which is said to have been devised by Āryabhaṭa. According to Gupta and Hayashi, et al., *Vīrasenācārya*'s formula considers Āryabhaṭa I's value to be a good ideal as it is deductible from the equation ax-by-c = 0. Harishankar has given details of a Kannada mathematical work *Vyavahāra Gaṇita* by Rajaditya, the poet and scholar (c 1190 AD).

The last topic is one the dates of Mahāvīrācārya and Śrīdhara. Gupta has given various opinions of various historians about Śrīdhara's date.

Thus the book under review is indispensable for the historians of mathematics.

There have remained some errors of printing which may be removed in the next edition.

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***Gaṇita-Yukti-Bhāṣā* of Jyeṣṭhadeva,** 2 Vols, Vol I: Mathematics, Vol II: Astronomy, Hindusthan Book Agency, New Delhi, 2008*.

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The *Gaṇita-Yuktibhāṣā*, a Malayalam work in mathematics and astronomy, is now available in two volumes, and contain the Malayalam text critically edited with English translation by K.V.Sarma, along with explanatory notes in English supplied by K.Ramasubramanian, M.D.Srinivas and MS.Sriram..Vol. I is devoted exclusively to mathematics, and Vol II to astronomy with one epilogue in each volume. The mathematics portion was edited sixty years back with notes by Ramavarma (Maru) Tampuran and A.R.Akhilesvara Iyer(Trichur, 1948), and both mathematics and astronomy portion(Sanskrit version) edited by K.V.Sarma (published by Shimla Institute of Advanced Studies, 2004) with an introduction in English. It is not clearly known which of the Sanskrit or Malayalam version, is original, though it is a fact that the translation and notes contain mainly the Sanskrit chapter break ups and Sanskrit technical terms. It would have been meaningful if the Sanskrit text could also have been published along with this edition for better appreciation.

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The present effort with English translation and modern interpretation is indeed laudable. There was originally some confusion about the authorship and the time of the work. The present edition has however correctly identified the author as Jeṣṭhadeva (1500-1600) on the basis of a Malayalam commentary on the *Sūryasiddhānta* (Ms. No.9886) in the Oriental Institute, Baroda. Jeṣṭhadeva was not only a pupil of Dāmodara (son of Parameśvara, a disciple of Mādhava) but also a junior colleague of Nīlakaṇṭha Somayājī , the versatile scholar known for his commentary (*Āryabhaṭīabhāṣya*) and many other works in the field of mathematics and astronomy. Mādhava,the father of the Kerala tradition was, in a way, the *parama-guru* of Jeṣṭhadeva who appears to be a competent scholar trained in the same tradition in the field. The Malayalam version of the text of Yuktibhāṣā along with English translation and notes in two volumes containing mathematics and astronomy are indeed a great contribution by itself. The Vol I has seven chapters and an epilogue on proofs (*upapattis*) on Indian Mathematics .

Chap I: *Parikarma*- deals with addition, subtraction, multiplication, division, square, squaring, square-root along with some special operational techniques following the methods of identities.

Chap II: *Daśapraśna* deals with how to find two numbers when the sum, difference, product, sum of squares, or difference of the squares of two numbers are known.

Chap III: *Bhinnagaṇita* deals with fractions and their conversion to the same denominations for addition and subtraction, along with their operations as usual for multiplication, division ,squares and square-root.

Chap IV: *Trairāśika* (rule of three) and *vyasta-trairāśika* (inverse rule of three).

Chap V : *Kuṭṭkāra*(pulveriser) of the type by=ax ±1 and by = ax ±c along with their solution by *valli* process (continued division) and application for finding mean motions of planets.

Chap VI: *Paridhi-vyāsa(*the relation between circumference and radius), *Śodhya-phalas* (itenerative process), *Sankalita* (repeated summation), *Cāpikaraṇa* (arc to arc-sines), *Antya-samskāra* (correction of final term in the series)

Chap VII: *Jyānayana*—derivation of Rsines, Rcosine and Rversine, accurate computation of Rsine and Rcosine at a desired point and from *sankalitas*, area of the cyclic quadrilateral, derivation of a shadow, surface area of a sphere, volume of a sphere and rational etc.

Epilogue: deals with nature of proofs in Indian mathematics.

Chaps VI and VII are of special interest in this volume. Chapter VI deals with a relation: $a^2 + b^2 = r^2$ within a circle where a = *koṭi* (base, *kojyā* or r cosine), b= *bhuja* (perpendicular, *jyārdha* or r sine) and r =*karṇa* (hypotenuse), and its application for finding the perimeter of a circle when the side of its circumscribed regular polygon was known. Summation of *bhuja-khaṇḍas* in series like *bhuja –sankalita* [$1 + 2 + 3 +\ldots + n = n(n+1)/2 = n^2/2$, when n= very large], *bhuja- varga-sankalita* [$1^2 + 2^2 + 3^2 +\ldots + n^2 = n^3/3$],.. *samaghāta-sankalita* [$1^k+2^k+3^k+\ldots+n^k= n^{(k+1)}/(k+1)$] for calculation of circumference has been systematically attempted. In a similar way, starting with *ādya-sankalita* [same as *bhuja-sankalita* = $n^2/2$], technical results like *dvitīyā-sankalita* [1/2 *bhuja- varga-sankalita*=$1/2(n^3/3) =n^3/3!$], *tṛtīya-sankalita* [1/3! *bhuja-ghana-sankalita* = $n^3/3!$], … and, *sankalita-sankalita* [1/k! *samghāta-samkalita* = $n^{(k+1)}/(k+1)!$] are defined. There is no doubt that these results are extension of Āryabhaṭa I's results (*ĀBh*.Ganita, 19-22) in Kerala school.

Circumference is expressed in terms of diameter . Attempts have been made to divide one-eighth or one-twelfth of the circumference into arc-bits to obtain the corresponding *bhujas* or *jyārdhas* (b_i s),and approximating the corresponding arc-bits with *bhuja-khaṇḍas* by applying iterative corrections and equating them to the sum of the *jyārdhas*, the results obtained are,

$C/8= r -r/3 + r/5\ldots$, or, $C= 4d (1-1/3 + 1/5..)$, and.

$C= \sqrt{(12d^2)} (1- 1/(3.3) +1/(3^2.5)- 1/(3^3.7) +\ldots$.

The results are considered in infinite series in which the denominators keep on slowly increasing, and performed a last correction, after nth term to find the value of the circumference to a high degree of accuracy, without having to evaluate a large number of terms.

The *Cāpikarṇa* deals with an arc, s= rθ, where the s, the arc of circle of radius r makes an angle θ,

Both *jyā* θ (r sineθ) and *kojyā* θ (r cosine θ) are so related that if one increases, the other will decrease and vice versa. Further when *jyā* is less *kojyā* is more and vice versa, Jyeṣṭhadeva also gives a result which is equivalent to:

$s = r\theta = r$ (jyā θ/ kojyā θ) – (r/3)(jyā θ/ kojyā $\theta)^3$+ (r/5)(jyā θ/ kojyā $\theta)^5$-…

Some results are extremely ingenuous and have closely followed Mādhava (c.1400 AD)and other scholars, which are indeed undoubtedly unique contributions of Kerala astronomers. Chap VII considered the derivation of 24 *r* sines (or Rsine table) by dividing the quarter-circumference of radius *r* into 24 points at an interval of 3^0 45′. The other functions like, *koṭi* (r cosineθ), *bhuja* (r sineθ), *utkramajyā* (= r – r cosine θ = r verseθ), , *bhuja-khaṇḍa*, *koti-khaṇḍa*, *jīva-khaṇḍa*, *khaṇḍa-jyā* and their mutual relationship were also considered and computed using various correction terms. He has also computed the correct formulae of the circumference of a circle making use of summation method, r sine of the sum of two angles, area of the triangle, some square properties, r sine- shadow, surface area and volume of a sphere & cyclic quadrilateral with geometrical or algebraical rational.

The Volume II containing chapters from VIII to XV(eight chapters) deal with astronomical problems as follows:

Chap VIII: Planetary theory and computation of mean and true planets,

Chap IX: *Bhū-vāyu-bhagola*, & *Ayanacalana* (Celestial sphere with great circles, their secondaries and parallels),

Chap X: Fifteen types of astronomical problems relating to spherical trigonometry and their solutions,

Chap XI: *Digjñāna* (Orientation), *Chāyāgaṇita* (Direction, snomonic shadow computation and timings), *Lagna* (Rising point of the Ecliptic), *Nati* (Parallaxes of Latitude) and *Lambana* (Longitude);

Chap XII: *Grahaṇa* (Eclipses) and Parallax correction,

Chap XIII: *Vyatīpāta* (Sun and Moon having same declination),

Chap XIV: *Dṛkkarma* (Visibility correction of Planets),

Chap XV:*Candraśṛngonnati* (Moon's Cusps and Phases of the Moon).

The Epilogue deals with Indian Planetary model by Nīlakaṇṭha Somayajī (c.1500 AD).

The eighth chapter deals with planetary theory based on the hypothesis that the planets including moon are supposed to move in circular orbits with reference to east-west and north-south directions or lines (apse lines), and their rates are uniform., angular velocities or the number of degrees or *yojanas* each planet moving per day is fixed. The constant orbital motion of a planet is disturbed by three factors, *mandocca* (apex of the slowest motion), *pāta* (ascending node of the planet's orbit), and *śīghrocca* (apex of the fastest motion). Both excentric and epicyclic models are adopted to explain these motions and to find true positions of planets. The *manda* correction (equation of centre) applied to mean sun gives the true sun and no other correction is necessary to the mean sun. For moon, *pāta* correction is necessary along with *manda* correction since it drives away from the path of the ecliptic sometimes to the north or south (*vikṣepa*). These two corrections are usually applied to find the true moon and no further correction is needed. For inferior planets (Mercury and Venus) the *manda* correction(equation of centre) was wrongly applied to the mean sun, instead of mean heliocentric planet , in the earlier Indian texts. Similar mistake was made by Ptolemy in his *Almagest*. It was Nīlakaṇṭha who first applied the *manda* correction to mean heliocentric planet instead of mean planet both for inferior and superior planets. This departure along with *pāta* correction led him to suggest a geometrical representation in which the planets move in eccentric orbits around the mean sun, which moves itself around the earth. The *śīghra* correction (from heliocentric to a geocentric direction) in a geocentric universe is correctly formulated for both inferior and superior planets. Jyeṣṭhadeva also appears to be quite comfortable in manipulating coordinate transformation from one system to another.

The ninth chapter gives details of geometrical parameters and definitions of celestial spheres, great circles and precession of equinoxes dealing with meridian, horizon, equator & ecliptic along with their poles and secondaries, parallel day circles, obliquity of the ecliptic (angle between equator and ecliptic), first point of Aries (commencing point, meeting point of equator and ecliptic) and its movement etc which are very close to modern system. The astronomical elements of horizontal, equatorial and ecliptic systems of which the reference to right ascension (*krānti-koṭi*) & declination (*nati*), in equatorial system is unique, and a few formulae like: $\sin \delta = \sin\lambda \times \sin\omega$ etc based on the properties of similar and spherical triangles are established. The backward motion of first point of Aries (*ayana calana*, precession of equinoxes) was also considered. The determination of any two elements at a time out of five elemets—altitude, hour angle, declination, azimuth and latitude giving ten types of problems in solution of astronomical triangles was given before by Nīlakaṇṭha in his *Tantrasamgraha* .

Jyeṣṭhadeva, however, deals in the tenth chapter of *Yuktibhāṣā* the solution of fifteen problems arising out of six different elements—longitude,R.A.,declination, obliquity of the ecliptic, *krānṭi-koṭi* and *nata* (inverse R.A. and inverse declination w.r.t sun) , last two elements not defined by other scholars before.

The eleventh chapter gives details of method for finding east-west and north-south lines from the gnomon-shadow , time from the shadow, corrections due to change in declination of the sun over a day. A interesting feature is found in its inclusion of the effects of the finite size of the solar disc and the solar parallax in the determination of the latitude of a place. It is far more systematic than earlier texts but lots of information remained unexplained. It refers also procedures for finding *kālalagna* (time elapsed after the rise of the Vernal equinox over the horizon), and corrections to the latitude and longitude, found crucial in eclipse corrections.

The twelfth chapter deals eclipses of the sun and moon in which the problems of finding actual distances of sun and moon from the centre of the earth from their corresponding mean distances, apparent sizes of the solar and lunar discs, as well as parallaxes were the main issues for considerations. For calculating actual distances, two corrections due to eccentricities of the orbits, and evection and deficit in the equation of the centre specially for the moon are needed. The second correction of the moon (evection) was referred to first by Ptolemy, subsequently appears in *Laghumānasa* of Manjulācārya, Nīlakaṇṭha's *Tantrasamgraha*, and even *Yuktibhāṣā*. This problem was discussed before by K.S. Shukla (1945).The chapter also discusses procedures for calculation of half-durations, eclipsed portion at required time, including parallax etc.

The thirteenth chapter deals with a peculiar topic (*Vyatipāta*) when the declinations of sun and moon are same but their rates of change have opposite signs. Though detail method for calculating declination of moon is given, it is bound by astrological implication in Kerala tradition that no good work should be taken up on the *vyatipāta* day.

The method for finding the *lagna* corresponding to the instant at which a planet with latitude rises in the eastern horizon is discussed in the fourteenth chapter.

The fifteenth chapter, dealing with the finding of angle between lunar crescent and the horizon, is essentially used to measure the distance between centres of lunar and solar discs (*bimbāntara*) on the celestial sphere and the effect of parallax. The method ends abruptly and remains incomplete. .

The *Yuktibhāṣā*, as the name suggests, is a book of rationale or explanation supplied to earlier knowledge of astronomy.The explanation is lucid, though no diagram is supplied for explanation, as if the models are already known. The modern diagrams and explanatory notes supplied by co-authors of Prof K.V.Sarma, viz Ramasubramaniam, Srinivas amd Sriram , are very pain-staking and a genuine attempt which is exemplary. The explanatory notes have not followed the style of a commentator by giving a line by line explanation, rather effort is made to put the subject on a unique and modern foundation. In places it is difficult to appreciate the text, and the explanation in many cases in the text remains unclear and vague. It is also difficult to delineate the contributions of Jyeṣṭhadeva from other Kerala scholars, specially from Nīlakaṇṭha, which is obvious since both of them were students of Dāmodara. Inspite of some small drawbacks, I am sure the work is to be appreciated to a great extent. We are also looking forward to the other works of Kerala astronomers, especially of Mādhava, Parameśvara, and Nīlakaṇṭha from the same group which will help us to assess the contributions of Kerala scholars and their standing in world mathematics and astronomy in the sixteenth century. The book is a must for every scholar in the field and should deserve a place in all scholarly libraries in India and abroad.

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