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THE LUNAR MODEL IN ANCIENT INDIAN ASTRONOMY

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The Indian planetary model shows great finesse, originality and completeness. In sharp contrast to this, the Indian lunar model appears to be in a half-developed state. This odd inconsistency is the subject of this paper. In this article we delve into the ancient texts to discover the roots of this lunar puzzle. It is found that the source of the problem can be traced to a 150-year old misinterpretation of the ancient verses. This misinterpretation, perpetrated by the colonial scholars of yesteryear, has resulted in the currently held tainted view of the Indian lunar model. The correct interpretation of the verses has a surprising outcome. It leads to a lunar model that is not only more consistent and far more accurate than previously thought, but also distinctly familiar.

Key words: Almagest, Al-Shatir, Copernicus, Epicycle, Lunar, Ptolemy, Second Correction, *Sūrya-Siddhānta*

1. INTRODUCTION

Ancient man, when he first turned his attention to the night sky, must have discovered the monthly cycle of the moon fairly early. Thus it comes as no surprise that the oldest calendric unit is the lunar month. According to western historians the ancient Babylonians were the first to undertake a systematic collection of moon data. Using cuneiform symbols, at a period earlier than 2000 BC, they faithfully recorded their lunar observations day after day, month after month, for centuries together.

Indian astronomy too can justifiably claim high antiquity. Notwithstanding attempts by western scholars to prove otherwise, Indian astronomical data on the equinoxes, the stars, the sun and others, all give

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indication of great age - upwards of six thousand years (Sen & Shukla, 1985; Tilak, 1893; Brennad, 1988; Narayanan, 2010, 2011). The antiquity of Indian astronomy is also reflected in its elaborate and elegant cosmological framework and in its planetary models both of which are marked not only by mathematical and geometrical ingenuity but also by the substantial accuracy of its data. This complex and complete system is obviously the result of a long and sustained study of the heavens aided by an equivalent progress in mathematical knowledge.

In this venerable picture of Indian astronomy however there appears a proverbial fly-in-the-ointment; one glaring inconsistency that stands out inexplicably. The ancient Indian Lunar Model, as currently understood, appears to be in a half-developed state with a consequent high error of as much as 3 degrees. In terms of the Moon's angular diameter this error amounts to 6 Moon-widths - not a trifling amount.

How comes this inconsistency, one may reasonably ask. For example, consider the five visible planets. Since the motion of these planets is centered on the Sun and not the earth, they appear at times to move in the opposite direction (retrograde) as seen from the earth. The Indian planetary model accounts for the planets' regular motion as well as the retrograde motion in an ingenious manner. It is quite apparent that a substantial amount of effort and ingenuity has gone into the creation of this complex model to match observational data. The same holds for the pulsating-epicycle technique that forms the backbone of all Indian planetary models. While the Greek, Islamic and European astronomers too have employed epicycles in their models, the *pulsation* feature is unique to Indian astronomy. It represents a geometrical and mathematical finesse of the most elegant variety. Thus, one is well within the limits of propriety in asking this question - when all these other Indian models have been so meticulously crafted and unified, why has the Indian lunar model been left half-developed?

The question assumes altogether a more serious character when we consider the fact that the Moon occupies a central place in the Indian calendric astronomy. The most fundamental unit of the Indian calendar is the lunar day (called *tithi*). Also, in the luni-solar Indian calendar new months are begun at Sun-Moon conjunctions. Furthermore, eclipse calculations depend critically on accurate prediction of the Moon's location. Overall, it becomes apparent that an error-prone Moon calculation would throw the entire calendar into chaos.

In this article we look at various aspects of this lunar puzzle to shed some light on this strange inconsistency in Indian astronomy. On the way we will also examine some Greek, Islamic and European lunar models and compare and contrast amongst them. The Indian text we will mainly refer is the *Sūrya-Siddhānta* (Burgess, 1858), the most revered of all ancient Indian works on astronomy. The element of the original text has been estimated to be older than 3000 BC (Brennad, 1988; Narayanan, 2010, 2011).

We begin by examining the motion of the object in question, our closest neighbor in space, the Moon.

2. The Wayward Moon

Sir Isaac Newton once famously quipped that analyzing the Moon's motion was the one thing that gave him a headache! And little wonder, since the motion of the Moon is highly irregular due to the dual influences of the earth and the Sun. Their combined gravitational effect produces great fluctuations, long term and short term, in the motion of the Moon.

How pronounced are these fluctuations? Fig. 1a shows the variation of the anomalistic period of the Moon for 200 orbits. The anomalistic period is the time taken by the Moon to move from one perigee to the next. Its average value is about 27.55 days. As seen in the figure, the anomalistic period changes continuously from orbit to orbit. It varies from about 27 days to nearly 28 days – almost a full day, which is a considerable variation for a 28-day cycle. Similarly Fig. 1b shows the variation of lunar apogee distance from the earth for 200 orbital cycles. The average distance is about 405,400 km. It can be seen that there is considerable variation of this distance from month to month. Quite a few short and long term variations can be discerned in these figures. Clearly, the motion of the Moon is highly irregular and predicting its location accurately and consistently will be no mean task.

Having understood the Moon's wayward motion and that it can be next to impossible to accurately predict its motion, the question naturally arises – how did the ancient astronomers deal with the errant Moon? In the next couple sections we will take a brief look at progress in lunar science down the ages.

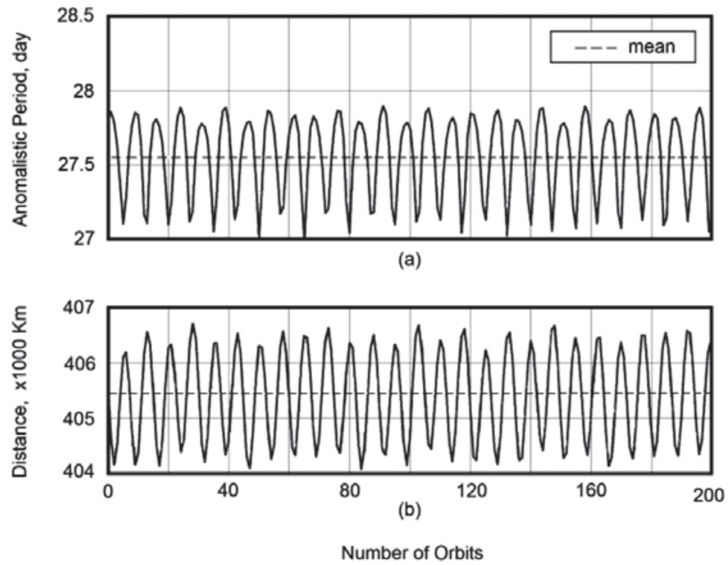

Fig. 1. The Irregular Motion of the Moon

Progress in lunar science can be broadly divided into two distinct periods: 1) the early period, from ancient times till the time of Sir Isaac Newton, and 2) the latter period, beginning with the works of Newton and culminating in the modern developments of the science.

3. Lunar Theory from Ancient Times till Newton

With clear skies all year round the climate of Mesopotamia is certainly very favorable for astronomical observations. According to western historians the earliest recorded observations of the Moon were carried out by the ancient Babylonians in Mesopotamia at about 2000 BC or so. By the middle of the millennium before Christ they were familiar with the 19-year Metonic cycle. By that time they also appear to have developed an arithmetical technique to predict the Moon's motion, a technique that approximates to what we call today the 'elliptic-inequality of the Moon'. As regards Moon data, the Babylonians were able to obtain very accurate estimates for three important quantities of the Moon, namely, the mean sidereal speed, the synodic month and the anomalistic month. These accurate values were borrowed by the Greeks and the Hebrews and were used throughout the Middle-Ages.

Among the Greeks, Hipparchus (140 BC) was the first to develop a geometric theory of the Moon's motion. Using a simple epicycle, he developed a model for the Sun and the Moon. His lunar model was later refined by the great Greek astronomer Ptolemy, thereby improving its longitudinal accuracy. However, Ptolemy's lunar model was defective in that the Earth-Moon distance it predicted was quite removed from reality. We will examine these Greek models in detail in later sections of this paper.

More than a thousand years passed before further improvements were made to Ptolemy's lunar model. Noticing that the model gave erroneous Earth-Moon distances, the Syrian astronomer, Ibn Al-Shatir (1304-1375), replaced Ptolemy's crank mechanism with a double-epicycle model which greatly improved the range of Earth-Moon distances while maintaining the longitudinal accuracy of the original model.

Among the Europeans, Nicolas Copernicus, of heliocentric theory fame, also developed a lunar model. Very curiously, his model was a replica of the Al-Shatir model. We shall examine the Al-Shatir/Copernicus model in later sections. Tycho Brahe, and later, Johannes Kepler, attempted to improve Ptolemy's model but did not achieve much success. They however did make some important discoveries regarding the Moon's motion. Tycho, in particular, discovered what is called the *Annual Equation* and the *Variation* of the Moon, both remarkable discoveries.

4. Modern Theory of Lunar Motion

It is generally accepted that modern lunar theory originated with the work of Sir Isaac Newton. Though the Moon's irregularities were well known since ancient times, it was Newton who first explained them and also calculated their amounts.

Modern lunar theory treats the subject as a 3-body problem. It considers the motion of the Moon with respect to the earth while taking into account the Sun's influence on both the Earth and the Moon. The differential equations that result cannot be integrated fully. A solution however can be affected by considering only terms of the first and second orders and dropping all others. The approximate solution thus obtained for the Moon's longitude, though still rather intimidating, can now be conveniently cast as sets of related terms (Hugh, 1885) as shown in Eqn. 1.

Actual Lunar Longitude = Mean Longitude ….. *(Uniform Circular Motion)*
+ 1st set of correction terms …… *(Elliptic Inequality)*
+ 2nd set of correction term …… *(Evection)*
+ 3rd set of correction term …… *(Variation)*
+ 4th set of correction term …… *(Annual Equation)*
+ 5th set of correction term …… *(Parallactic Inequality)*
+ 6th set of correction term …… *(Reduction)*
+ (other minor terms) …(1)

Thus we see that the actual longitude of the Moon at any instant is the sum of the mean longitude and a series of ever diminishing corrections. Starting in the mid-18thcentury with only a couple dozen corrections, at the beginning of the twentieth century over 1400 such correction terms were identified in the full lunar theory. At the present time their number has mushroomed to tens of thousands of terms which are collectively employed to approach the centimeter-level accuracy obtained from laser ranging observations of the Moon.

Till the advent of the telescope, astronomical observations were always made with the naked eye, which can perceive angular magnitudes down to about 1 minute of arc. Since we are considering the history of lunar science, we will leave out any correction terms that are less than 1 minute of arc or so. That leaves us with six of the largest terms, as shown in Eqn. 1. These six terms have names associated with them and are often called *named inequalities*. Their relative magnitudes are shown in Fig. 2. These named

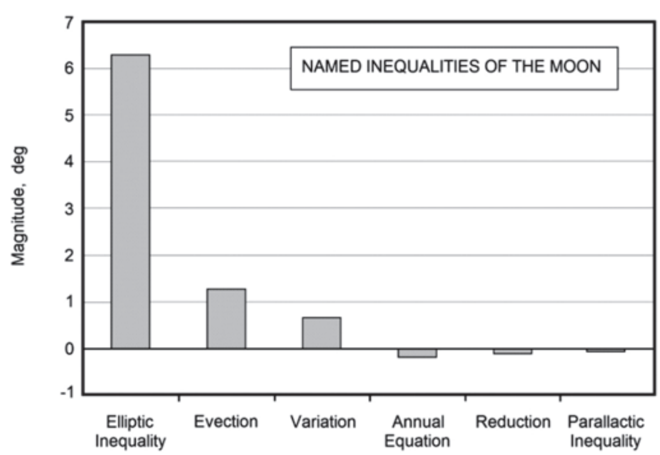

Fig. 2. Relative Magnitudes of the Moon’s Inequalities

inequalities are described in brief below and can be found in greater detail on the web (Wikipedia.org).

4.1. The Elliptic-Inequality

Also called the *Equation of Center*, this was well known to the ancients. The Moon does not move in a circle but in an elliptic trajectory around the earth. It moves faster as it nears perigee and slows down as it nears apogee. The elliptic-inequality represents the deviation in longitude from the circular due to the elliptic nature of the Moon's orbit. The ancients used eccentrics or epicycles to account for this inequality. In modern terms, this inequality has a series of terms the largest of which is +6.29 degrees.

4.2. The Evection

The *Evection* was known to the Greek astronomer Ptolemy though its cause was discovered only in the 17th century. It has a period of about 31.8 days. This can be represented in a number of ways, for example as the result of an approximate 6-monthly libration in the position of perigee with an accompanying 6-monthly pulsation in the size of the Moon's orbital eccentricity. Its maximum value is about +1.274 degrees.

4.3. The Variation

Discovered by Tycho Brahe, this is a speeding-up of the Moon as it approaches new-Moon and full-Moon, and a slowing-down as it approaches first and last quarter. Its maximum value is about +0.6583 degrees.

4.4. The Annual equation

Also discovered by Tycho, this was explained by Newton in terms of changes to the Moon's orbit. The lunar orbit becomes slightly expanded in size and longer in period when the earth is at perihelion, closest to the Sun. Conversely, it becomes slightly contracted in size and shorter in period when the Sun is most distant, at its apogee. The maximum value of this inequality is about -0.186 degrees.

4.5. The Reduction

The actual orbit of the Moon is not exactly in the ecliptic plane but is inclined to it by about 5 degrees. The *Reduction* represents the effect of expressing the Moon's motion in the plane of the ecliptic. Its largest term is about -0.1144 degrees.

4.6. The Parallactic Inequality

The *Parallactic Inequality* makes Tycho's *Variation* a little asymmetric as a result of the finite distance and non-zero parallax of the Sun. Its effect is that the Moon is a little behind at first quarter, and a little ahead at last quarter. Its principal term is about –0.0347 degrees.

4.7. Later Developments in Lunar Theory

For a couple hundred years after Newton, researchers were occupied mainly with the task of proving the inverse-square law of gravitation. They did so for the Moon with great success, culminating in Brown´s Lunar Theory and associated Tables (1919), which were used in the American Ephemeris and Nautical Almanac until 1968 and in a modified form until 1984. Brown's improved lunar ephemeris was also used during the Moon landing.

In more recent times, astronomical research has received a great impetus in the form of modern digital computers. Previously intractable differential equations could now be handled by numerical integration. These numerical techniques take into account not only gravitational forces but also tidal, geophysical and libration effects. Also in recent times (1970s to 1990s), some researchers have gone back to the analytical methods of the 18th and 19th centuries, but this time with the assistance of the computer. Using computer-assisted algebra Jean Chapront and Michelle Chapront-Touze from the Bureau des Longitudes have taken developments beyond what was done earlier using manual efforts alone. Their semi-analytical theory, called ELP, is not derived from purely theoretical considerations alone but with the added introduction of numerical values for orbital constants.

These new computer techniques have been complimented by developments in observational techniques such as lunar laser ranging (LLR), which involves laser beams from earth being bounced off reflectors placed on the surface of the Moon by astronauts. LLR can give centimeter-level accuracies for the Moon's longitude.

4.8. Lunar Calculations in this Paper

In the present paper we use a shortened version of Chapront's ELP theory to determine the Moon's longitude (Meeus, 2000). It provides an accuracy of about 10 arc-seconds which is good enough for our present purposes.

5. The Greek Lunar Model

From these modern developments we return back to the ancient. We start by examining the Greek lunar models of Hipparchus and Ptolemy.

Hipparchus (140 BC) was the first Greek astronomer to construct a geometrical model for describing the motions of the Sun and the Moon. He employed a simple epicycle scheme as shown in Fig. 3. The system consists of a smaller circle (the epicycle) the center of which moves on a larger circle (the deferent). The earth is located at the center of the deferent. The Moon revolves around the epicycle. Both the epicycle and the Moon move at the same angular velocity in their orbits, though in opposite directions. When

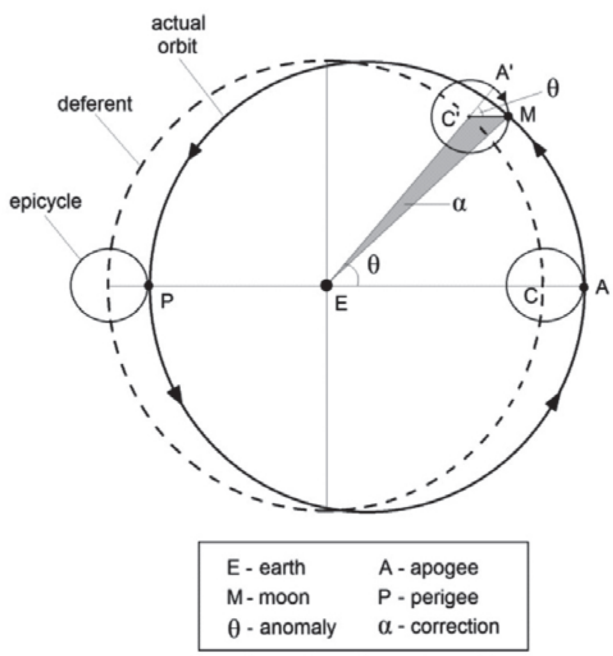

Fig. 3. The Simple Epicycle Scheme

the Moon is at position A, it is at the apogee of the orbit since at this point it is farthest from the earth. As the epicycle moves counter clock wise (CCW) on the deferent and reaches point C′ from C, the Moon moves clock wise (CW) on the epicycle and reaches point M. Both have traversed the same angular distance, called the anomaly (θ), though in opposite directions. The Moon's longitude at this instant is the sum of the mean anomaly (θ) and the correction ($-\alpha$). This simple model proved fairly successful at predicting the Sun's motion (Narayanan, 2011). However, when applied to the Moon, the results were understandably poor. The highly irregular motion of the Moon could not be adequately captured by a simple epicycle.

Three hundred years later, in 140 AD, Ptolemy, the greatest of the Greek astronomers and author of the *Almagest,* took up the lunar challenge. Starting from Model-I (same as Hipparchus' model), he created a new lunar model by introducing a central crank mechanism in the simple epicycle scheme of Fig. 3. This new model, called Model-II, was moderately successful in predicting the Moon's longitude. He then further modified Model-II to Model-III by introducing a fluctuating apogee instead of a fixed one. Though fairly successful at predicting the Moon's longitude, the cranking mechanism in the center produced significant errors in the Earth-Moon distance. The model was thus useless for anything other than predicting the longitude. While some historians have hailed Model-III as a great success, others have considered it a dismal failure. Ptolemy himself was aware of the model's limitations and never employed it in his own eclipse calculations. Nevertheless, since historians consider Model-III as the most significant achievement by the Greeks in lunar science, we will take a detailed look at it.

Fig. 4 shows the working details of Ptolemy's Model-III as given in Book-V of the *Almagest* (Ptolemy, 1952). The earth (E) is at the center of system. S′ and M′ represent the mean-Sun and mean-Moon respectively. The epicycle is centered on the mean-Moon. The angle δ is the mean elongation which is the angular separation between the mean-Sun and mean-Moon. As the epicycle (or mean-Moon) advances eastward (CCW), away from the mean Sun (δ increasing eastward), the crank rod AEB turns westward (CW) at an equal rate but in the opposite direction (δ increasing westward). Meanwhile, the true Moon advances westwards (CW) on its epicycle (ν

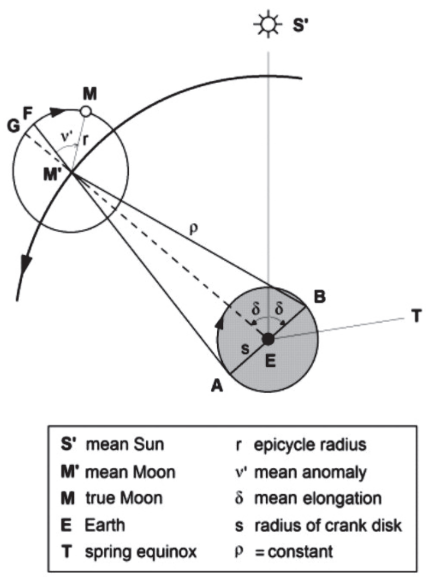

Fig. 4. Ptolemy's Lunar Model with Central Crank (Model-III)

increasing westwards), the angle being reckoned from the line AF, where AB is a diameter of the crank circle. An important restriction in the model is that the distance $M'B = \rho$ is maintained constant throughout the motion. Finally it must be remembered that the mean-Sun also moves eastward (CCW), though at a much slower speed than the mean-Moon. This completes the kinematic picture of Model-III.

One of the major changes in Model-III from Hipparchus' model (or Model-I) is that as the mean-Moon moves away from the mean-Sun, the epicycle is pulled in towards the earth. At quadrature the Earth-Moon distance becomes a minimum. This 'pull-in' distorts the orbital path so much that the resultant orbit is an elongated oval (see Fig. 12), which is a major drawback for this model. The other big change is that the anomaly (ν) is now measured from an apogee (F) that is constantly changing. As the central crank disk turns, it can be seen that the point F will fluctuate accordingly, sometimes being to the right of G, the true apogee, and sometimes to the left. In Hipparchus' model the anomaly was measured from the true apogee (G). A third change from Hipparchus' model is that Model-III now includes the influence of the Sun by means of δ, the mean elongation.

The *Almagest* provides the following data for Model-III:

Rate of δ = 13.17638222° per day
Rate of ν = 13.06498286° per day
Mean Sun Speed = 0.985635278° per day

The physical measurements in the model are as follows:

R = Distance from earth to Moon at mean syzygy = 60 units
s = 10.3167 units
r = 5.25 units
ρ = R - s = 49.6833 units

From this data we can see that the maximum possible Earth-Moon distance is R + r = 65.25 units, while the minimum is R– 2(s) – r = 34.12 units, a ratio of almost 2:1. Since the size of the Moon as seen from the earth depends on the distance, this implies that the Moon's apparent diameter ratio must be around the same range, which is certainly not the case in reality.

In section 8, we will run a computer simulation of Model-III and examine the results.

6. The Lunar Models of Ibn Al-Shatir and Copernicus

For more than a thousand years Ptolemy's astronomical system reigned supreme. It was eventually transmitted to the Arabs and the Islamic world and from thence to Europe. In the Arab world the first person to confront Ptolemy's Model-III was Al-Shatir. Ibn Al-Shatir (1304–1375 AD) was a Syrian astronomer and mathematician who worked as timekeeper at the Umayyad Mosque in Damascus. After performing detailed observation of eclipses he concluded that the actual angular diameters of the Sun and the Moon did not agree with Ptolemy's predictions. He soon set about making major reforms to the Ptolemaic system.

One of the first things he did was to re-establish the geocentric design with the earth firmly at the center of the system. In the process he eliminated the Ptolemaic Equant and eccentric and replaced it with a secondary epicycle. For the Sun, the secondary epicycle appears not to have improved matters much over Ptolemy's model. But for the Moon there was a definite improvement in terms of the lunar distance. For the planets, Al-Shatir chose the epicycle sizes to produce a system that gave the same result as Ptolemy's.

One remarkable fact about Al-Shatir's system is that two centuries later it was found duplicated, almost exactly, in the works of Nicolas Copernicus, the founder of the heliocentric system. Did Copernicus have access to Al-Shatir's work? While the jury is still out on this one, it does appear highly likely that Copernicus may have copied Al-Shatir's work. The discovery that a mistake Al-Shatir made in his model for mercury was also duplicated in Copernicus' model seems to strongly favor that conclusion.

Fig. 5 shows the double-epicycle lunar model of Al-Shatir (Roberts, 1957) which, with very minor differences, is the same as the Copernicus (Stephen, 2002) lunar model. The mean Moon (M′) moves on the deferent in an eastwards direction (CCW) with the mean sidereal speed. The primary epicycle, which carries the center of the secondary epicycle, rotates westwards (CW) at the anomalistic rate. The secondary epicycle, carrying the true

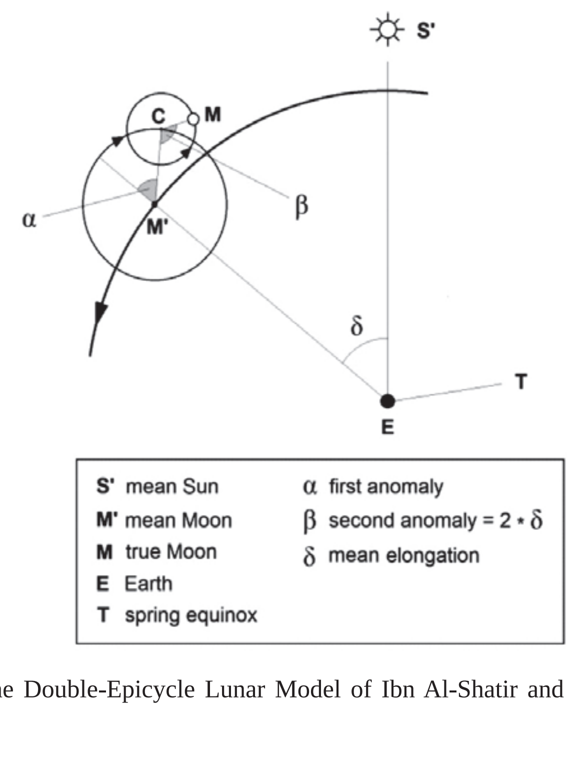

Fig. 5. The Double-Epicycle Lunar Model of Ibn Al-Shatir and Copernicus

Moon, rotates eastwards (CCW) and has a speed that is twice that of the mean elongation rate. One consequence of this arrangement is that at mean syzygy, when mean-Sun and mean-Moon are in conjunction, the actual Moon will always be at the perigee of the secondary epicycle.

The lunar model parameters for Al-Shatir and Copernicus are given in Table 1.

Table 1. Orbital Parameters for the Al-Shatir and Copernicus Lunar Models

Item	Al-Shatir	Copernicus
First epicycle radius/Deferent radius	0.109722	0.1097
Second epicycle radius/Deferent radius	0.023611	0.0237
Mean Sun motion (°/day)	0.985601218	0.98558966
Mean Moon motion (°/day)	13.17639452	13.17639452
First epicycle motion (°/day)	13.06493657	13.06498372
Second epicycle motion (°/day)	24.38149538	24.381612

In section 8, we will run a computer simulation of this model and examine the results.

7. The Indian Theory of Lunar Motion

The Indian theory of lunar motion, like its solar counterpart, employs the pulsating epicycle scheme (Narayanan, 2011) as its basis. The procedure to calculate the Moon's longitude has three major steps. The first calculates the mean position of the Moon at the required instant of time. The second step applies a first-correction, for elliptic-inequality, to this mean position. The third applies a further (second) correction for the effect of the Sun. Let us examine these steps in detail.

7.1. Mean Motions

The *Sūrya-Siddhānta* states that the Moon's mean motion produces 57,753,336 revolutions in a *yuga* of 4,320,000 years, each year being of 365.2587565 days. Thus the Moon's mean motion works out to be 13.176352 °/day. Similarly, the number of revolutions of the Moon's apogee is stated to be 488203 in a *yuga* and that turns out to be 0.111383 °/day. A solar year is given as 365.2587565 days from which we calculate the Sun's mean motion as 0.985602655 °/day.

7.2. The First Correction: Elliptic Inequality

The first correction to the mean motion is that for elliptic-inequality. This is found by the epicycle method as described earlier and shown in Fig. 6. The Indian epicycle however is different from the Greek/Islamic/European epicycles in that its radius increases and decreases (pulsates) in time.

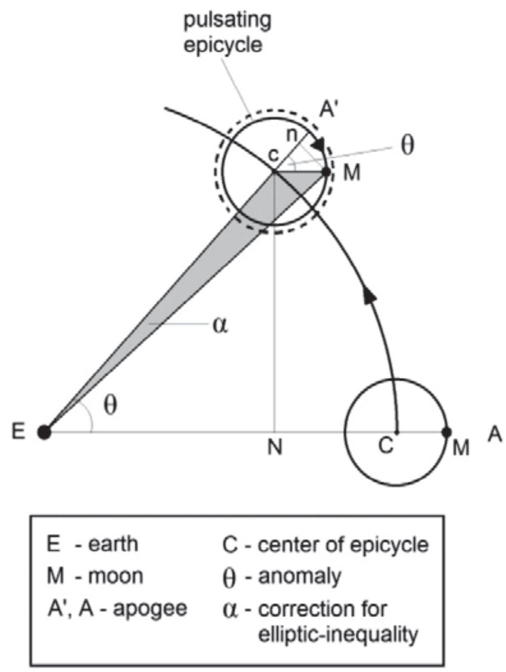

Fig. 6. The Pulsating Epicycle in the Indian Lunar Model

In the Indian system the deferent radius has a standard value of 3438 units. This remarkable number is obviously not an arbitrarily chosen quantity like the Greek (60 units) or the European (10000 units). It is the number of minutes in one radian (57.3 degrees). If nothing else, it points to the originality of the Indian system.

From Fig. 6 we note that θ is the mean-anomaly, which is the angular separation between the mean Moon and the apogee A. As the mean-Moon (or epicycle) moves by an angle θ in anticlockwise direction away from the apogee, the Moon on the epicycle has moved by the same angle but in a clockwise direction. The radius of the epicycle is not constant but is a function of θ. The *Sūrya-Siddhānta* gives the circumference of the epicycle as 32 degrees at the end of even quadrants and 20 minutes less at the end of the odd. The radius at these locations then works out to be 305.6 and 302.42 units respectively. Using simple geometry we can now calculate α, the correction to be applied to the mean anomaly θ, to obtain the true longitude of the Moon.

Let us now test out the accuracy of this 'true' longitude that is obtained after applying the first correction. We first calculate the true longitude of the Moon as per the *Sūrya-Siddhānta* method, described above, for a period of 14 months. During this time interval the Sun has completed a full circle of 360 degrees with respect to the Moon's apogee. Next, we calculate the actual longitude of the Moon during the same time interval using the modern Short-ELP theory (Meeus, 2000). Fig. 7 shows the difference, or error, between the two longitudes. It can be seen that the calculated (*Sūrya-Siddhānta*) longitude has a high error, as much as 3 degrees.

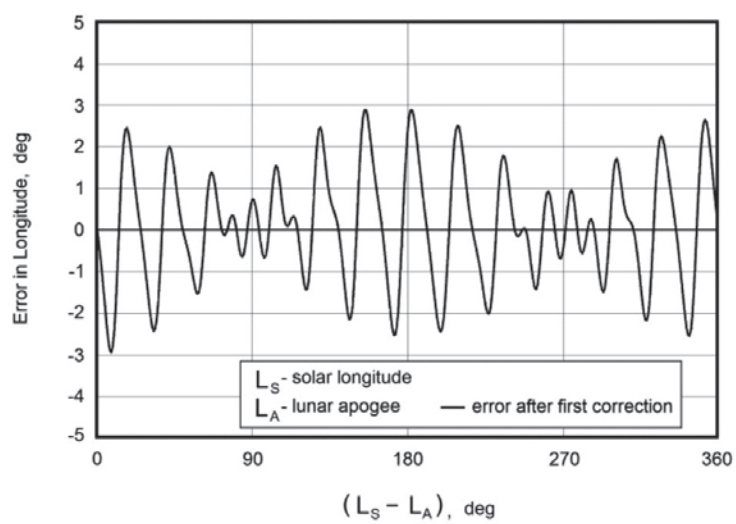

Fig. 7. Error in the Indian Lunar Model after the First Correction

In the popular view, the Indian lunar model is complete at this point. A great majority of scholars are of the opinion that the ancient Indian lunar model has only one correction, that for elliptic inequality, as described above. This opinion has come about largely due to the efforts of the colonial scholars who misinterpreted the second lunar correction as given in the *Sūrya-*

Siddhānta and thereby rendered it unusable. We shall return to this topic later.

From Fig. 7 we note a couple of interesting things. Firstly, the 'true' longitude has a substantial error, up to about 3 degrees. Secondly, the error appears to be dependent on the angular separation between the Sun and the Moon's apogee. When the Sun and lunar apogee are in a line (separation is 0 or 180 degrees), the error is a maximum. On the other hand, when the separation is 90 or 270 degrees, the error is a minimum.

This interesting finding above throws some light on the effect of the Sun on the Moon's orbit. Recall that the first correction is for the elliptic-inequality. This first correction should have given us a close approximation to the Moon's elliptic orbit around the earth, had the Sun been absent from the picture. From Fig. 7 we observe that the Sun appears to have a maximum distorting effect on the Moon's orbit when it is in line with the lunar apsis and a minimum when it is perpendicular to it. This is shown schematically in Fig. 8.

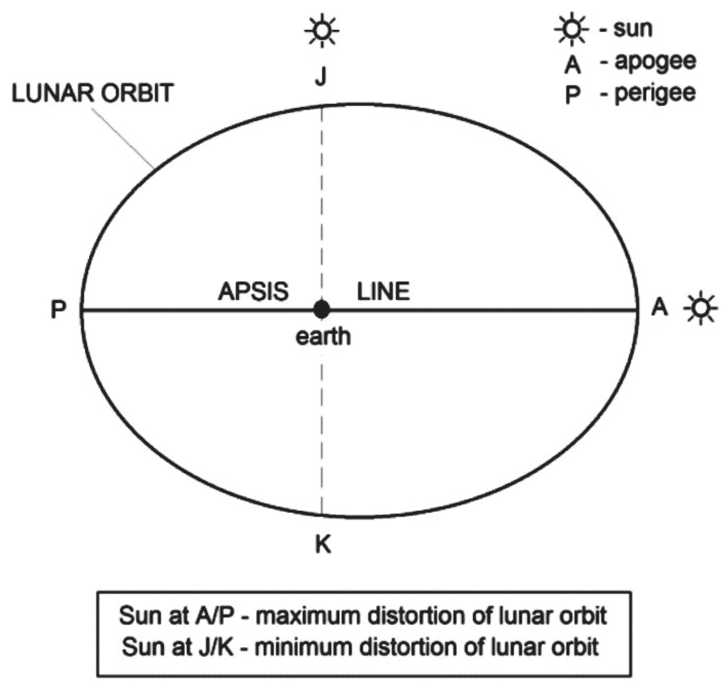

Fig. 8. Effect of Angular separation between Sun and Lunar Apogee on the Lunar Orbit

Armed with this knowledge, we can now proceed to the second correction for lunar longitude as given in the *Sūrya-Siddhānta*.

7.3 The Second Lunar Correction: Sun-related Correction

A transliteration of the original Sanskrit verse in the *Sūrya-siddhānta* (*SS*) for the second lunar correction is shown below:

> *SS*.ii.46:
>
> Sun-sine multiplied by planet-daily-motion divided by
>
> minutes-in-circle. So-obtained minutes apply to planet Sun-like...
>
> (Eng. Tr. by the author)

Though the verse is in the usual terse style of the *Sūrya-Siddhānta*, a few things are obvious:

1. This is yet another correction (a second correction) to be applied to the planet (the Moon, in this case).
2. This correction is related to the Sun.
3. The Sun is mentioned twice, firstly with regard to sine of some angle and secondly with regard to a procedure of some sort to be applied at the end.

It appears that we must find the sine of a Sun-related angle and multiply that with the instantaneous daily motion of the Moon and divide by 21600 (minutes in a circle). This result has to be applied to the Moon's longitude by a procedure that is related to the Sun.

To help us understand the verse better, let us examine what some ancient Indian astronomers have written about the second-correction to the Moon in their own works.

Mañjula (930 AD):

> The daily motion of the Moon, diminished by 11 and multiplied by the cosine of the longitude of the Sun diminished by that of the Moon's apogee is the multiplier of the sine and cosine of the longitude of the Moon diminished by that of the Sun, …etc. apply to the Moon.
>
> (*Laghumānasa*, iv. 1-2, Eng. Tr. Shukla, p. 137)

Disregarding constants, this verse says the following:

$$\text{Second Correction} = \text{Sin}(Lm - Ls) \times (\text{Daily motion}) \times \text{Cos}(Ls - La) \quad (2)$$

Where,

Lm = Longitude of Moon after first correction

Ls = Longitude of Sun

La= Longitude of lunar apogee

Śṛīpati (1050 AD):

> From the Moon's apogee subtract 90°, diminish the Sun by the remainder left; Take the sine of the result; Multiply it by 160' and divide by the radius; Save the result. Call it *Cara-phala*. Multiply *Cara-phala* by versed sine of Moon anomaly; Divide by the diff bet the Moon's distance and radius. Call this *Parama-phala*. Multiply the sine of Moon diminished by the Sun with the *Parama-phala*; Divide by radius. Apply this to Moon.
>
> (*Siddhāntasékhara*, xi. 2-4)

Once again, disregarding constants, Śṛīpati's method seems to suggest the following:

$$\text{Second Correction} = \text{Sin (Lm – Ls)} \times E \times \text{Sin [Ls – (La-90°)]}$$

…which can be rewritten as:

$$\text{Second Correction} = \text{Sin (Lm – Ls)} \times E \times \text{Sin [90° – (Ls-La)]}$$

$$= \text{Sin (Lm – Ls)} \times E \times \text{Cos (Ls-La)} \qquad \ldots(3)$$

The 'E' is a complicated expression that is a function of the Moon's anomaly (Lm-La). Note that the daily-motion too is a function of anomaly, as we'll see in later sections. Thus it appears that E may be the daily motion as calculated by Śṛīpati's method. If so, we can see that Eqns. 2 and 3 have the same form.

Nīlakaṇṭha (1500 AD):

> The sine and cosine of the difference between the Sun and Moon is multiplied by half of the cosine of the difference between the longitudes of the Sun and the apogee of the Moon…etc…whatever is obtained from the sine has to be multiplied by the radius and divided by the result. Apply to the true Moon positively or negatively…etc.
>
> (*Tantrasaṅgraha*, viii. 1-3; Eng. Tr., Ramasubramanian & Sriram, 2011)

Disregarding constants, Nīlakaṇṭha's method appears to be this:

$$\text{Second Correction} = \text{Sin (Lm – Ls)} \times B \times \text{Cos (Ls - La)} \qquad \ldots(4)$$

'B' is a complex expression that appears to differ from Śṛīpati's 'E', although a more thorough analysis is required to conclude that. Our main concern in this paper is the general form and not the exact expression. We observe that Nīlakaṇṭha's expression for the second correction has the same form as Mañjula's and Śripati's expressions.

Having examined the works of these ancient authorities, it becomes clear that the *Sūrya-siddhānta* has a similar algorithm, as given below:

Second Correction = Sun-Sine × Daily Motion × Sun-like procedure

We can thus make the following conjectures:

1. By 'Sun-sine' is meant the sine of the angle between the Sun and the Moon.
2. By 'Sun-like' procedure is meant the multiplication of the cosine of the angular separation between the Sun and the lunar apogee.

In section 8, we will verify computationally that this interpretation does indeed produce the Moon's longitude with great accuracy. In the meantime let us look at how the meaning of this *Sūrya-siddhānta* verse for the Moon's second correction was twisted out of context by the colonial scholars.

7.3.1 The Colonial Misinterpretation

The colonial scholars (Burgess, 1858) have translated the verse as follows:

Second Correction = (Equation-of-Sun × Daily Lunar Motion) / 21600 …(5)

The Equation-of-Sun is related to the equation-of-time and is the correction that is obtained for the Sun by the simple epicycle process as given in the *Sūrya-siddhānta*. Let us calculate the maximum possible value for the second correction of the Moon as given by Eqn. 5.

Max value of Equation-of-Sun (Narayanan, 2011) = 2.17°
Max value of Daily Motion of Moon ~ 925 minutes

Thus, Max Second Correction = 2.17 × 925/21600 = 0.093°

It would strike anyone as very odd that a terse text like the *Sūrya-siddhānta* would devote six whole verses to describe a procedure that applies a correction whose maximum is less than one-tenth of a degree. We have seen from Fig. 7 earlier that the maximum error after the first process was about 3 degrees. Why would the *Sūrya-siddhānta* describe at length a procedure that applies, at most, a correction of only one-tenth of a degree to this 3-degree error? Clearly this interpretation of the verse is in error.

The matter turns out even more ludicrous when we consider that as per the *Sūrya-siddhānta* this same equation (Eqn. 5) is to be applied as a second correction for all planets. For example, the maximum daily-motion of Saturn is about 0.13°/day. Thus, according to the colonial interpretation, the maximum second correction for Saturn would be 0.00078°!!

7.4 Calculating the Actual Daily Lunar Motion

After that digression, let us get back on track to the *Sūrya-Siddhānta's* method for calculating the second correction for the Moon. The first step involves calculating the daily lunar motion. The calculation is interesting, if somewhat perplexing.

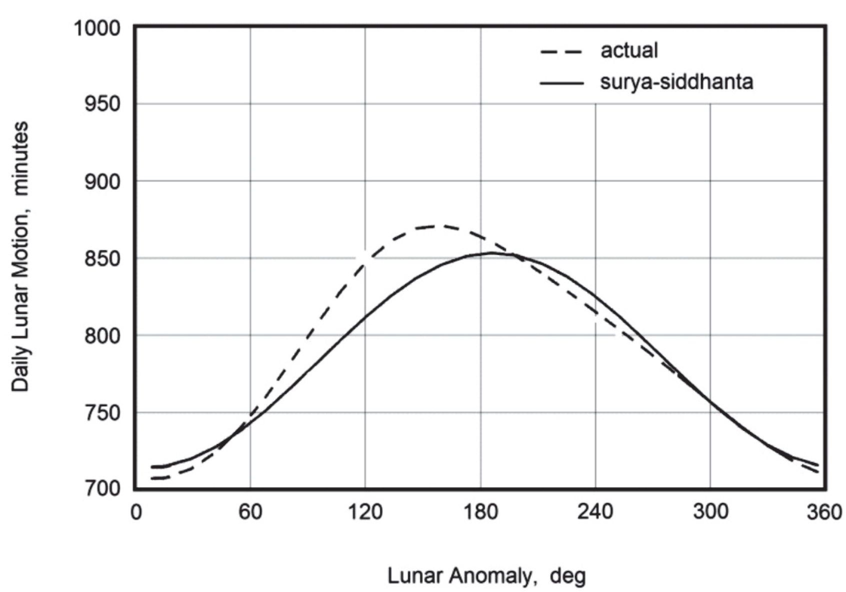

Fig. 9a. Actual and Calculated Daily Motion for one Lunar Orbit

As a starting point, we are required to calculate the daily-mean-motion (DMM) of the Moon, in minutes.

$$\text{DMM} = (\text{daily Sidereal Motion} - \text{daily Movement of Apogee}) \times 60 \qquad \ldots(6)$$

The actual daily motion (DM) is then calculated as follows:

$$\text{DM} = \text{DMM} - \text{DMM} \times \text{Cos}(\theta) \times \text{Ci}/\ 360.0; \qquad \ldots(7)$$

…where θ is the anomaly, and Ci is the instantaneous epicycle circumference in degrees.

We notice from Eqns. 6 and 7 that the daily-motion calculation appears to be purely Moon-oriented. There is no Sun-related term in them. However, as we know, the Sun does influence the Moon's motion greatly. Thus we may expect that the daily motion obtained by Eqn. 7 will not be the actual true daily motion. This is confirmed by Fig. 9a which shows a comparison of the daily-motion as obtained by Eqn. 7 and the actual daily motion for a single monthly cycle. It can be seen that there is a large mismatch. While the *Sūrya-siddhānta's* calculated daily-motion curve appears symmetric along the apsis line, the actual daily-motion curve does not appear so. Due to the Sun's influence, the actual daily-motion over a month will rarely, if ever, be symmetric along the apsis line.

From a single orbit, when we move to a large number of orbits (200), the daily-motion picture changes dramatically. This is shown in Fig. 9b. From this figure we note, firstly, that over a large number of orbits, the actual daily-motion data is spread symmetrically about the apsis and secondly, that the calculated daily motion as per the *Sūrya-siddhānta* is a sort of median daily motion.

If this was not amazing enough, the next concept will leave the reader astounded. The daily motion, it turns out, is the foundation of a second epicycle system.

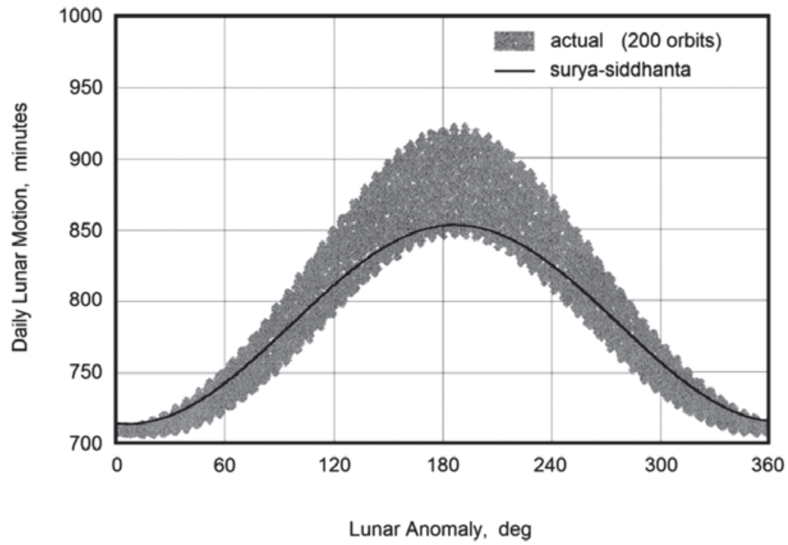

Fig. 9b. Actual and Calculated Daily Motion for 200 Lunar Orbits

7.5 The Secondary Epicycle

Consider the epicycle scheme as shown in Fig. 6. In the figure triangles ENC and CnM are similar. Thus we may write as follows:

$$CN/EC = Mn/CM \text{ Or,} \qquad Mn = (CM/EC) \times CN$$

…where Mn is the correction to be applied to the mean.

Now, CN is the sine of angle θ as expressed in Indian Chord table, the radius being constant at 3438. Thus we may write Eqn. 9 as follows:

$$Mn = (CM/EC) \times \text{Sin } (\theta)$$

Replacing the radius-ratio with circumference-ratio, we obtain:

$$Mn = (\text{Circumference of epicycle / Circumference of Deferent}) \times \text{Sin } (\theta) \quad \ldots(8)$$

Expressed in degrees, the circumference of the deferent = 360° or 21600 minutes. Thus Eqn. 8 may be written as follows:

$$Mn = (\text{Circumference of epicycle in minutes } /21600) \times \text{Sin } (\theta) \quad \ldots(9)$$

For comparison, we quote the earlier Eqn. 5 here again:

Second Correction = (Cos (Ls – La) × Daily-Motion / 21600) × Sin (Lm – Ls)

Comparing the two, Eqn. 9 and Eqn. 5, we see immediately that the second correction for the Moon, as given in the *Sūrya-siddhānta*, has the form of an epicycle. The circumference Ci of this new epicycle has the instantaneous value Ci = Cos (Ls–La)×Daily-Motion. The anomaly θ of the second epicycle is the angular separation between the Moon and the Sun. We note the interesting fact that this second epicycle is also of the pulsating variety.

Since it has two epicycles, it is natural to wonder whether the Indian model is similar to Al-Shatir's double-epicycle model. On careful scrutiny we see that it is not. In the Indian model the second correction has the same form as the first correction, meaning that the second epicycle too is centered on the deferent. The complete schematic of the Indian lunar model is shown in Fig. 10.

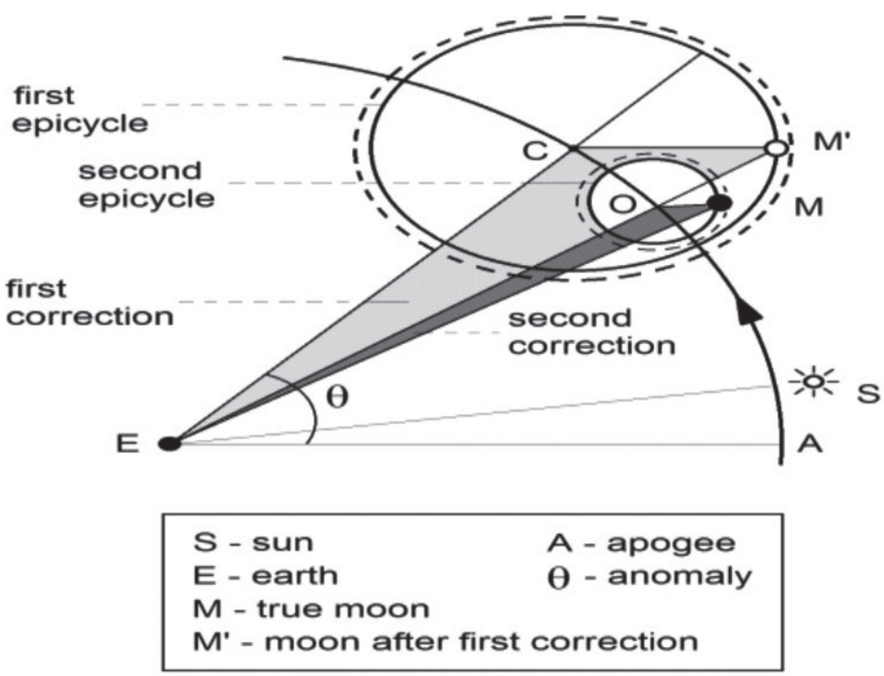

Fig. 10. The Indian Lunar Model

From the figure we see that the second epicycle is centered on the deferent on a line joining the earth and the Moon's location after the first correction. The anomaly for the second epicycle (angle MOM′) is equal to the angular separation between the Moon (M′) and the Sun S (angle SEM′). Also, due to the presence of a cosine function in the expression, the minimum value of the instantaneous circumference of the second epicycle will be zero. That is, the radius of the second epicycle will be zero when the Sun is perpendicular to the Moon's apsis line, since Cos (90°)=0, and consequently the second correction will be zero at that point.

Let us now determine the second correction as described above and examine how well it compensates for the error remaining after the first correction. The results are shown in Fig. 11. It can be seen that the second correction compensates for the first error exceptionally well. Thus we may rest assured that our interpretation of the ancient verses has been on the right track.

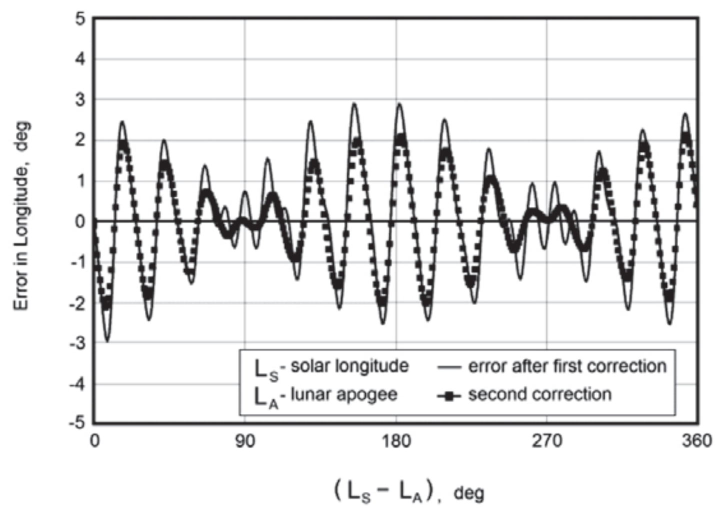

Fig. 11. The Second Lunar Correction in the Indian Model

8. Computational Results

Based on the above descriptions of the various lunar models, computer programs were prepared and executed. The results for the three lunar models: (1) Ptolemy (2) Al-Shatir/Copernicus and (3) the *Sūrya-siddhānta* are presented in the sections below.

8.1. Orbital shape

As expected, a computer simulation of Ptolemy's Model-III resulted in a greatly elongated oval orbit, as shown in Fig. 12. The mean lunar circular orbit is shown for reference. There is a small overlap in the orbit due to the shifting apsis line of the Moon from orbit to orbit. As mentioned earlier, in this model, at quadrature, the epicycle is pulled in greatly towards the earth resulting in the elongated appearance. The Al-Shatir and *Sūrya-siddhānta* orbits were much closer to reality, as seen in the next sections.

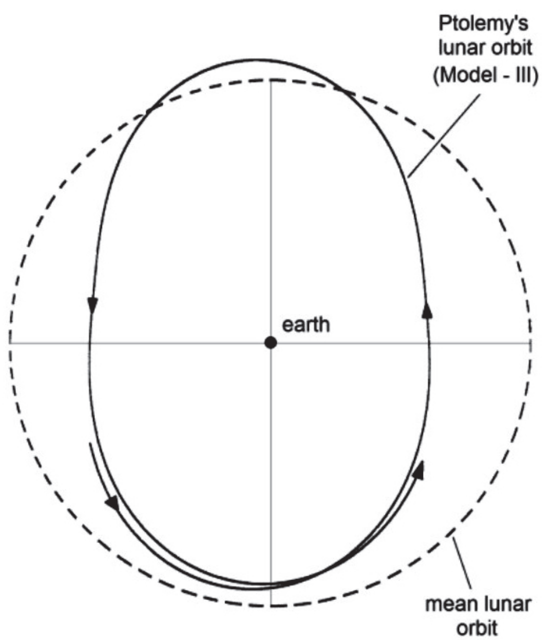

Fig. 12. Ptolemy's Lunar Orbit

8.2. Lunar Diameter

Maximum and minimum lunar angular diameters were computed for the three models and are shown in Fig. 13 along with the modern values. Ptolemy's Model-III produced a maximum angle of nearly 1 degree, 58.8 minutes to be exact, while the minimum was 30.77 minutes. The corresponding values for the Al-Shatir model, obtained from reference 11, were 37.97 and 29.033 minutes. Those for Copernicus, from reference 7, were 37.55 and 28.75 minutes.

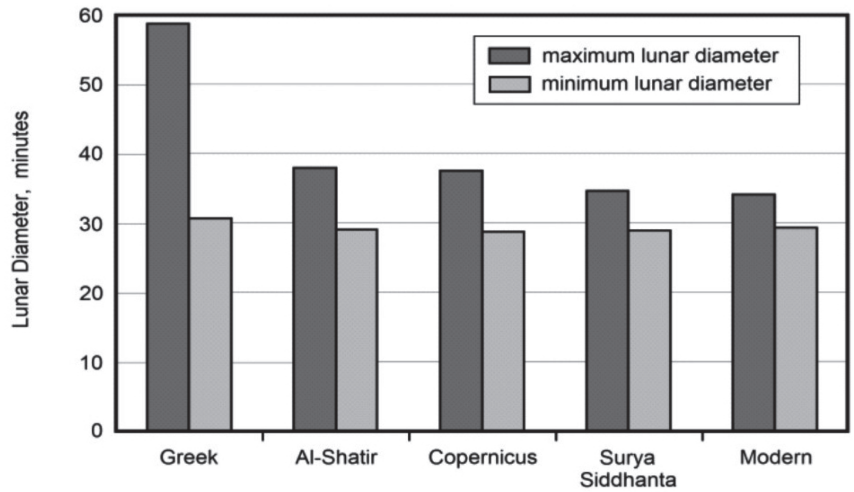

Fig. 13. Maximum and Minimum Apparent Lunar Diameter for Various Models

Unlike the other lunar models, the *Sūrya-siddhānta* explicitly describes how to determine the angular diameter of the Moon at any instant. The calculation is straightforward and proceeds as follows:

Angular Diameter =

Mean-Moon-Diameter × (Actual-Daily-Motion/Mean-Daily-Motion) …(10)

Note that the Actual-Daily-Motion is determined during the calculation of the second correction. The Mean-Moon-Diameter is determined as follows. It is stated in the *Sūrya-siddhānta* that one *yojana* on the Moon's mean orbit equals one minute of angular width as seen from the earth. Next, the Moon's physical diameter is given as 480 *yojanas*. Thus we obtain the result that the mean Moon angular diameter is 480/15 = 32 minutes.

The *Sūrya-siddhānta* computer model was run for a simulation interval of 5800 days and the angular diameter calculated using Eqn. 10. The maximum and minimum angular diameters obtained were 34.55 and 28.91 minutes respectively. It can be seen from Fig. 13 that amongst the various models presented in this paper, the *Sūrya-siddhānta's* values of lunar diameter are closest to actuality.

8.3. Longitudinal Error

Fig. 14a shows error histograms for the three Greek models of Ptolemy for a simulation period of 5800 days. Model-1 displayed an error range of -4.25° to 4.62°. Model-II showed -4.2° to 4° while Model-III produced a range of -3° to 2.94°. The gradual improvement from Model-I to Model-II can be seen in this figure.

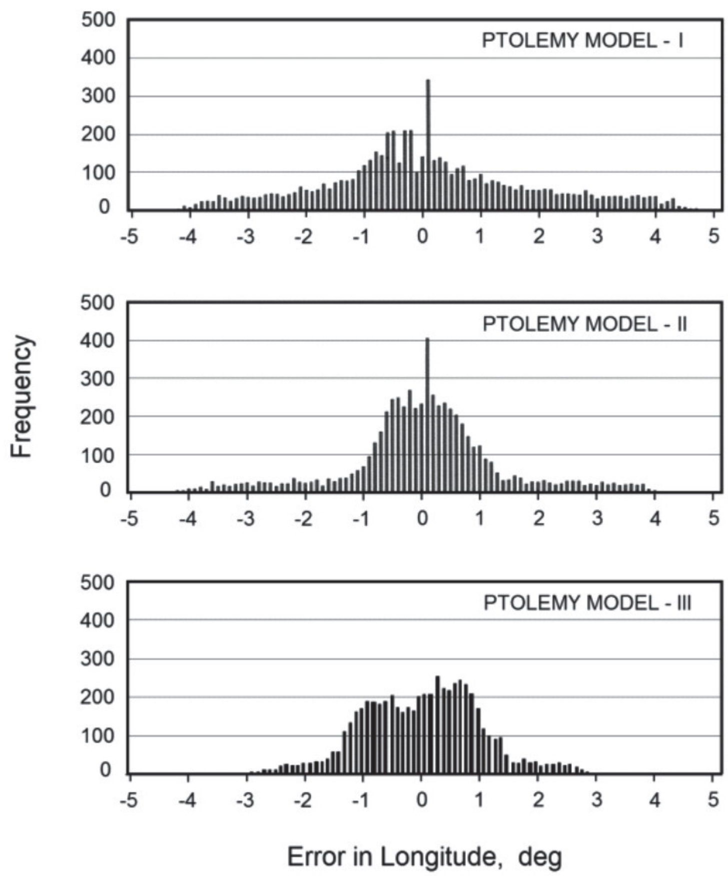

Fig. 14a. Error Histograms for Ptolemy's Lunar Models

Fig. 14b shows similar error histograms for Al-Shatir's model and that of the *Sūrya-siddhānta*. The Al-Shatir model displayed an error range of -2.75° to 2.55°. The *Sūrya-siddhānta* had the smallest error range of all, -1.3° to 1.32°.

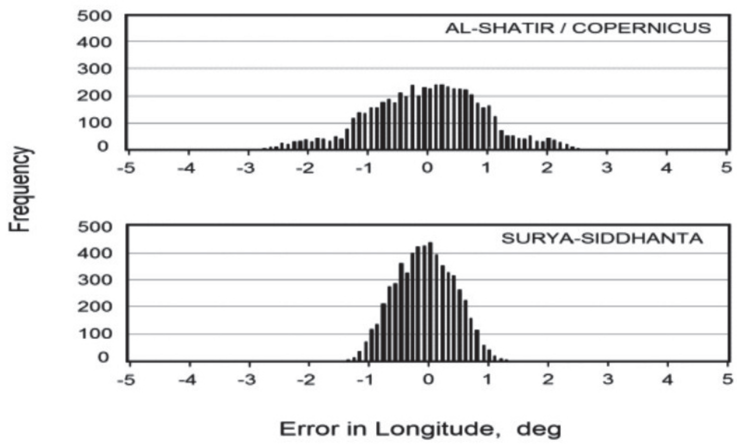

Fig. 14b. Error Histograms for the Al-Shatir/Copernicus and *Sūrya-siddhānta* Lunar Models

Fig. 15 shows the progressive decrease of longitudinal error in the *Sūrya-siddhānta* lunar model as we move from mean motion to the first and then second corrections, over a period of 5800 days.

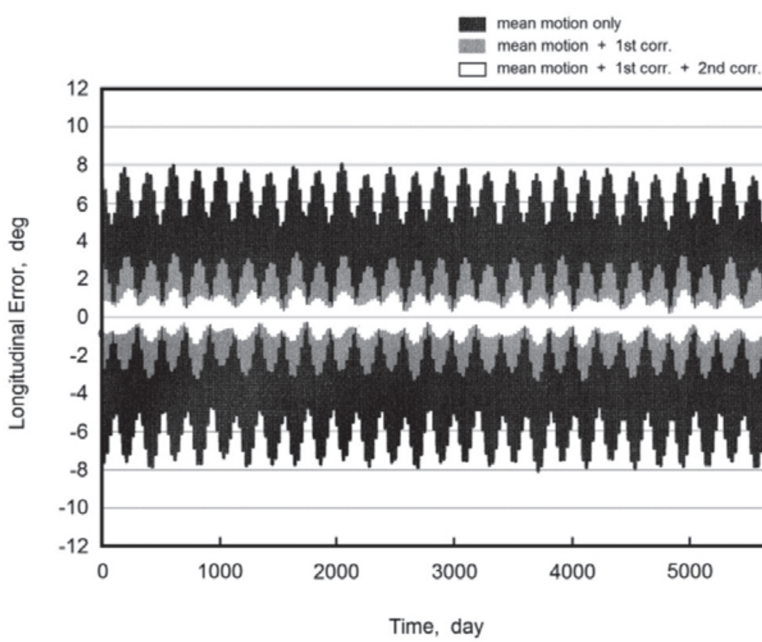

Fig. 15. Progressive Decrease of Error with Corrections in the Indian Lunar Model

8.4. Standard Deviation of Longitudinal Error

The Standard Deviation of a set is an indication of the dispersion from the mean. A low value indicates that data points tend to be close to the mean while high values indicate data points that are more spread out. As seen in Figs. 14a and 14b, the histograms are roughly symmetrical about the zero error point, which can be considered as the mean. The Standard Deviation of the error values for each model thus indicates their deviation from zero error. Each simulation run included 5800 data points. Table 2 shows the Standard Deviation obtained for each Model.

Table 2. Standard Deviation of Error for Various Lunar Models

Greek Model-I	Greek Model-II	Greek Model-III	Al-Shatir	*Sūrya-Siddhānta*
1.13°	0.94°	0.598°	0.587°	0.274°

Since the Moon's average angular diameter is about 0.5 degrees, these results imply that most of the time the Greek and Al-Shatir model errors are greater than the Moon's diameter. On the other hand, the *Sūrya-siddhānta's* error Standard Deviation is only about half the Moon's diameter, which means that the *Sūrya-siddhānta's* predicted value of the Moon's longitude falls inside the actual Moon's orb most of the time.

9. Discussion

Though Ptolemy's great work '*Almagest*' is very well-known in the west, not many in-depth studies appear to have been done to determine the accuracy of its models. In the author's knowledge, Petersen (1969), Newton (1977) and Van Brumellan (1993) are the only ones that have tested out Ptolemy's lunar models. Of these, Petersen's work is the most in-depth. However the longitude of the Moon he obtained was always less than the actual longitude and therefore he suspected a systemic error in his model formulation. His time range was 20 years with a total of 1461 data points. The error Standard Deviation values he obtained for Models I, II and III were 1.37°, 0.83° and 0.57° respectively. The corresponding maximum errors he obtained were 4.1°, 3.4° and 2.5°.

In the current study, for a time range of 20 years and 5800 data points, the error values obtained for Ptolemy's models were nearly symmetrical about zero, that is, there were equal numbers of negative and positive errors. Thus Petersen may have been correct to suspect that there was something wrong, either in his formulation, or in his Moon data. The error values and Standard Deviations obtained in this study for Ptolemy's models are somewhat greater than those found by Petersen.

The western penchant for instinctively praising anything Grecian can be seen in lunar astronomy also. One often comes across phrases like 'remarkable accuracy' to describe Model-III. Others extol the resultant orbit of Model-III as 'close to an ellipse' and hence the reason for the 'remarkable accuracy'. Let us briefly examine this. Considering the maximum and minimum earth-Moon distances in Ptolemy's model to be the major and minor axes of an ellipse, the eccentricity works out to be 0.85. The actual eccentricity of the Moon's orbit is only 0.055. Also, Ptolemy's 'major axis' is always aligned towards syzygy and 'minor axis' towards quadrature. The actual apsis line of the Moon's orbit has no such fixed directions. As we saw in this paper, Ptolemy's Model-III is neither 'remarkably accurate' nor practical with its enormous error in earth-Moon distance. Overall, it is a rather crude effort, perhaps one among dozens such in history.

Coming to the Indian lunar theory, the author has come across several articles where it is stated that the influence of the Sun on the lunar orbit was understood by the Indian astronomers only around 900 AD, about the time of Mañjula. However, as seen in this paper, this is not true. The *Sūrya-siddhānta's* instructions regarding the second correction are identical in form with those of Mañjula and later astronomers.

Indian astronomy has long been denied its rightful place in the astronomical achievements. As shown in this paper, Prof. Whitney and the editorial team of the translation of *Sūrya-siddhānta* of 1858 have misinterpreted the second lunar correction. With its dual pulsating epicycles, the Indian lunar model is the most complex as well as the most accurate of all ancient models.

Whitney and his team further considered the text as stating that all heavenly bodies move in perfect circles and that the orbit calculations were merely tools to account for the influence of the *Manda* and *Śīghra* disturbances. This is not true, as seen by the calculation for ascertaining the

Moon's angular diameter. As per the *Sūrya-siddhānta*, the Moon's angular diameter is proportional to its velocity (daily motion rate), which implies that the Moon is nearer to the earth when faster and farther away when slower. Thus it cannot possibly be moving in a perfect circle around the earth.

It is interesting to compare the size of the secondary epicycle in the Al-Shatir, Copernicus and *Sūrya-siddhānta* models. While the Al-Shatir and Copernicus models have a fixed size, that of the *Sūrya-siddhānta* is variable. Also, while the former model sizes seem to be arbitrary (perhaps arrived at by hit-and-trial), that of the *Sūrya-siddhānta* is derived from an actual orbital parameter (the daily motion). This strongly indicates that the *Sūrya-siddhānta* model is an original creation and not a borrowed idea.

Finally, one may wonder how the ancient astronomers came upon the Sun-related second correction term in the Indian lunar model. The exact nature of this second correction can only be discerned by someone having access to a vast collection of Moon-data.

10. Concluding Remarks

Among the Greek, Islamic, European and Indian lunar models, the Indian model is the most complex and the most accurate in longitude and angular diameter.

The Indian lunar model consists of not one but two epicycles. Both epicycles are of the pulsating type.

The second lunar correction as given in the *Sūrya-siddhānta* has the same form as those given by later Indian astronomers. It is not unlikely that the second lunar correction originated in India at an early phase.

Unlike the other lunar models the secondary epicycle diameter in the Indian model is not an arbitrary constant. It is derived from an orbital parameter. This strongly suggests that the ancient Indians were not borrowers but the original creators of their model.

Bibliography

Brennand, W., *Hindu Astronomy, 1896* (reprinted Caxton Publications, Delhi 1988).

Burgess, Ebenezer, *Sūrya-siddhānta - A Text Book of Hindu Astronomy*, 1858.

Godfray, Hugh, *An Elementary Treatise On The Lunar Theory*, Fourth Ed., Macmillan and Co, 1885.

http://en.wikipedia.org/wiki/Lunar_theory

Meeus, J, *Astronomical Algorithms,* Willmann-Bell Inc, 2000.

Narayanan, A., 'Dating the *Sūrya-Siddhānta* using computational simulation of proper motions and ecliptic variations', IJHS, 45.4 (2010) 455-476.

Narayanan, A., '*The Pulsating Indian Epicycle of the Sun*', *IJHS*, 46.3 (2011).

Newton, R.R., *The Crime of Claudius Ptolemy,* The Johns Hopkins University Press, Baltimore, 1977.

Petersen, Viggo, 'The Three Lunar Models of Ptolemy', *Centaurus*, 14.1(1969)142-171.

Ptolemy, *Almagest*, Translated by Catesby Taliaferro, Great Books of the Western World, Encyclopedia Britannica, 1952.

Ramasubramanian, K and Sriram, M.S.,*Tantrasaṅgraha of Nīlakaṇṭha Somayājī,* Hindustan Book Agency, New Delhi, 2011.

Roberts, Victor,'The Solar and Lunar Theory of Ibn ash-Shatir – A Pre-Copernican Copernican Model', *Isis*, 48.4 (1957) 428-432.

Stephen Hawking, *On the Shoulders of Giants*, Running Press, Nicolaus Copernicus, 2002.

Shukla, K.S., *A critical Study of the Laghumānasa of Mañjula*, Indian National Science Academy, 1990.

Sen, S.N. and Shukla, K.S., *History of Astronomy in India*, Indian National Science Academy, 1985.

Tilak, B.G., *The Orion or Researches into the antiquity of the Vedas*, Bombay, 1893.

Van Brummelen, Glen, *Mathematical Tables in Ptolemy's Almagest*, *PhD thesis*, Simon Fraser University, 1993.