dr. Salmon and reproduced it with some extensions.

Regarding his contribution in Geometry we may mention that he published, in 1893, a book entitled '*An Elementary Treaties on the Geometry of Conics*' (Das Gupta, 2013, p.161) meant for the beginners. It ran into many editions.

The French mathematician Gasperd Monge (1749-1818) was an outstanding geometer who interwove analysis and geometry which is evident from his book entitled '*Application of Analysis to Geometry*' (1795). He may be rightly called the man behind the flourishing of 'Differential Geometry'. He also authored the book '*Géométrie descriptive*' which came out in 1795. Another great contribution of Monge was his geometrical interpolation of the solutions of partial differential equations. No wonder the works of such a mathematician would have immense influence on Asutosh. Being inspired by Monge, he produced some papers from 1887-1890 showing that Geometry and Calculus were interwoven. He believed that Differential Equation did not merely provide tools for solving mathematical problems. He was of the opinion that the solutions were capable of providing simple geometrical representation. In this context we may refer to his observation regarding the work of Italian mathematician Gaspare Mainardi (1800-1879). He was the first mathematician to solve the problem of determining the oblique trajectory of a system of confocal ellipses. But Asutosh was not satisfied with the solution as it was too complicated. He published a paper entitled 'On the Differential Equation of a Trajectory', (Mookerjee, 1887, pp. 117-120), immediately after receiving his M A degree. In this paper he discussed Mainardi's solution (reproduced by Boole). Asutosh noted the solution to be "so complicated that it would be a hopeless task to have to trace the curve from it." He expressed his dislike by naming the solution as "unsymmetrical and inelegant" and provided a very elegant solution. He proved that equation of the curve provided by Mainardi's solution could be replaced by a pair of remarkably simple equations which admitted of an interesting geometrical interpretation. These simple equations have been included by A R Forsyth (1858-1942) in his book on Differential Equations in later editions.

Having dealt with an ingenious way the problem regarding Mainardi's solution, he was led to the publication of a general theorem on the differential equations of trajectories. This paper entitled 'A general theorem on the differential equations of trajectories' (Mookherjee, 1888, pp.

72-99), showed his power of generalization and elegant expression.

Asutosh published three papers relating to Monge's general differential equation of all conics during the period 1887-1889. He mentioned that Monge neither discussed the method by which he derived the differential equation nor did he provide any geometrical interpretation regarding the equation. He simply stated the differential equation. Boole, in his book on Differential Equations, made the following statement:-

> "But here our powers of geometrical interpretation fail, and results such as this can scarcely be otherwise useful than as a registry of integrable forms." Sinha, 2009, p.175.

Inspite of being inclined to French Mathematicians, he was well apprised of the British School of Mathematics. He criticized the geometrical interpretation of Monge's differential equation to all conics by British Mathematicians Prof. J J Sylvester (1814-1897) and Lieutenant Col. AJC Cunningham (1842-1928). The geometrical interpretation of Prof. Sylvester was that

> 'The differential equation of a conic is satisfied at the sextactic points of any given curve.' Sinha, 2009, p.175

Lieutenant - Col. Cunningham gave his interpretation in the following way:-

> 'The eccentricity of the osculating conic of a given conic is constant all around the latter (Sinha, 2009, p.175).'

We have already mentioned that according to Asutosh neither of them was the true geometrical representation of Monge's differential equation. Asutosh gave the geometrical interpretation on Monge's differential equation as follows:-

> 'The radius of curvature of the aberrancy curve vanishes at every point of every conic (Sinha, 2009, p.180).'

This interpretation was accepted by all mathematicians. J Edwards incorporated it in his book on Differential Calculus.

On 14th September, 1887, A Cayley (1821-1895) wrote a letter to Asutosh from Cambridge, supporting his criticism regarding Sylvester's interpretation. Cayley categorically stated that 'it is of course all perfectly right.' Cunningham wrote

> 'Professor Asutosh Mookerjee has proposed a really excellent mode of geometric interpretation of differential equations in general . . . . . . ' (S e n, 2013, p.29).

Asutosh Mookerjee acknowledged the professional support he received from British mathematicians A Cayley (1821-1895) and J J Sylvester (1814-1897).

The works of both the French mathematicians J L Lagrange (1736 – 1813) and A M Legendre (1752 – 1833) had influenced Asutosh to explore elliptic functions as an area of research. J L Lagrange's '*Theorie des functions analytiques*' (1797) and '*Lecon sur le Calculus des functions*'(1804) are two outstanding books in the field of elliptic functions. Legendre published his three-volumed '*Traité des fonctions elliptiques*' during the period 1825-1828. It may be mentioned that elliptic functions were the produce of his toil of forty years. He is aptly known as the founder of 'Elliptic Functions'. In this context it should be mentioned that CGJ Jacobi (1804-1851) made immense contribution in this field. The study of the works of these two giants inspired Asutosh to take up research on elliptic functions. The outcome was the publication of his third paper entitled "A note on elliptic functions" (Mookerjee1886, pp. 212-217). In this paper, he, a student of fifth year established a certain addition theorem in the theory of elliptic functions with the help of a new method where he used the properties of ellipse. The German mathematician Alfred Enneper (1830-1885) has referred to this paper in his book '*ElliptischeFunktionen*.' Regarding this third paper, Professor Cayley remarked that it was praiseworthy how he reached a real result by considering an imaginary point. Subsequently he published two papers where he applied elliptic functions to problems of 'Mean Values'.

The French mathematician Siméon Denis Poisson (1781-1840), both a physicist and a geometer, had great influence on Asutosh Mookerjee. Prof. Poisson also possessed inclination to integration. Asutosh noticed that Poisson's method of integration helped the subject (integration) advance forward. He (Poisson) took help of differential equation in his method of integration. It may be noted from his 'Diary' that Asutosh read two books of Poisson namely '*Theorie Mathematique de la Chaleur*' (1835) and '*Nouvelle Theorie de L'action Capillire*'(1831) very minutely. The study of the books helped Asutosh, already obsessed with geometry, to understand Poisson's view on analytic and synthetic approach to geometry. Poisson, in his memoir '*Suite du Memoire sur les Integrales Definies*' first considered the integral known as 'Poisson's Integral'. To evaluate the integral, Asutosh used a technique different from Poisson. He obtained a formula of transformation for evaluation. Asutosh stated (Sinha, 2009, p.223).

> "this method has also the advantage of shewing how the indefinite integral itself may be evaluated."

The process of evaluation of the integral also yielded the symbolic value of Π (pi). In this paper Asutosh also establishes an elegant geometrical interpretation of calculus (Poisson's Integral) thus showing that he was a mathematician seeking to prove that calculus and geometry are interwoven. We notice a departure of Asutosh from his usual image of being obsessed with geometry and calculus in the two papers we are going to consider. The papers under our purview are 'On Clebsch's Transformation of the Hydrokinetic Equations' (Mookerjee 1890, pp.56-59) and 'Note on Stoke's theorem and Hydrokinetic Circulation' (Mookerjee 1890, pp.59-61). These are the two research papers of Asutosh on Fluid Mechanics. Being influenced by Alfred Clebsch (1833-1872), a German mathematician, he contributed the two papers mentioned above. Clebsch made important contributions to algebraic geometry and invariant theory. He also contributed in the general theory of curves and surfaces, their uses in Geometry etc. He also studied elliptic functions and worked in the field of elasticity. Asutosh was drawn towards Clebsch for their common interest. During his student life in Presidency college, Asutosh had gone through the book on fluid dynamics, entitled '*A treaties on the Mathematical Theory of Motion of Fluids*' by Horace Lamb. Influence of both Clebsch and Lamb inspired Asutosh to contribute two papers on hydrokinetics. He considered hydrokinetic equations in three cases:-

1. Irrotational motion, 2. Steady rotational motion, 3. General rotational motion

He showed how the method of applying Clebsch's transformation to the third case can be materially simplified.

In the second paper entitled "Note on Stoke's Theorem and hydrokinetic circulation", Asutosh presented a new proof of Stoke's formula for hydrokinetic circulation with the help of Clebsch's transformation. In the second paper he specifically stated

> "It is worth noting that as no physical conception enters into the above proof, it holds good, whether we regard the theorem as a purely analytical one or as merely furnishing a formula for hydrokinetic circulation."

We observe that both the papers are 'purely analytical' and no 'physical conception' has been applied; judging from these two points, Asutosh's contribution may be attributed as remarkable as Clebsch.

It is noteworthy that Asutosh's outstanding contributions to mathematics were due to his own brilliance, without any guidance from any one while he was only a college student. Prof. Ganesh

Prasad rightly observed,

> 'After Bhāskara II he was the first Indian to enter into the field of mathematical research as distinguished from astronomical research and did much which was truly original (Prasad, 2013, pp. 25)".

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