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Combinatorics as Found in the *Gommaṭasāra* of Nemicandra*

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Abstract

Prastāra, *naṣṭa*, *uddiṣṭa*, *la-ga-kriyā*, *saṅkhyā* and *adhvayoga* are six *pratyayas* (combinatorial tools). They have been in systematic use in India for the study of prosody since the time of Piṅgala (sometime before 200 BC). Fascinating is in the *Gommaṭasāra* (*Jīvakāṇḍa*) of Nemicandra (c. 981 AD) that the tuples of a cartesian product, formed by selecting only one element at each time from each one of the n finite ordered sets, at the time of discussion on carelessness (*pamāda*, Skt. *pramāda*), were brought in purview of those *pratyayas* with the omission of *la-ga-kriyā* and *adhvayoga* and with the addition of a new *pratyaya* (tool) *parivartana* (rotation). For the case when $n = 5$, more particularly $n = 3$, he gives the rules for each of those five tools. *Akṣa* (axle) and *akṣapada* (terms on the axle) are the new terms. The former begins its service from *parivartana* (rotation) onwards and the latter fulfilled the new requirements of *naṣṭa* and *uddiṣṭa* in consideration of combinatorics of tuples.

Key words: Combinatorics, Jaina school of Indian mathematics, *Naṣṭa*, Nemicandra, *Parivartana*, *Prastāra*, *Saṅkhyā*, Tuple, *Uddiṣṭa*

1. Introduction

Ācārya[1] Nemicandra "Siddhānta Cakravartī"[2] was a Jaina monk. There is a famous colossal statue of Bāhubalī at Śravaṇabelgola, near Mysore, in India. It was erected by his disciple Cāmuṇḍarāya, a celebrated commander-in-chief and wise minister of the *Gaṅga* dynasty for the period from 953 AD to 985 AD. Nemicandra is identified to have been in attendance in its first consecration ceremony, held on 13th March of 981 AD (Jadhav 2006, pp. 75-81).

The Jaina school of Indian mathematics[3] is suggested to have been divided into two classes. One is the canonical class and the other is the exclusive class (Jadhav 2004, 35-40 and Jadhav 2013, 34-52).[4] Mathematicians of the canonical class are the authors of those works that describe the Jaina canons including those on the system of cosmology and cosmography and the system of *karma*. Mathematical material found embedded in their works occurs in the form of rules and results and at some places in the functional form. It is available in abundance. The *Bhagavatī Sūtra* (Shah, 2008, pp. 1-21; Jadhav 2009, pp. 35-55) of Sudharma Svāmī (300 BC or earlier), the *Tiloyapaṇṇatti* (Sarasvathi 1961-1962, pp. 27-51) of Yativṛṣabha (some period between 176 AD and

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1 Ācārya is an official position in the ecclesiastic organization. It simply means preceptor or teacher in India.

2 It is a title of scholarship. It was conferred on him. It can be interpreted to be "Monarch-like master of the ‹six› *siddhānta*(s)". See Jaini 1927b, v. 397, p. 255.

3 The introductory articles on the school are the following two ones. Datta 1929, pp. 115-145 and Jain, L. C. 1967, 265-292.

4 The first of the present authors proposes to write a paper titled "The Jaina school of Indian mathematics" that will be mainly aimed at describing the two divisions of the school.

609 AD), the *Dhavalā*, a commentary on the *Ṣaṭkhaṇḍāgama* of Puṣpadanta and Bhūtabalī of some period between 87 AD and 156 AD, of Vīrasena (816 AD) (Singh, A. N. 1942, pp. 1-21), and the *Gommaṭasāra* and the *Trilokasāra* of Nemicandra (c. 981 AD) (Datta 1935, pp. 25-44; Jadhav 2006, pp. 75-81) are a few of the works of the canonical class. The authors of the exclusive class were originally mathematicians. Their works are written exclusively on mathematics. Some of the works of this class are the *Pāṭīgaṇita* (Shukla 1959) and *Triśatikā* (Dvivedi 1899) of Śrīdhara[5] (c. 799 AD), the *Gaṇitasārasaṃgraha* (Padmavathamma 2000) of Mahāvīra (c. 850 AD), the *Vyavahāra-gaṇita* (Padmavathamma et al. 2013) of Rājāditya (12th century AD) and the *Gaṇitasārakaumudī* (SaKHYa 2009) of Ṭhakkara Pherū (1265 -1330 AD).

The *Gommaṭasāra* (‘Essence ‹extracted from and developed on the previous sources on the *karma* system and composed› for *Gommaṭa* ‹i. e. Cāmuṇḍarāya [Jadhav 1999, pp. 19-24]›’) is Nemicandra’s celebrated treatise. It is composed in Prakrit language into two sections. One is *Jīvakāṇḍa* (‘Section regarding soul’) and the other is *Karmakāṇḍa* (‘Section regarding *karma*’). These two parts together offer a synopsis of spiritual stages and self-quest which are the well known Jaina ways of considering the soul (*jīva*).

At the time of discussion on carelessness (*pamāda*, Skt. *pramāda*) in the *Gommaṭasāra* (*Jīvakāṇḍa*) (Jaini 1927a, vv. 34-42, pp. 27-31), he sets forth a combinatorics, for n=5, pertaining to ordered tuples of n elements formed by selecting only one element at each time from each one of the n finite ordered sets. *Saṅkhyā* (‹total› number ‹of the tuples›), *prastāra* (spreading ‹the elements of those sets in order to obtain tuples›), *parivartana* (rotation ‹of the elements of those sets while the tuples being formed›), *naṣṭa* (‹the structure of a› lost ‹tuple›) and *uddiṣṭa* (‹the position of a› mentioned ‹tuple›) are five tools of that combinatorics.

Those who have explained that combinatorics for the readers of the *Gommaṭasāra* (*Jīvakāṇḍa*) in the traditional manner are its commentators and modern editors. Keśavavarṇī (1359 AD) (Upadhye and Shastri 1997, pp. 35-42 and pp. 63-74) and Ṭoḍaramala (1720 -1767 AD) (Jain, G. L. and Jain, S. L. 1919) are two of its commentators. Khūbacandra Jaina (1916 AD) (Jain, Khūbacandra 1997, pp. 4-11 and pp. 26-30), G. L. Jain and S. L. Jain (Jain, G. L. and Jain, S. L. 1919), J. L. Jaini (Jaini 1927a, vv. 34-42, pp. 27-31), A. N. Upadhye and K. C. Shastri (Upadhye and Shastri 1997) are some of its editors. Jaini’s edition is in English while those of the others are in Hindi.

Bibhutibhusan Datta (1888 -1958 AD) is the first historian of mathematics who made efforts, in his paper titled “Mathematics of Nemicandra” (Datta 1935, pp. 38-44; also see Datta and Singh 1992, pp. 245-248) appeared in 1935, to explain that combinatorics to the scholars of history of mathematics. But two fallacies occur in his article. One is that *parivartana* (rotation) is the second kind of *prastāra* (spreading) (Datta 1935, p. 39; also see Datta and Singh 1992, p. 245). The other is that spreading can be formed by 2 (a!) alternative ways (Datta 1935, p. 39), where a is the number of the ordered non-singletons and $a! = a \times (a - 1) \times ... \times 3 \times 2 \times 1$. These fallacies, especially the former one, were brought to our notice by an anonymous scholar who translated, in 1936, Datta’s above paper into Hindi from English (Datta 1936, p. 55). Further, without any elaboration L. C. Jain too refers to the above combinatorics (Jain, L. C. 1982, p. 40). He informs us that the same combinatorics was applied in the *Śrī Prastāra Ratnāvalī* of Muni Ratnacandra “for finding out the combinations in relation to carelessness (*pramāda*) and character

[5] The reasons to regard Śrīdhara to be of Jaina faith may be seen in the followings. Sastri, N. C. Jain 1947, pp. 31-42; Sastri, N. C. Jain 1974, pp. 187-195; Jain, Anupam and Jain, Jaychand 1988, pp. 49-54.

(*śīla*)" (Jain, L. C. 1982, pp. 40-41).[6] L. C. Jain is said to have completed an Indian National Science Academy project on the *Prastāra Ratnāvalī*. So far the present authors could approach neither the *Śrī Prastāra Ratnāvalī*[7] nor L. C. Jain's above project report on it. Even the date of Muni Ratnacandra is not known to them.

"Some of his <i.e., Nemicandra's> results in combinations we have not so far found in any Hindu work before the fourteenth century of the Christian era (Datta 1935, p. 27)." This remark made by Datta on the above combinatorics is noteworthy although he did it without citing any detail.

The first of the present authors has published, in Hindi, two papers (Jadhav 1996, pp. 45-51, and Jadhav 1997, pp. 19-34) on that combinatorics. One more paper has been published. It is in English and is under the joint authorship of the present authors (Jadhav and Jain 1997, pp. 103-109). Three of those papers were with different purposes. Among them the main one was to enhance the understanding of that combinatorics.

As far as known to the present authors, a few materials, which include Jaini's edition and Datta's above paper, on that combinatorics are available in English. On the other hand, a number of papers are available on combinatorics found in ancient and medieval Indian treatises. Few of them, except the ones referred to above, contain a mention of the combinatorics found in the *Gommaṭasāra* (*Jīvakāṇḍa*). For this reason the first and foremost purpose of the present paper is to provide that combinatorics to English readers. This paper is also aimed at understanding that combinatorics on its own terms as well as on modern mathematical terms. The paper will view its tools in a historical perspective in the light of a brief survey of Indian combinatorics. It is also aimed at viewing that combinatorics in a contextual perspective.

To fulfill the purpose of understanding that combinatorics on modern mathematical terms, the terms such as ordered set, ordered pair, ordered tuple have been employed in this paper from the beginning.

In the above two Hindi papers a single-suffix notation was used to express the elements of an ordered set. For the sake of generalization and a smooth linking of those five tools of combinatorics we shall follow a double-suffix notation in this paper for the same purpose. It does not mean that a notation of this sort was prevalent in Nemicandra's time.

2. A brief survey of Indian combinatorics up to the fourteenth century

Before we take in the combinatorics as found in the *Gommaṭasāra* (*Jīvakāṇḍa*), we would like to make a brief survey regarding Indian combinatorics up to the fourteenth century of the Christian era.

Permutations, combinations, and variations are enumerated among the major concepts of combinatorics. The number, ${}^hP_k$, of permutations of h objects taken k at a time is given by $h!/(h-k)!$ where $1 \leq k \leq h$. When $k = h$, we have simply $P(h)=h!$. Unlike permutations, combinations do not have any order in their formation. The number, ${}^hC_k$, of combinations of h objects taken k at a time is given by $h!/k!(h-k)!$ where $k \leq h$. Variations combine features of combinations and permutations. They are permutations of combinations. The number, ${}^hV_k$, of variations of h objects taken k at a time is given by ${}^hC_k.P(k)$. The same with repetition is given by h^k where either $1 \leq k \leq h$ or $1 \leq h \leq k$.

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[6] The following information may be useful for some future study. *Śrī Prastāra Ratnāvalī*, by Muni Ratnacandra. Bikaner: *Vikrama Saṃvat* 1981. See Jain, L. C. 1982, foot note number 95 on p. 40.

[7] Jain, L. C. 1994. See its back cover page where this information is referred to in L. C. Jain's biographical sketch.

Indian interest in combinatorics initiated in a very early age. In the *Suśruta Saṃhitā* (600 BC), a treatise on medicine, *Suśruta* has correctly given the total number of various kinds of taste, ${}^hC_1 + {}^hC_2 + {}^hC_3 + {}^hC_4 + {}^hC_5 + {}^hC_6$, to be equal to 63 where h(=6) is the number of basic tastes. He is supposed to have arrived at the result by listing all the combinations in their entirety [Chakravarti 1932, p. 79; also see Datta and Singh 1992, p. 232]. The same numerical result is referred to in the *Bhagavatī Sūtra* (300 BC or earlier), a treatise from the canonical class of the Jaina school of Indian mathematics, for total combinations, where h(=6) is the number of aspects about food assimilated by hellish beings (Shah, R. S. 2008, pp. 11-18). It also discusses as to in how many ways k souls can enter h(=7) hells. The results correctly calculated using ${}^{h+k-1}C_k$ for from $k = 1$ up to $k = 10$ are 7, 28, 84, 210, 462, 924, 1716, 3003, 5005, and 8008 respectively (Shah, R. S. 2008, pp. 17-20).

In the *Chandaḥ-Sūtra*, a treatise written on Sanskrit prosody[8] before 200 BC, Piṅgala describes a scheme of forming all the variations with two sorts of syllables, $h = 2$, one *guru* (long, *g*) and the other *laghu* (short, *l*), to produce required rhythms. He states that the number of the trisyllabic, $k = 3$, variations is eight (Datta and Singh 1992, pp. 242-243; also see Biggs 1979, 130 and Sarma 2003, p. 128). We are able to arrive at this result using h^k. Those variations are *ggg, lgg, glg, llg, ggl, lgl, gll, lll*.

The *Anuyogadvāra Sūtra* (3rd century AD), a treatise from the canonical class of the Jaina school of Indian mathematics, states that the number of all possible permutations of 6 substances excluding the direct order and the reverse order, $P(6)$-2, is 718 (Chakravarti 1932, p. 86; also see Shah, R. S. 2007, p. 97).

The method applied by Varāhamihira (c.485-587 AD) for finding the numerical values of ${}^hC_k$ for various values of h and k was peculiar (Datta and Singh 1992, pp. 233-234; and Gupta 1992, pp. 45-47) from the one we know today or known to the rest of the Indian mathematicians in past. The rule for finding the numerical value of ${}^hC_k$ for various values of h and k referred to in general terms by Śrīdhara (c. 799 AD) is $(h/1).((h - 1)/2)...((h - k + 1)/k)$ (Shukla 1959, v. 72, p. 97 and its English translation, p. 58) whereas by Mahāvīra (c. 850 AD) is $(h.(h - 1)...(h - k + 1)/(1.2.3...k)$ (Padmavathamma 2000, v. 6.218, p. 348). Both of them belonged to the exclusive class of the Jaina school of Indian mathematics.

Śīlāṅka Sūri (c. 862 AD), who belonged to the canonical class of the Jaina school of Indian mathematics, has quoted three rules, two of them in Sanskrit and the third one in Prakrit, that too in Ardha Māgadhī Prakrit, from some earlier sources and explained them. One pertains to *bheda-saṃkhyā-parijñānāya* (for finding the total number of permutations) and the other two to *prastārānayanopāya* (for finding their actual spreading) (Datta 1929, pp. 134-135).

In the seventh chapter of the *Chando'nuśāsana*, a treatise written in Prakrit prosody, Hemacandra (1088-1172 AD), a scholar of Jaina faith, enumerates six *pratyayas* (combinatorial tools, literal meaning: ideas or cognitions) regarding variations with syllables. They are *prastāra* (spreading ‹to obtain forms›), *naṣṭa* (literal meaning: lost or erased, finding the structure of a form of which position or serial number is given), *uddiṣṭa* (literal meaning: mentioned or particularized, finding the position or serial number of a form of which structure is given), *sarvaikādi-ga-la-kriyā* (i. e., *sarvaikādi-guru-laghu-kriyā,* calculating the number of forms with long syllables and short syllables), *saṅkhyā* (‹total› number ‹of the forms obtained after complete spreading›), and *adhvayoga* (determining the amount of space required for

[8] Meter is the regular and rhythmic arrangement of syllables as per particular patterns. Its theory is prosody.

writing the entire list of the forms) (Alsdorf 1991, pp. 20-31). Their knowledge was required not only for a good understanding of prosody but also for its progress.

The first three combinatorial tools are found in the same sequence (Alsdorf 1991, p. 32) in the *Chandaḥ-Sūtra* of Piṅgala (200 BC) without any labels assigned to them (Sastri, Asoke Chatterjee 1987, eighth chapter in the text of the *Piṅgala-Chandaḥ-Sūtra*, pp. 11-12). The rest of the combinatorial tools may seem to have been developed and elaborated during the period after Piṅgala to Hemacandra but Jayant Shah shows that those six combinatorial tools were traditionally identified by the prosodists right from Piṅgala (Shah, Jayant 2013, pp. 1-53). They are also referred to by Kedāra (c. 1100 AD) in the *Vṛttaratnākara* with the reduction of the title of the fourth combinatorial tool to *la-ga-kriyā* (Alsdorf 1991, p. 31). We are also able to see them in the *Mṛtasañjīvanī*, a commentary on the *Chandaḥ-Sūtra* of Piṅgala by Halāyudha (c. 950 AD). Sometime between 200 BC and 100 AD those combinatorial tools excluding *adhvayoga* were dealt on the subject of the prosody in the *Nāṭyaśāstra* of Bharata (Shah, Jayant 2013, pp. 1-53).

Two of the prosodists that India produced between Piṅgala and Hemacandra are Virahāṅka (somewhere between 600 and 800 AD) and Gopāla (prior to 1135 AD). Not only the concept of the sequence of Fibonacci numbers was known to these two prosodists and Hemacandra but also the rule for their formation was known to three of them (Singh, Parmanand 1985, pp. 230-244).

Jayadeva (before 900 AD) was a well known writer on Sanskrit prosody. He seems to Parmanand Singh to have been an author of Jaina faith (Singh, Parmanand 1988, pp.43-44). He has dealt six of the *pratyayas* in his *Jayadevacchandaḥ* (Shah, Jayant 2013, pp. 27-53; also see Singh, Parmanand 1988, pp.43-48).

Indian astronomer and mathematician Brahmagupta (628 AD) as well is said to have described *prastāra*, *saṅkhyā*, *naṣṭa*, and *uddiṣṭa* in the chapter on prosody in the *Brāhma-sphuṭa-siddhānta* (Singh, Parmanand 2001, p. 38). Mahāvīra (c. 850 AD), belonging to the exclusive class of the Jaina school of Indian mathematics, provides rules, in relation to the prosody, for finding *saṅkhyā*, *prastāra*, *naṣṭa*, *uddiṣṭa*, *laghu-guru-kriyā*, and *adhvayoga* (Padmavathamma 2000, vv. 6.333½-6.336½, p. 420). The chapters 26 and 34 of the *Līlāvatī* of Bhāskara II (1114 - 1200 AD) are devoted to combination and concatenation respectively (Patwardhan et al. 2001, vv. 118-122, pp. 101-104 and vv. 268-279, 177-182).

In the *Saṅgītaratnākara* of Sārṅgadeva (c. 1225 AD), the combinatorial tools are systematically dealt with, both in connection with *tānas* (patterns of musical phrases) and *tālas* (patterns of musical rhythms) (Sridharan et al. 2010, pp. 55-112).

Aṅka Pāśa (concatenation of numbers) is the name of the chapter 13 of the *Gaṇitakaumudī* of Nārāyaṇa Paṇḍita (c. 1356 AD). In it, he applies *prastāra*, *saṅkhyā*, *naṣṭa*, *uddiṣṭa*, and *la-ga-kriyā* to combinatorics that includes permutations, combinations, partitions, and sequences. He also describes some other combinatorial tools such as *ūrdhva-paṅkti-yoga* (the sum of digits at a particular place in all ‹permutations›), *āvṛtti* (the number of ‹permutations› having a particular digit at a particular place), *sarva-yoga* (the sum of all ‹permutations›), and *aṅka-pāta* (total number of digits in all ‹permutations›). He extends most of them to the partitions of a number as well (Singh, Parmanand 2001, pp. 18-78; also see Kusuba 1993).

3. The context of the combinatorics found in the *Gommaṭasāra* (*Jīvakāṇḍa*)

The *Jīvakāṇḍa* is the first part of the *Gommaṭasāra*. It deals with the soul in 734 *gāthās*

(i. e., verses). The topics that are introduced in it into 20 chapters (*parūbaṇā*, skt. *prarūpaṇā*) for discussion are seven, namely (1) *guṇa* (Skt. *guṇa* ‹*sthāna*›, ‹14› qualitative ‹stages of spiritual development›), (2) *jīva* (Skt. *jīva* ‹*samāsa*›, ‹14 kinds of› soul-‹class›), (3) *pajjattī* (Skt. *paryāpti*, ‹6 kinds of› completion or capacity to develop), (4) *prāṇa* (‹10 kinds of› vitality), (5) *saṇṇā* (Skt. *sajñā*, ‹4 kinds of› impulse or animate feeling), (6) *maggaṇā* (Skt. *mārgaṇā*, ‹14 characteristics of› soul-quest), and (7) *uvaoga* (Skt. *upayoga*, ‹12 kinds of› conscious attentiveness) (Jaini 1927a, v. 2 and its translation, p. 3). Of them *maggaṇā* alone forms 14 chapters while one chapter each is assigned to the others. There are two appendices. One is *antarbhāva* (inner-activities ‹of *jīva samāsa*, *paryāpti*, *prāṇa*, *sajnā*, *upayoga* with reference to *guṇa sthāna* and *mārgaṇā*›) (Jain, Khūbacandra 1997, p. 298) and the other is *ālāva* (Skt. *ālāpa*, further distinctions) (Jaini 1927a, v. 706, p. 338).

The chapter on *guṇa sthāna* (qualitative stages ‹of spiritual development›) runs into 62 verses from verse 8 to verse 69. Those 14 qualitative stages of spiritual development are due to *moha*[9] (delusion) and *joga* (Skt. *yoga*, vibratory activity, of the soul, which causes inflow of *karma* matter into the soul) (Jaini 1927a, v. 3 first hemistich, p. 3). Those stages are (1) *miccho* (Skt. *mithyātatva*, ‹the soul having› delusion ‹and hence having wrong belief›), (2) *sāsaṇa* (Skt. *sāsana* i.e., *sāsādana*, downfall ‹from right belief to wrong belief›), (3) *missa* (Skt. *miśra*, ‹stage where right belief is› mixed ‹with wrong belief›), (4) *aviradasammo* (Skt. *aviratasamyaktva*, vow-less right belief), (5) *desa-virada* (Skt. *deśa-virata* i.e., *saṃyatāsaṃyata*, partial vow), (6) *pamatta virada* (Skt. *pramatta virata*, imperfect vow), (7) *idara* (Skt. *itara* i. e., *apramatta virata*, perfect vow), (8) *apuvva* (Skt. *apūrva* ‹*karaṇa*›, new-‹thought-activity›), (9) *aṇiyammhi* (Skt. *anivṛtti* i. e. *anivṛtti* ‹*karaṇa*›, advanced-‹thought-activity›), (10) *suhama* (Skt. *sūkṣma* ‹*sāmparāya*›, slightest ‹delusion›), (11) *uvasanta* (Skt. *upaśānta* ‹*moha*›, ‹stage of› suppressed ‹delusion›), (12), *khīṇa moha* (Skt. *kṣīṇa moha*, ‹stage of› destroyed delusion), (13) *sajogakevalijiṇa* (Skt. *sayogakevalijina*, vibratory-omniscient-conqueror), (14) *ajogī* (Skt. *ayogī* ‹*kevalijina*›, non-vibratory-‹omniscient-conqueror›).

What is described in those 14 stages one after another is the purely-inner progress of the soul from delusion to non-vibratory-omniscience. After the last stage, the soul becomes *siddha* (liberated) [Jaini 1927a, vv. 9-10, pp. 8-9; also see Upadhye and Shastri 1997, vv. 9-10, pp. 40-42]. The combinatorics that we are going to discuss in this paper is of importance to the sixth stage only.

In the sixth stage i.e., the stage of imperfect vow, owing to the destruction and suppression of total-vow-preventing passion, there is perfect control but owing to the operation (*udaya*) of *saṃjalaṇaṇokasāya* (Skt. *saṅjvalananokaṣāya*, perfect-conduct-preventing passion and quasi-passion) there is *pamāda* (Skt. *pramāda*, carelessness or heedlessness) that produces impurity (*mala*) in control (Jaini 1927a, v. 32, p. 25; also see Jain, Khūbacandra 1997, v. 32, p. 24 and Upadhye and Shastri 1997, v. 32, pp. 60-61).

[9] There are 28 *mohas* (delusions). They are classified into *darśana-moha* (right-belief-delusion) and *cāritta-moha* (right-conduct-delusion). *Mithyātatva* or *mithyātva* (wrong-belief), samyag*mithyātva* or simply *miśra* (a fusion of right-belief and wrong-belief) and *samyaktva prakṛti* (right-belief clouded by slight wrong-belief) are three *darśana-mohas* (right-belief-delusions). *Anantānubandhi kaṣāya* (error-feeding passion), *apratyākhyānāvaranīya kaṣāya* (partial-vow-preventing passion), *pratyākhyānāvaranīya kaṣāya* (total-vow-preventing passion), *saṅjvalana kaṣāya* (perfect-conduct-preventing passion) and *nokaṣāya* (quasi-passion i. e., slight or minor passion) are five subdivisions of *cāritta-moha* (right-conduct-delusion). Each of the first four contains *krodha* (anger), *māna* (pride), *māyā* (deceit) and *lobha* (greed). *Hāsya* (laughter), *rati* (indulgence), *arati* (ennui), *śoka* (sorrow), *bhaya* (fear), *jugupsā* (disgust), *strī-veda* (feminine inclination), *puṃ-veda* (masculine inclination) and *napuṃsaka-veda* (neither feminine nor masculine sexual inclination) are nine *nokaṣāyas* (quasi-passions). See Jaini 1918, p. 132f. Here common Sanskrit terms are used instead of the original Prakrit.

Let us denote the number of the elements in the ith finite ordered set, P_i, by m_i where $i = 1, 2, 3, ..., n$. Further denote the jth element in P_i by p_{ij} where $j = 1, 2, 3, ..., m_i$.

Nemicandra enumerates fifteen sorts of *pamāda* (Skt. *pramāda*, carelessness) into five ordered sets. 4 ($=m_1$) of those fifteen ones form the ordered set of *vikahā* (Skt. *vikathā,* censurable gossip, P_1). Another 4 ($=m_2$) of them form the ordered set of *kasāya* (Skt. *kaṣāya*, passion, P_2). 5 ($=m_3$) of the leftovers form the ordered set of *iṃdiya* (Skt. *indriya*, sense organs, P_3). 1 ($=m_4$) of the remaining two form singleton of *ṇiddā* (Skt. *nidrā*, sleep, P_4) and the remaining 1 ($=m_5$) does that of *paṇa^a* (Skt. *praṇaya*, attachment, P_5) (Jaini 1927a, v. 34, p. 27).

For the sake of convenience, we can enumerate those five ordered sets as follows:

$$P_1 = \{p_{11}, p_{12}, p_{13}, p_{14}\}$$

where

p_{11} = *strī kathā* (gossip regarding fair sex),

p_{12} = *bhojana kathā* (gossip regarding food),

p_{13} = *rāṣṭra kathā* (seditious gossip or politics), and

p_{14} = *avanipāla kathā* (gossip pertaining to king or gossip regarding scandal);

$$P_2 = \{p_{21}, p_{22}, p_{23}, p_{24}\}$$

where

p_{21}= *krodha* (anger),

p_{22}= *māna* (pride),

p_{23}= *māyā* (deceit), and

p_{24}= *lobha* (greedy);

$$P_3 = \{p_{31}, p_{32}, p_{33}, p_{34}, p_{35}\}$$

where

p_{31}= *sparśana* (tactile organ),

p_{32}= *rasanā* (tongue),

p_{33}= *ghrāṇa* (nose),

p_{34}= *cakṣu* (eye) and

p_{35}= *śrotra* (ear);

$$P_4 = \{p_{41}\}$$

where p_{41}= *nidrā* (sleep); and

$$P_5 = \{p_{51}\}$$

where p_{51}= *praṇaya* (attachment).[10]

P_4 and P_5 being singletons, neither they are mentioned by him nor their elements, however they are the integral parts of the tuples, are discussed in any dealing.

In fact, there are sixty three sorts of *pamāda* (Skt. *pramāda*, carelessness) including $m_1 = 25$, $m_2 = 25$, $m_3 = 6$, $m_4 = 5$, $m_5 = 2$ (Jaini 1927a, pp. 35-36; also see Jain, Khūbacandra 1997, p. 26). Only 15 of them were taken by Nemicandra for this discussion.

4. Combinatorics as found in the *Gommaṭasāra* (*Jīvakāṇḍa*)

The combinatorics that we are to understand in this section runs into 10 verses of the *Gommaṭasāra* (*Jīvakāṇḍa*) from verse 35 to verse 44 but our study in this paper would be confined to only 8 verses i. e., from verse 35 to verse 42. Nemicandra begins with the following verse.

संखा तह पत्थारो परियट्टण णट्ठ तह समुद्दिट्ठं।

एदे पंच पयारा पमदसमुक्कित्तणे णेया।।

saṃkhā taha patthāro pariyaṭṭaṇa ṇaṭṭha taha samuddiṭṭhaṃ |

ede paṃca payārā pamadasamukkittaṇe ṇeyā ||

(Jaini 1927a, v. 35, p. 28)

"*Saṃkhā* (Skt. *saṅkhyā*, ‹total› number ‹of the tuples formed by selecting only one element at each

[10] See Jain, Khūbacandra 1997, p. 25 (as enumerated by the editor below v. 34). Here common Sanskrit terms are used instead of the original Prakrit.

time from each one of the n finite ordered sets›), *patthāra* (Skt. *prastāra*, spreading ‹the elements of those sets in order to obtain tuples›), *pariyaṭṭaṇa* (Skt. *parivartana*, rotation ‹of the elements of those sets while the tuples being formed›), *ṇaṭṭha* (Skt. *naṣṭa*, ‹finding the structure of a› lost ‹tuple›) and *samuddiṭṭha* (Skt. *samuddiṣṭa*, simply: *uddiṣṭa*, ‹finding the position of a› mentioned ‹tuple›), these five types ‹of tools› should be known in a description of *pamada* (Skt. *pramāda*, carelessness)."

In this combinatorics, tuples of n elements are formed by selecting only one element at each time from each one of the n finite ordered sets. We have already noticed in the latter part of the section three that Nemicandra took $n = 5$.

4.1. *Saṅkhyā* (Number)

If the total number of all possible tuples that can be formed by taking only one element from each one of n finite ordered sets be denoted by S, then

$$S = \prod_{i=1}^{i=n} {}^{m_i}C_1.$$

It gives the method for computing the total number of all tuples without constructing them. He sets it in the rule as stated below:

सव्वे पि पुव्वभंगा उवरिमभंगेसु एक्कमेक्केसु।

भेलंति त्ति य कमसो गुणिदे उप्पज्जदे संखा।।

savve pi puvvabhaṃgā uvarimabhaṃgesu ekkamekkesu |

bhelaṃti tti ya kamaso guṇide uppajjade saṃkhā ||

(Jaini 1927a, v. 36, p. 28; also see Jain, Khūbacandra 1997, v. 36, p. 27)

"‹Since› all the preceding combinations (*puvvabhaṃgas*, Skt. *pūrvabhaṅgas*) are combined one by one with the succeeding combinations (*uvarimabhaṃgas*, Skt. *uparimabhaṅgas*), by the successive multiplication ‹of their numbers› is produced *saṃkhā* (Skt. *saṅkhyā*, ‹total› number ‹of tuples›, S)."

Illustration

$$S = \prod_{i=1}^{i=5} {}^{m_i}C_1 = 4\times 4\times 5\times 1\times 1 = 80.$$

4.2. *Prastāra* (Spreading)

He refers to two kinds of spreading. He states the rule for each of them. They can be named the first kind of spreading and the second kind of spreading. Using them we construct all possible tuples.

4.2.1. First kind of spreading

He describes the first kind of spreading as follows:

पढमं पमदपमाणं कमेण णिक्खिविय उवरिमाणं च।

पिंडं पडि एक्केकं णिक्खित्ते होदि पत्थारो।।

paḍhamaṃ pamadapamāṇaṃ kameṇa ṇikkhiviya uvarimāṇaṃ ca|

piṃḍaṃ paḍi ekkekaṃ ṇikkhitte hodi patthāro ||

(Jaini 1927a, v. 37, p. 29)

"‹Distribute› the number (*pamāṇa*, Skt. *pramāṇa*) of ‹the elements of› the first ‹ordered set of› carelessness (*paḍhama pamada*, Skt. *prathama pramāda*, P_1) ‹into unities›; put down ‹its elements› in order and then the succeeding (*uvarima*, Skt. *uparima*) ‹ordered set of carelessness over each of them›, one by one, in the form of a set (*piṃḍa*, literal meaning: accumulation). When done ‹in the same manner›, there happens to be the spreading (*patthāra,* Skt. *prastāra*)."

Generalization

"Distribute each one of the elements of P_1 respectively. Then place P_2 over each one of those elements. In this way we would get the ordered set of ordered pairs. In order to obtain the ordered tuples containing three elements, place P_3 over each one of those pairs. Do this process till the ordered tuples of n elements are obtained."

Illustration

Distribute each one of the elements of P_1 respectively.

$$p_{11}, p_{12}, p_{13}, p_{14}.$$

Place P_2 over each one of the above elements.

$$p_{11}^{P_2}, p_{12}^{P_2}, p_{13}^{P_2}, p_{14}^{P_2}$$

or
$$p_{11}p_{21}, p_{11}p_{22}, p_{11}p_{23}, p_{11}p_{24},$$
$$p_{12}p_{21}, p_{12}p_{22}, p_{12}p_{23}, p_{12}p_{24},$$
$$p_{13}p_{21}, p_{13}p_{22}, p_{13}p_{23}, p_{13}p_{24},$$
$$p_{14}p_{21}, p_{14}p_{22}, p_{14}p_{23}, p_{14}p_{24}.$$

If P_3 is placed over each one of the above pairs, the tuples, each containing three elements, will be obtained. Placing P_4 over each one of those tuples and thereafter placing P_5 in the same way, we get

$$\underset{1}{\underline{p_{11}p_{21}p_{31}p_{41}p_{51}}}, \underset{2}{\underline{p_{11}p_{21}p_{32}p_{41}p_{51}}}, \underset{3}{\underline{p_{11}p_{21}p_{33}p_{41}p_{51}}}, \underset{4}{\underline{p_{11}p_{21}p_{34}p_{41}p_{51}}}, \underset{5}{\underline{p_{11}p_{21}p_{35}p_{41}p_{51}}},$$
$$\underset{6}{\underline{p_{11}p_{22}p_{31}p_{41}p_{51}}}, \underset{7}{\underline{p_{11}p_{22}p_{32}p_{41}p_{51}}}, \underset{8}{\underline{p_{11}p_{22}p_{33}p_{41}p_{51}}}, \underset{9}{\underline{p_{11}p_{22}p_{34}p_{41}p_{51}}}, \underset{10}{\underline{p_{11}p_{22}p_{35}p_{41}p_{51}}},$$
$$\underset{11}{\underline{p_{11}p_{23}p_{31}p_{41}p_{51}}}, \underset{12}{\underline{p_{11}p_{23}p_{32}p_{41}p_{51}}}, \underset{13}{\underline{p_{11}p_{23}p_{33}p_{41}p_{51}}}, \underset{14}{\underline{p_{11}p_{23}p_{34}p_{41}p_{51}}}, \underset{15}{\underline{p_{11}p_{23}p_{35}p_{41}p_{51}}},$$
$$\underset{16}{\underline{p_{11}p_{24}p_{31}p_{41}p_{51}}}, \underset{17}{\underline{p_{11}p_{24}p_{32}p_{41}p_{51}}}, \underset{18}{\underline{p_{11}p_{24}p_{33}p_{41}p_{51}}}, \underset{19}{\underline{p_{11}p_{24}p_{34}p_{41}p_{51}}}, \underset{20}{\underline{p_{11}p_{24}p_{35}p_{41}p_{51}}},$$
$$\underset{21}{\underline{p_{12}p_{21}p_{31}p_{41}p_{51}}}, \underset{22}{\underline{p_{12}p_{21}p_{32}p_{41}p_{51}}}, \ldots, \underset{39}{\underline{p_{12}p_{24}p_{34}p_{41}p_{51}}}, \underset{40}{\underline{p_{12}p_{24}p_{35}p_{41}p_{51}}},$$
$$\underset{41}{\underline{p_{13}p_{21}p_{31}p_{41}p_{51}}}, \underset{42}{\underline{p_{13}p_{21}p_{32}p_{41}p_{51}}}, \ldots, \underset{59}{\underline{p_{13}p_{24}p_{34}p_{41}p_{51}}}, \underset{60}{\underline{p_{13}p_{24}p_{35}p_{41}p_{51}}},$$
$$\underset{61}{\underline{p_{14}p_{21}p_{31}p_{41}p_{51}}}, \underset{62}{\underline{p_{14}p_{21}p_{32}p_{41}p_{51}}}, \ldots, \underset{79}{\underline{p_{14}p_{24}p_{34}p_{41}p_{51}}}, \underset{80}{\underline{p_{14}p_{24}p_{35}p_{41}p_{51}}}$$

$p_{11}p_{21}p_{31}p_{41}p_{51}$ is stated in the *Jīvatattvapradīpikā*, a Sanskrit commentary of the fourteenth century on the *Gommaṭasāra* (*Jīvakāṇḍa*), as follows:

> *strīkathālāpī krodhī sparśanendrivaśaṅgataḥ nidrāluḥ snehavān iti prathamapramādālāpaḥ*|
> [Upadhye and Shastri 1997, p. 67]

> "‹One who is affected by› the first (*prathama*) tuple (*ālāpa*) of carelessness (*pramāda*) is interested in talking on the topic of fair sex (*strīkathālāpī*, p_{11}), angry ‹in passion› (*krodhī*, p_{21}), surrendered to his tactile organ (*sparśanendrivaśaṅgata*,p_{31}), indolent (*nidrā*, p_{41}) and attached to accessory (*snehavān* i. e., *praṇayavān*, p_{51})."

4.2.2. Second kind of spreading

He describes the second kind of spreading as follows:

> णिक्खित्तु विदियमेत्तं पढमं तस्सुवरि विदियमेक्केक्कं।
> पिंडं पडि णिक्खेओ एवं सव्वत्थ कायव्वो।।
>
> *ṇikkhittu vidiyamettaṃ paḍhamaṃ tassuvari vidiyamekkekkaṃ* |
> *piṃḍaḥ paḍi ṇikkheo evaṃ savvattha kāyavvo* ||
>
> (Jaini 1927a, v. 38, p. 29; also see Jain, Khūbacandra 1997, v. 38, p. 27)

> "Place the first (*paḍhama*, Skt. *prathama*) ‹ordered set of carelessness, P_1,› in the form of a set (*piṇḍa*, Skt. *piṇḍa*), as many times as the number of ‹the elements in› the second (*vidiya*, Skt. *dvitīya*) ‹ordered set of carelessness, P_2›; on the top of each set (*piṇḍa*, Skt. *piṇḍa*), place one by one ‹the elements of› the second (*vidiya*, Skt. *dvitīya*). And this is to be properly done with all ‹further sets›."

Generalization

"Place P_1 as many times as there are the elements in P_2. On the top of each P_1, place one by one the elements of P_2. This gives the ordered set of ordered pairs. Place this set as many times as there are the elements in P_3. Place, on the top of each, one by one the elements of P_3. This is to be properly done till the ordered tuples containing n elements are obtained."

Illustration

Place P_1 as many times as there are the elements in P_2.

$$P_1, P_1, P_1, P_1 \qquad .$$

Place, on the top of each P_1, one by one the elements in P_2.

$$P_1^{p_{21}}, P_1^{p_{22}}, P_1^{p_{23}}, P_1^{p_{24}}$$

or
$$p_{11}p_{21}, p_{12}p_{21}, p_{13}p_{21}, p_{14}p_{21},$$
$$p_{11}p_{22}, p_{12}p_{22}, p_{13}p_{22}, p_{14}p_{22},$$
$$p_{11}p_{23}, p_{12}p_{23}, p_{13}p_{23}, p_{14}p_{23},$$

$$p_{11}p_{24}, p_{12}p_{24}, p_{13}p_{24}, p_{14}p_{24} \quad .$$

If this set is placed as many times as there are the elements in P_3 and on the top of each of them are placed one by one the elements of P_3, the tuples, each containing three elements, will be obtained. Performing the same operation further with P_4 and then with P_5, we get

$$\frac{p_{11}p_{21}p_{31}p_{41}p_{51}}{1}, \frac{p_{12}p_{21}p_{31}p_{41}p_{51}}{2}, \frac{p_{13}p_{21}p_{31}p_{41}p_{51}}{3}, \frac{p_{14}p_{21}p_{31}p_{41}p_{51}}{4},$$

$$\frac{p_{11}p_{22}p_{31}p_{41}p_{51}}{5}, \frac{p_{12}p_{22}p_{31}p_{41}p_{51}}{6}, \frac{p_{13}p_{22}p_{31}p_{41}p_{51}}{7}, \frac{p_{14}p_{22}p_{31}p_{41}p_{51}}{8},$$

$$\frac{p_{11}p_{23}p_{31}p_{41}p_{51}}{9}, \frac{p_{12}p_{23}p_{31}p_{41}p_{51}}{10}, \frac{p_{13}p_{23}p_{31}p_{41}p_{51}}{11}, \frac{p_{14}p_{23}p_{31}p_{41}p_{51}}{12},$$

$$\frac{p_{11}p_{24}p_{31}p_{41}p_{51}}{13}, \frac{p_{12}p_{24}p_{31}p_{41}p_{51}}{14}, \frac{p_{13}p_{24}p_{31}p_{41}p_{51}}{15}, \frac{p_{14}p_{24}p_{31}p_{41}p_{51}}{16},$$

$$\frac{p_{11}p_{21}p_{32}p_{41}p_{51}}{17}, \frac{p_{12}p_{21}p_{32}p_{41}p_{51}}{18}, \ldots, \frac{p_{13}p_{24}p_{32}p_{41}p_{51}}{31}, \frac{p_{14}p_{24}p_{32}p_{41}p_{51}}{32},$$

$$\frac{p_{11}p_{21}p_{33}p_{41}p_{51}}{33}, \frac{p_{12}p_{21}p_{33}p_{41}p_{51}}{34}, \ldots, \frac{p_{13}p_{24}p_{33}p_{41}p_{51}}{47}, \frac{p_{14}p_{24}p_{33}p_{41}p_{51}}{48},$$

$$\frac{p_{11}p_{21}p_{34}p_{41}p_{51}}{49}, \frac{p_{12}p_{21}p_{34}p_{41}p_{51}}{50}, \ldots, \frac{p_{13}p_{24}p_{34}p_{41}p_{51}}{63}, \frac{p_{14}p_{24}p_{34}p_{41}p_{51}}{64},$$

$$\frac{p_{11}p_{21}p_{35}p_{41}p_{51}}{65}, \frac{p_{12}p_{21}p_{35}p_{41}p_{51}}{66}, \ldots, \frac{p_{13}p_{24}p_{35}p_{41}p_{51}}{79}, \frac{p_{14}p_{24}p_{35}p_{41}p_{51}}{80}.$$

Example of the person affected by $p_{13}p_{24}p_{31}p_{41}p_{51}$, according to J. L. Jaini, may be something like this. "A man takes part in seditious talk and is prepared to commit sedition for money so that he may go and live with his wife in his village. Here the elements of seditious gossip ‹,p_{13}›, greed ‹,p_{24}›, and the object of touch ‹,p_{31},› are obvious. Sleep ‹,p_{41},› and attachment ‹,p_{51},› are the two elements common to all combinations ‹, i.e., tuples,› of *Pramāda*. They merely indicate that the soul who is affected by carelessness (*Pramāda*) is a mundane soul. Sleep and attachment to some extent or other are always present in all careless mundane souls, i. e., in all souls who are not in the seventh or higher spiritual stages [Jaini 1927a, p. 32]."

4.3. *Parivartana* (Rotation)

Before it is taken up, it may be noted that the terms *akṣa* (literal meaning: axle) and *akṣapada* refer to 'ordered set' and 'elements on the axle or in the ordered set' respectively.

4.3.1. Rotation with respect to the first spreading

तदियक्खो अंतगदो आदिगदे संकमेदि विदियक्खो।

दोण्णि वि गंतूणंतं आदिगदे संकमेदि पढमक्खो।।

tadiyakkho aṃtagado ādigade saṃkamedi vidiyakkho |

doṇṇi vi gaṃtūṇaṃtaṃ ādigade saṃkamedi paḍhamakkho ||

[Jaini 1927a, v. 39, p. 30; also see Jain, Khūbacandra 1997, v. 39, p. 28]

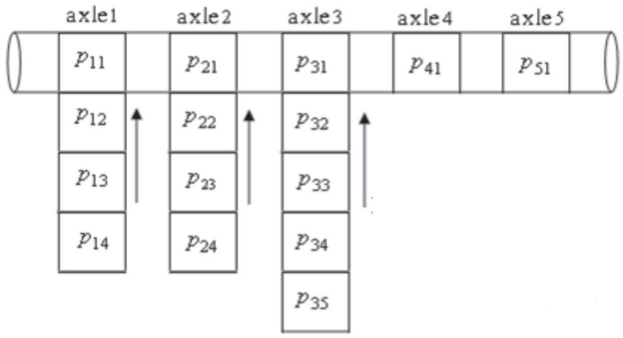
"When the third axle (*tadiyakkha*, Skt. *tṛtīyākṣa*) comes to the end ‹of its complete revolution› and goes again to its beginning, the second axle (*vidiyakkha*, Skt. *dvitīyākṣa*) makes ‹its elements (*padas*)› proceed. When both come to the end and go again to the beginning, the first axle (*paḍhamakkha*, Skt. *prathamākṣa*) makes ‹its elements› proceed."

The term *pada* (element) used above in the paraphrase would be corroborated from the verse referred to in the section 4.4.

Generalization

"When the *n*th ordered set comes to the end of its complete revolution and goes again to its beginning, the (n - 1)th ordered set makes its elements proceed. When both come to the end and go again to the beginning, the (n - 2)th ordered set makes its elements proceed. The process remains on till the last element of the first ordered set is exhausted."

See the following figure to realize rotation of the elements of an axle. Imagine that each vertical line of cells labelled 'axle *i*' on its top surrounds a cylinder and can rotate like a disk of a combination lock.



4.3.2. Rotation with respect to the second spreading

पढमक्खो अन्तगदो आदिगदे संकमेदि विदियक्खो।
दोण्णि वि गंतूणंतं आदिगदे संकमेदि तदियक्खो।।

paḍhamakkho antagado ādigade saṃkamedi vidiyakkho |

doṇṇi vi gaṃtūṇaṃtaṃ ādigade saṃkamedi tadiyakkho ||

(Jaini 1927a, v. 40, p. 31; also see Jain, Khūbacandra 1997, v. 40, p. 29)

"When the first axle (*paḍhamakkha*, Skt. *prathamākṣa*) comes to the end ‹of its complete revolution› and goes again to its beginning, the second axle (*vidiyakkha*, Skt. *dvitīyākṣa*) makes ‹its elements (*padas*)› proceed. When both come to the end and go again to the beginning, the third axle (*tadiyakkha*, Skt. *tṛtīyākṣa*) makes ‹its elements› proceed."

Generalization

"When the first ordered set comes to the end of its complete revolution and goes again to its beginning, the second ordered set makes its elements proceed. When both come to the end and go again to the beginning, the third ordered set makes its elements proceed. The process remains on till the last element of the *n*th ordered set is exhausted."

4.4. *Naṣṭa* (‹Finding the structure of a› lost ‹tuple›)

This tool finds out the structure of a tuple lost or erased by the questioner. In other words, given the serial number of a specified tuple with respect to a particular spreading, its structure has to be found out without constructing all tuples with respect to the spreading. Let us denote the structure of a tuple by J. It is already known to us that the number of p_{ij}'s contained in J is n. J is to be found by analyzing its serial number, A, in the given spreading. He gives the following rule to do so.

सगमाणेहिं विभत्ते सेसं लक्खित्तु जाण अक्खपदं।
लद्धे रुवं पक्खिव सुद्धे अंते ण रुवपक्खेवो।।

sagamāṇehiṃ vibhatte sesaṃ lakkhittu jāṇa akkhapadaṃ |

laddhe ruvaṃ pakkhiva suddhe aṃte ṇa ruvapakkhevo ||
(Jaini 1927a, v. 41, p. 31; also see Jain, Khūbacandra 1997, v. 41, p. 29)

"When ‹the given serial number is› divided by the number (*māṇa*, Skt. *māna*) of ‹the elements on› the respective (*saga*, Skt. *svaka*, literal meaning: own) ‹axle beginning with the last axle in the case of the first kind of spreading and beginning with the first axle in the case of the second kind of spreading›, know the remainder (*sesa*, Skt. *śeṣa*), ‹if any›, to be the element on the axle (*akkhapada*, Skt. *akṣapada*). Unity (*ruva*, Skt. *rūpa*) is added to the quotient (*laddha*, Skt. *labdha*) ‹if there is remainder›, but if there is no remainder, the last ‹element is taken› and there is no addition of unity. ‹Do the same for all axles.›"

Elaboration in general terms

"When A_{i-1} (where i = 1, 2, 3, ..., n) is divided by m_i (where $i = n$, ..., 3, 2, 1 when the kind of the given spreading is the first and i = 1, 2, 3, ..., n when the kind of the given spreading is the second), we have the quotient q_i (where i = 1, 2, 3, ..., n) and the remainder r_i (or more explicitly $r_i(A_{i-1}/m_i)$) (where i = 1, 2, 3, ..., n). r_i ($\neq 0$) determines $j(p_{ij})$ where $j(p_{ij})$ refers to j of the corresponding p_{ij} contained in P_i. If $r_i = 0$, it will mean that the last is the ordinal number of the element contained in P_i. $A_{i-1} = q_i + 1$ when $r_i \neq 0$. $A_{i-1} = q_i$ when $r_i = 0$. $A_0 = A$ for the case when i = 1."

4.4.1. Finding the structure of a tuple in the first kind of spreading

$$J(A)=\left(r_n\left(\frac{A_{n-1}}{m_1}\right), r_{n-1}\left(\frac{A_{n-2}}{m_2}\right), \ldots, r_2\left(\frac{A_1}{m_{n-1}}\right), r_1\left(\frac{A_0}{m_n}\right)\right)$$

Illustration: Find $J(18)$ when the first kind of spreading is applied.

Here, $A = 18$, $n = 5$.

$A_0 = 18$, when divided by $m_5 = 1$, yields $q_1 = 18$ and $r_1 = 0$.

$A_1 = 18$, when divided by $m_4 = 1$, yields $q_2 = 18$ and $r_2 = 0$.

$A_2 = 18$, when divided by $m_3 = 5$, yields $q_3 = 3$ and $r_3 = 3$.

$A_3 = 3 + 1$, when divided by $m_2 = 4$, yields $q_4 = 1$ and $r_4 = 0$.

$A_4 = 1$, when divided by $m_1 = 4$, yields $q_5 = 0$ and $r_5 = 1$.

Therefore, the remainders in the reverse order are (1,0,3,0,0). By replacing the zeros by the last elements of the respective axles, we have

$$J(18) = (1,4,3,1,1) = (p_{11}, p_{24}, p_{33}, p_{41}, p_{51})$$

4.4.2. Finding the structure of a tuple in the second kind of spreading

$$J(A) = \left(r_1\left(\frac{A_0}{m_1}\right), r_2\left(\frac{A_1}{m_2}\right), \ldots, r_{n-1}\left(\frac{A_{n-2}}{m_{n-1}}\right), r_n\left(\frac{A_{n-1}}{m_n}\right) \right)$$

Illustration: Find $J(18)$ when the second kind of spreading is applied.

Here, $A = 18$, $n = 5$.

$A_0 = 18$, when divided by $m_1 = 4$, yields $q_1 = 4$ and $r_1 = 2$.

$A_1 = 4+1$, when divided by $m_2 = 4$, yields $q_2 = 1$ and $r_2 = 1$.

$A_2 = 1+1$, when divided by $m_3 = 5$, yields $q_3 = 0$ and $r_3 = 2$.

$A_3 = 0+1$, when divided by $m_4 = 1$, yields $q_4 = 1$ and $r_4 = 0$.

$A_4 = 1$, when divided by $m_5 = 1$, yields $q_5 = 1$ and $r_5 = 0$.

Therefore, the remainders are (2, 1, 2, 0, 0). By replacing the zeros by the last elements of the respective axles, we have

$$J(18) = (2,1,2,1,1) = (p_{12}, p_{21}, p_{32}, p_{41}, p_{51})$$

4.5. *Uddiṣṭa* (‹Finding the position of a› mentioned ‹tuple›)

This tool finds out the position of a tuple mentioned by the questioner. In other words, given the structure of a specified tuple with respect to a particular spreading, its position has to be found out without constructing all tuples with respect to the spreading. Thus the process of *uddiṣṭa* is the reverse of that of *naṣṭa*. The position of a tuple is its serial number in the given spreading. We have already denoted it by A. It is found by synthesizing its structure, J, in the given spreading. He gives the following rule to do so.

संठाविदूण रुवं उवरीदो संगुणित्तु सगमाणे।
अवणिज्ज अणंकिदयं कुज्जा एमेव सव्वत्थ।।

saṃṭhāvidūṇa ruvaṃ uvarīdo saṃguṇittu sagamāṇe |

avaṇijja aṇaṃkidayaṃ kujjā emeva savvattha ||

[Jaini 1927a, v. 42, p. 32; also see Jain, Khūbacandra 1997, v. 42, p. 30]

"Having put down unity (*ruva*, Skt. *rūpa*), multiply it by the number of ‹the elements on› the respective ‹axle› from above (i.e., beginning with the first axle in the case of the first kind of spreading and beginning with the last axle in the case of the second kind of spreading), and subtract ‹the number of› the uncounted (*aṇaṃkidaya*, Skt. *anāṅkita*) ‹elements of the respective axle›. Do the same for all ‹axles›."

Below is formulated, in general terms, this rule in accordance with the kind of the given spreading.

4.5.1. Finding the position of a tuple in the first kind of spreading

$$A(J) = \prod_{i=1}^{i=n} 1.m_i - (m_i - j(p_{ij}))$$

Illustration: Find $A(p_{11}p_{24}p_{33}p_{41}p_{51})$ when the first kind of spreading is applied.

Here, $J = p_{11}p_{24}p_{33}p_{41}p_{51}$, $n = 5$.

$$\begin{aligned} A(p_{11}p_{24}p_{33}p_{41}p_{51}) &= \Bigg[\bigg[\Big[\big[1.m_1-(m_1-j(p_{11}))\big]\, m_2-(m_2-j(p_{24}))\Big]\; m_3- \\ &\quad (m_3-j(p_{33}))\bigg]\, m_4-(m_4-j(p_{41}))\bigg]\, m_5-(m_5-j(p_{51})) \\ &= \Bigg[\bigg[\Big[\big[1.4-(4-1)\big]4-(4-4)\Big]5-(5-3)\bigg]1-(1-1)\Bigg]1-(1-1) \\ &= 18 \end{aligned}$$

4.5.2. Finding the position of a tuple in the second kind of spreading

$$A(J) = \prod_{i=n}^{i=1} 1.m_i-(m_i-j(p_{ij}))$$

Illustration: Find $A\,(p_{12}p_{21}p_{32}p_{41}p_{51})$ when the first kind of spreading is applied.

Here, $J = p_{12}p_{21}p_{32}p_{41}p_{51}$, $n = 5$.

$$\begin{aligned} A(p_{12}p_{21}p_{32}p_{41}p_{51}) &= \Bigg[\bigg[\Big[\big[1.m_5-(m_5-j(p_{51}))\big]\, m_4-(m_4-j(p_{41}))\Big]\; m_3- \\ &\quad (m_3-j(p_{32}))\bigg]\, m_2-(m_2-j(p_{21}))\bigg]\, m_1-(m_1-j(p_{12})) \\ &= \Bigg[\bigg[\Big[\big[1.1-(1-1)\big]1-(1-1)\Big]5-(5-2)\bigg]4-(4-1)\Bigg]4-(4-2) \\ &= 18 \end{aligned}$$

5. Concluding remarks

5.1. Viewing the section four in the light of the section two we are able to see that the forms obtained in the combinatorics as found in the *Gommaṭasāra* (*Jīvakāṇḍa*) are none of permutations, combinations, variations, and partitions. In fact, they are tuples. In all the earlier papers[11] they were undeservedly regarded to be combinations (*bhaṅgas*). In this paper, from the very beginning we have recognized those forms as tuples.

Further we are able to observe that they belong to the cartesian product of P_1, P_2, P_3, P_4, P_5. In fact, the first kind of spreading without any condition forms $P_1 \times P_2 \times P_3 \times P_4 \times P_5$ while the second kind of spreading does $P_5 \times P_4 \times P_3 \times P_2 \times P_1$ with the reservation that the structures of the tuples are written in reverse order.

Going by the 10 verses, extending from verse 35 to verse 44, of the *Gommaṭasāra* (*Jīvakāṇḍa*) we find that Nemicandra does not refer to any term for tuple. Here it may be noted that the term *bhaṅga* (combination) occurred in the verse referred to in the section 4.1, does not refer to tuple. In fact, S is defined to be in terms of ${}^{m_i}C_1$. In its commentaries like the *Karṇāṭavṛti* composed by the 14th century author Keśavavarṇī and the *Jīva-tatva-pradīpikā* composed by the 16th century author Nemicandra we find that the term *ālāpa* is employed for tuple (Upadhye and Shastri 1997, pp. 67-68). It must have been known to our Nemicandra (c. 981). He is found to have made use of it, however, in the sense of 'distinction' during the philosophical discussion (Jaini 1927a, v. 706, p. 338).

5.2. In the section two we have noticed that six combinatorial tools have been in systematic use in India for the study of prosody and they go back to the time of Piṅgala (sometime before 200 BC). Not only the prosodists but also the mathematicians like Brahmagupta (628 AD), Mahāvīra (c. 850 AD) and so forth referred to them in their respective treatises.

Fascinating is in the *Gommaṭasāra* (*Jīvakāṇḍa*) that the tuples of a cartesian product were brought in purview of those combinatorial tools with the omission of *la-ga-kriyā* and *adhvayoga* and with the addition of *parivartana* (rotation). Since combinatorics of tuples has nothing to do with the prosody, *la-ga-kriyā* was bound to be omitted. *Adhvayoga* was left out, perhaps for the reason that it varies person to

[11] For combination see Datta 1935, pp. 38-44; Datta and Singh 1992, pp. 245-248. For *bhaṅga*, *ālāpa* see Datta 1936, pp. 54-57. For combination, *ālāpa* see Jain 1982, p. 40. For *sañcaya* see Jadhav 1996, pp. 45-51; Jadhav 1997, pp. 19-34. For combination, tuple see Jadhav and Jain 1997, pp. 103-106.

person. Here it may be noted that *la-ga-kriyā* is the process (*kriyā*) of classifying the variations of syllables into those having zero long (*guru*) syllable and *n* short (*laghu*) syllables, those having one long (*guru*) syllable and (*n*-1) short (*laghu*) syllables, those having two long (*guru*) syllables and (*n*-2) short (*laghu*) syllables and so on (see Sarma 2003, p. 129, and Shah, Jayant 2013, pp. 9-15). *Adhvayoga* determines the amount of space required for writing the entire list of the forms in separate lines one below the other. It is equal to 2*S*-1 *aṅgulas*. It allows the writer to leave space between successive forms (Shah, Jayant 2013, pp. 50-53). The process of *prastāra* (spreading) in case of combinatorics of tuples is so complicated that one new tool, *parivartana* (rotation), was introduced to understand the tuples in the way they appear. *Akṣa* (axle) and *akṣapada* (elements on the axle) are the new terms. The former begins its service from *parivartana* (rotation) onwards and the latter fulfilled the new requirements of *naṣṭa* and *uddiṣṭa* in consideration of combinatorics of tuples. This kind of accomplishment assures us that the important tools originally invented in and for Indian prosody were not only applied more widely than their original context but also supplemented in accordance with the need of the areas of their application with the progress of time and irrespective of the schools of mathematical thoughts in India.

5.3. *Parivartana* (rotation) is essentially a helpful tool. It can be said so for the reason that without *parivartana* (rotation) too combinatorics of tuples is a complete theory. *Parivartana* (rotation) provides an alternative way to understand the process of the formation of tuples. This seems to be the reason why Bibhutibhusan Datta took *parivartana* (rotation) to be the second kind of *prastāra* (spreading). *Parivartana* (rotation) has been made so important that the further two tools, *naṣṭa* and *uddiṣṭa*, are described in terms of *akṣa* (axle) or *akṣapada* (elements on the axle) or the both occurred in *parivartana* (rotation). These terms make *parivartana* (rotation) to be consistent with its succeeding combinatorial tools *naṣṭa* and *uddiṣṭa*. On the other hand, none of them occur in *prastāra* (spreading).

5.4. "Some of Nemicandra's results in combinatorics were not found in any Indian work composed before the fourteenth century." This is our version of Datta's remark of which original one we have already noticed in the section one. Since we do not find any detail around his remark, it cannot be said for certain for what he made it. We have already noticed that knowledge of six combinatorial tools was required not only for a good understanding of prosody but also for its progress. Nemicandra (c. 981) made an unusual use of them on tuples with the omission of two of them and with the addition of one new tool. Nārāyaṇa Paṇḍita (c. 1356 AD) made many unusual applications of those six combinatorial tools with the omission of some of them and with the addition of a number of tools on permutations, combinations, partitions, and sequences (Singh, Parmanand 2001, pp. 18-78; also see Kusuba 1993). Datta seems to have pointed out that the kind of unusual applications of combinatorial tools made in the fourteenth century by Nārāyaṇa Paṇḍita was made prior to that century only in the *Gommaṭasāra* (*Jīvakāṇḍa*).

5.5. In the end of the section three we have noticed that as a result of perfect-conduct-preventing passion and quasi-passion being operational in the stage of imperfect vow, it is carelessness (*pamāda*, Skt. *pramāda*) that produces impurity in control. Using combinatorics Nemicandra showed how one is able to know the particulars regarding the forms (i.e., tuples) made from the sets of carelessness. Being distinct, each tuple is culpable to produce distinctive impurity in control. As far as the stage of imperfect vow is concerned, it was very well known prior to Nemicandra in Jaina philosophy (Jain, H. L. et al. 2000, pp. 177-180; also see Jaini 1930) but the way in which he demonstrated the case of impurity in control due to carelessness using combinatorics is not found in any treatise anterior to the *Gommaṭasāra*.

5.6. Since we do not find such a treatise that contains a mention of this combinatorics from the exclusive class of the Jaina school of Indian mathematics or from any non-Jaina school of Indian mathematics, it can be deduced that combinatorics of tuples was developed in the canonical class of the Jaina school of Indian mathematics as it was required for the exposition of carelessness that produces impurity in control and was applied in the same class as the *Gommaṭasāra* (*Jīvakāṇḍa*) of Nemicandra and the *Śrī Prastāra Ratnāvalī* of Muni Ratnacandra substantiate.

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