BOOK REVIEW

Ṭhakkura Pherū Gaṇitasārakaumudī: The Moonlight of the Essence of Mathematics, edited with Introduction, Translation, and Mathematical Commentary by SaKHya, Manohar, New Delhi, 2009. Pages xlvi+279, Price Rs 995/-.

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The book, under review, presents a critically emended text, an English translation, and a detailed mathematical commentary of the *Gaṇita-sāra-kaumudī* (=*GSK*) which was composed by Ṭhakkura Pherū in the early part of the 14th century AD. A valuable introduction, a bibliography, four technical appendices, and three useful indices have also been included in the present book. The main portion of the volume has been divided into four parts as follows:

Part I, Introduction (pp. ix-xlvi) which consists of two sections respectively on Pheru's life and works, and Mathematics of *GSK*.

Part II, Text (pp. 1-41) which includes the Prakrit text of the *GSK* in five chapters in the transliterated roman script.

Part III, Translation (pp. 43-73) which is almost piece by piece translation that is nearly literal.

Part IV, Mathematical Commentary (pp. 75-193) which is indeed very elaborate.

The four appendices are on (i) Concordance of the *GSK* and other works, (ii) Type of problems, (iii) Index to numbers in the text, and (iv) Glossary. Then follows the Bibliography and the three indices respectively of Mathematical terms, Things mentioned in the text, and of Sanskrit/Prakrit authors and titles. These appendices and indices are of great help to scholars. But there is no general index! So that, for example, if one is interested in knowing as to where or in what context the Indo-Persian work of Muḥarrir (*see* Bibliography, p. 259) has been used in the present book, one has to search page by page for the information.

An important fact to note is that the present edition of *GSK* is solely based on the printed version (published about half a century ago) which inturn was based on a single manuscript. Moreover, the where abouts of the single original manuscript are also reported to be unknown (p. 6) so that re-examination and confirmation of text and interpretations seem to be difficult.

Ṭhakkura Pherū son of Chanda (or Candra) and grand son of the wealthy banker Seth Kalasa, was a Śvetāmbara Jaina and held a high position in the treasury of Alauddin Khalji, Sultan of Delhi from 1296 to 1316. The history of accesion and adventures of this king is instructive. After the murder of the scandalous Qaiqabad (the last sultan of the Slave Dynasity), the Delhi nobles installed Jalaluddin Khalji as Sultan in 1290. But he along with his sons met the same fate at the hands of his very ambitious nephew and son-in-law Alauddin Khalji who thus ascended the throne of Delhi in 1296. His army overran and conquered various Hindu kingdoms of Gujarat, Ranthambhor, Chitor, Malwa, Ujjain, Dhar, Mandu, and Chanderi. With similar conquests of kingdoms of South India, the victorious army returned to Delhi in 1311. The vast booty included 96000 maunds of gold and many boxes of jewels and pearls which were added to the treasury of Khalji. The abundance of his wealth indeed needed an able *Khajanchi* like Pherū to manage. Pherū served him and his successors.

In addition to *GSK*, Pherū's writings include at least six more works earliest of which is dated 1291. His *Vāstusāra* on traditional architecture was completed in 1315 and was quite popular. Its printed versions contain many additional *gāthās*. H.D. Velankar's famous *Jinaratna-kośa* (Poona, 1944) does not mention Pherū's *Jyotiṣasāra* and *GSK*. The unique manuscript containing 7 works of Pherū was discovered by A.C. and B.C. Nahatas around 1946 in a Jaina Library in Kolkata. It was a paper manuscript whose copying was finished in March 1347. They brought out a combined edition of the 7 works (Jodhpur, 1961) but changed order of presentation.

The group name SaKHYa stands for the four scholars S.R. Sarma, T. Kusuba, T. Hayashi, and M. Yano. Of these the Japanese trio is concentrated in Kyoto and forms the largest group-core which is working on Sanskrit texts on mathematics and astronomy. Prof. Sarma joined the groups in 2002 and the result is the present work under review. It is devoted to the memory of the great David Pingree with whom all above were closely linked.

The *GSK* (also called *Gaṇitasāra*) is claimed to be "not only the first full-fledged, mathematical text composed in *Apabhraṃśa*, but it also extends the range of mathematics beyond the traditional frame work of the earlier Sanksrit texts" (pp. xvi-xvii). Of course, Pherū has acknowledged the borrowing of his material from earlier teachers and shows strong influence of the works of Śrīdhara (c. 750 AD) and Mahāvīra (c. 850 AD). Pherū has also included new material and topics in *GSK*. These include mathematical riddles, magic squares, some problems of solid geometry, and rules for mutual conversion of Vikrama and Hijrī dates.

Some rules and problems of *GSK* are still said to be "not understood" and some others need more or fuller exposition (see pp. 120, 169, 181-182). In some cases the present reviwer's remarks and interpretations may be considered. The problem of number of cows (p. 83) has no significance unless the answer is found by L.C.M. (*niruddha*) already known to Mahāvīra. The problem of the erased digit (*GSK* 4.60) will be alright if only the unstated condition be remembered i.e. the sum of the digits of the given number be divisible by 9. The reason for choosing the correction factor 10/9 (stated to be "not clear", p. 152) may be explained as follows. Modification of an empirical rule by adjusting it to some suitable value of π is an ancient practice e.g. Śrīdhara's formula for the volume of a sphere (pp. 144 & 151)

$$V_1 = (d^3/2).(19/18) \qquad \qquad \ldots(1)$$

is an adjustment of empirical value ($d^3/2$) to $\pi = 19/6$ from $\pi = 3$. The very popular ancient rule for the area of a circular segment (of chord a and height h)

$$A_0 = (a+h)\ h/2 \qquad \qquad \ldots(2)$$

was supposed to be based on $\pi = 3$ for which it gives exact value in the case of a semicircle. It was adjusted to the Jaina value $\pi = \sqrt{10}$ in the form equivalent to the formula

$$A = (a+h)\ (h/2).\ \sqrt{10}/9 \qquad \qquad \ldots(3)$$

which is frequently found in India (p. 147). But if the rule (to be modified) is supposed to involve square of π or of the circular circumference C, then the Jainisation factor $\sqrt{10/9}$ should also take its square form 10/9. This was done by Mahāvīra to modify the empirical rule

$$V = (9/16)d^3 \qquad \qquad \ldots(4)$$

to find "accurate" volume of a sphere (p. 151) although his Jainisation made it bad to worse (10 V/9). That (4) involves π^2 is illustrated sufficiently by the present reviewer (*Journal of Asiatic Society*, Kolkata Vol. 30, 128-140; *Historia Scientiarum* No. 42, 33-44, etc.)

Pherū applied the factor 10/9 to modify the rule

$$S = C^2/4 \qquad \qquad \dots(5)$$

for surface of a sphere as well as to $V = d^3/2$, although he was confused in the latter case which involves π and not π^2.

Significantly, although Pherū stated rule (4) in slightly different form elsewhere (see pp. 191-192), the accompanying numerical example ($d = 6$) there was still solved actually by his Jainised formula

$$V_2 = (d^3/2).(10/9) \qquad \qquad \dots(6)$$

which gives the mentioned answer $V = 120$. It may be noted that had Pherū applied the proper Jainisation factor $\sqrt{10/9}$ here (and not 10/9), he would have got a far better and accurate rule for the volume of a sphere.

SaKHya claims that *GSK* "throws valuable light on the development and popularization of mathematics in northern India in the early 14th century" (p. xviii). It is surprising as to why more manuscripts of the work or some simpler expositions of it are not found (like the *Patan Manuscript*).

Anyway, the present book is an excellent edition and translation of the *GSK* with so much additional matter and notes. It should find a place in all libraries related to History of Science and Orientology etc. Some one may bring out the text in Devanāgarī script with Sanskrit *Chāyā* and possibly a Hindu translation and a cheaper paper back edition for it.