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Mathematics and Astronomy in Medieval India*

A two–day Seminar on History of Science with special emphasis on Mathematics and Astronomy in Medieval India was organized at IIT, Madras by Indian National Science Academy in collaboration with Department of Theoretical Physics, University of Madras and Department of Physics IIT Madras on Oct.25-26, 2010. The purpose of the seminar was to create awareness among the younger university students towards the researches being carried out in history of science in India and to appreciate the relevance in university curriculum. This was amply highlighted by Professor R Gadagkar, Chairman, Research Council for History of Science in his introductory remark. Dr. AK Bag presented a detailed account of the origin and growth of history of science in India and the researches done under the auspices of Indian National Commission for History of Science ever since its inception in 1965. He emphasized the need to cultivate and encourage the young scholars for taking up researches in this area. Professor MS Sriram, the local coordinator of the seminar gave an account of the work being done in this area in south India, specially the role of medieval astronomers and mathematicians from Kerala. A visit to KV Sarma Foundation was organized where rare and invaluable manuscripts and unpublished monographs in the area of mathematics and astronomy were displayed. These were the contributions and collections of Professor Sarma during his life time.

During the two-day seminar the following papers were presented:

Mādhava – A great Kerala Mathematician of Medieval Times: A K Bag, traced the origin and development of Kerala school of mathematics and astronomy with special reference to Mādhava (c. 1340-1425Ad) (born in a village Sangamagrāma near the temple of Samgameśvara, modern Irinjalakkuda, lat 10 20’ N near Kochi-Cochin) who was at the top of the lineage of the medieval Kerala mathematicians followed by Parameśvara, Nīlakaṇṭha, Jyeṣṭhadeva, Śankara, Nārāyaṇa, Acyuta Piṣāroṭi and others. From a number of citations available in the works of later scholars it is shown how Mādhava had achieved the value of π (correct to 10^{th} place) in mathematics and used Interpolation Techniques for fixation of π, sine and co-sine series.

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Yuktibhāṣā and the case for Calculus : PP Divakaran raised the question of whether the methods used by Mādhava (as described in the later texts) to derive the π and sine series can be considered as the origin of the discipline, we now call calculus. He compared the detailed description provided in Yuktibhāṣā about the derivation of these series as well as the formula for the surface area and the volume of the sphere from today's calculus point of view. The approach to sine series is however quite distinctive. He tried to trace the source of the Calculus of the Nīlakaṇṭha school.

The Origin and Growth of Mathematics in India: K Ramasubramanian, provided a brief overview of the important contributions found in the Indian mathematical treatises, generally composed in Sanskrit in verses. He started with the *Sulvasūtras*, the oldest extant texts that deal with mathematics which explicitly stated and made use of the so-called Pythagorean theorem apart from giving various approximations to surds, in connection with the construction of altars and fire-places of different sizes and shapes. By the time of Āryabhaṭa (c. 499 AD), the Indian mathematicians were fully conversant with most of the mathematics that we currently teach in our schools. Among other things, Āryabhaṭa also presented the differential of sine function in its finite difference form and a method for solving linear indeterminate equation. The '*bhāvana*' law of Brahmagupta (c. 628) and the '*cakravāla*' algorithm described by Jayadeva and Bhāskarācārya (12th cent.) for solving quadratic indeterminate equation are some of the important landmarks in the evolution of algebra. The tradition seems to have flourished well into the medieval times, culminating in the discovery of calculus by the Kerala School of astronomers. Mādhava (c.1350) enuntiated the infinite series for $\pi/4$ (the so-called Gregory Leibniz series) and other trigonometric functions. The series for $\pi/4$ being slowly converging series, Mādhava had also given several fast convergent approximations to it. Interesting proofs of these results are presented in the famous Malayalam text *Gaṇita-Yuktibhāṣā* (c. 1530) of Jyeṣṭhadeva.

Microbiology, Cancer and Genetics in Classical Ayurvedic Literature - A Historical perspective: P Ram Ramanohar, highlighted the contribution of Āyurveda to the history of medical ideas, need and scope for a fresh enquiry into the subject from development of world medicine. A careful study of classical Āyurvedic literature reveals that Āyurveda had developed very interesting ideas and notions about various aspects of the human body in health and disease. The presentation focused on information in the ancient texts of Āyurveda on

Microbiology, Cancer and Genetics from a historical perspective. Āyurveda was aware of the role of worms and germs (microbes) in the causation of disease and about communicable diseases and how the microbes spread from one person to the other by contact or use of clothes, foot wear etc. Cancer was also known to ancient Āyurvedic physicians. The depth of information on cancer available in the ancient texts needs to be explored and brought to the limelight. The most obvious term in the texts *Caraka* and *Suśruta* that correlates with cancer is *arbuda*. *Arbuda* cannot be explained without referring to a related term *granthi*, together the terms *arbuda* and *granthi* make a clear distinction between malignant and benign tumours. Texts like *Caraka Saṃhitā* and *Suśruta Saṃhitā* make several references to genetic diseases, birth defects and also distinguish between congenital and genetic disease. Diseases like hemorrhoids and diabetes have been mentioned to have a genetic background.

Study of Magic Squares in India: M D Srinivas explained the concept of magic square which is a square array consisting of numbers whose rows columns and principal diagonal add up to the same number. India has a long history of magic squires going back to the very ancient times. For example, the work of the ancient seer Garga is supposed to contain several 3x3 magic squares. Later, a general class of 4x4 magic squares has been attributed to the Buddhist philosopher Nāgārjuna (c.2nd Century AD). In *Bṛhatsaṃhitā* of Varāhamihira (c.550 AD), a mention is made of the magic square (referred to as *sarvatobhadra* or "auspicious all around"). A 4x4 pan-diagonal magic square is inscribed in the entrance of a Jaina temple at Khajuraho (c. 12th century). The Prākṛta work, *Gaṇitasārakaumudī* of Ṭhakkura Pheru (c.1300), discusses the classification of nxn magic squares of three types (a classification which is employed later by Nārāyāṇa Paṇḍita also) viz. *Samagarbha*, *Viṣamagarbhya*, and *Viṣama*. Ṭhakkura Pheru also indicates methods of constructing *samagarbha* and *viṣama* magic squares. It was the renowned mathematician Nārāyāṇa Paṇḍita who gave a systematic account of magic squares in the fourteenth chapter of his work *Gaṇitasārakaumudī* (c. 1356 AD) entitled *Bhadra-gaṇita* or "auspicious mathematics".

Nārāyāṇa's treatment of varga-Prākṛti: Amartya Kumar Dutta, referred to the efficient analysis of the hard and subtle number-theoretic problem of finding all integer solutions of the *varga-prākṛti* – the quadratic indeterminate equation $D x^2 + 1 = y^2$. The problem was discussed by Nārāyāṇa Paṇḍita (c.1350 AD), whose contributions in this area got overshadowed by his brilliant predecessors Brahmagupta, Jayadeva and Bhāskara II. He briefly discussed the main results and algorithms developed by Indian algebraists, highlighted some subtle touches in the exposition of Nārāyāṇa on *varga-prākṛti* and some noteworthy features in the treatment of Nārāyāṇa.

Development of Combinatorics in India : From Piṅgala to Nārāyāṇa Paṇḍita: Raja Sridharan traced the study of combinatorics in India beginning with the work of Piṅgala (c. 4th century BC) on *Chandas* or Sanskrit Prosody. In *Chandaḥśāstra*, Piṅgala deals with the combinatorics underlying Vedic and Classical Sanskrit metres, a finite sequence of long and short syllables (*guru* and *laghu)*. Piṅgala presented a way of enumerating all metres of a given length in terms of *prastāra* (or spreading) which has become a paradigm for all enumeration problems in Indian combinatorics. Piṅgala also introduced a tabular figure called *meru* as a device to compute the number of metres with a given number of long or short syllables (*lagakriyā).* His *meru* turns out to be the earliest known version of the so called Pascal triangle. In his treatise on music, *Saṅgītaratnākara*, Sārṅgadeva (c. 1250) discusses combinatorial problems related to the enumeration of all possible of permutations of the seven basic musical notes (*tāna prastāra*). In the work of Nārāyāṇa Paṇḍita (c. 1350), the Indian contribution to combinatorics seems to have attained its culmination. In the chapter *Aṅkapāsa* of his *Gaṇitakaumudī*, Nārāyāṇa reformulates most of the earlier work on combinatorics in a general mathematical setting – where mathematical generalizations are considered without specific practical applications in view. During the presentation, he compared the combinatorial and factorial representations of integers.

Problem of Technology Change in Pre-Modern India: Ishrat Alam began by asserting that Indian technology was not static by any means by citing series of examples from ancient and medieval periods of Indian history. Both internal innovation and diffusion from outside were responsible in introducing changes in technology. Of course, the changes were neither even nor rhythmic. Indeed they were there. He cited relevant examples from agriculture, textile, mining and metallurgy, building, military technology, bridge construction, transport and navigational, sectors. He pointed out various factors possibly responsible for the sluggishness of internal innovation and the imperviousness to importation. He underpinned especially the availability of abundant skilled labour, caste system, the imperviousness of mercantile class, aristocracy, the rulers (except kings like Bhoja and Akbar) and their nobility, ideological orientation of the people, lack of recognizable relationship between with theoretically sciences and technology. He concurred with Irfan Habib's contention that ideologically tolerance of science in India was caused not by "unqualified belief in reason but by a deep belief in pantheism which preached respect for diversity". The doctrines of Ibn al-Arabi (d. 1240) had generated tremendous influence on Muslim mind and this possibly created tolerance of rationality which in turn created atmosphere for development of scientific ideas in the society.

Survey of Arabic Persian Sanskrit Sources on Astrolabe Extant in India and on the Indian School of Astrolabe-makers: S M Razaullah Ansari discussed the transmission of Islamic astronomy and mathematics and its impact on Indian astronomy during 12th - 19th centuries. In the this context he referred to the characteristic feature of Islamic practical astronomy, the establishment of various observatories in the Islamic countries, for instance, Marāgha Observatory in 13th century (Director, Nasīruddīn Tūsī) Samarqand Observatory in 15th century (founded by Sultan Ulung Beg) Istanbul Observatory (Director, Taqūīuddīn), etc. The most important instrument developed by Islamic astronomers was the *Astrolabe* (in Arabic *Asturlāb)* and reported that as many as 349 tracts were written on astrolabe (Matvievskaya and Rozenfeld 1983). The presentation highlighted the contribution of Medieval Indian astronomers to the promotion and popularity of this instrument. Isfahan school of astrolabe-makers sprang up in India also during the Mughal period. He reported briefly about the school of Alahā h Dād and his descendants (known as Lahore school). During the Sultanate (pre-Mughal) period, a court astronomer of Sultan Fīrūz Shāh Tuglaq (d. 1388), Mahendra Sūrī, wrote a book on astrolabe in Sanskrit, with the title: *Yantrarāja* ("the King-instrument"). This was the first book on astrolabe in Sanskrit that initiated a spade of Sanskrit tracts on astrolabe later on.

Venvaroha from a Modern Perspective: S. Madhavan explained, how the well-known *Candravākyas* of Vararuci (4^{th} Cent. AD) provide a method of finding the longitude of Moon for any Kaliday, at sunrise at Lanka and how corrections like *cara*, *desāntara* etc. can be effected to get the longitude of the Moon for any place at sunrise on any day. For a time other than sunrise one can use linear interpolation. But the results obtained thus are not accurate. To rectify this inaccuracy, Mādhava of Saṅgamagrāma (24^{th} Cent. AD) devises an ingenious method in which day is divided into nine parts and the longitude corresponding to the beginning of each part is obtained. From a modern point of view the method can be interpreted in terms of periodic functions with certain properties. Madhava also gives revised *Vākyas* in the place of *Candravākāyas* of Vararuci.

Sines and Interpolations Techniques in Indian Mathematics: V Madhukar Mallayya elaborated on how Indian astronomers developed various methods for computing sine tables that are needed for interpolation of desired functional values needed for correct astronomical purposes. Accuracy in a computed value depends on the accuracy in the tabular values that are involved in the process as well as on the method of interpolation. He explained various methods thus developed for computation of highly accurate tabular values and tabular differences and also for interpolating desired functional values. Methods for computing desired values without using tabular value were also known and explained in a few instances during his presentation.