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THE JEWISH CALENDAR AND ITS RELATION TO THE CHRISTIAN HOLIDAYS AS DESCRIBED BY A MUSLIM MATHEMATICIAN-ASTRONOMER IN 852 AD

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We bring to light a treatise written in Arabic in 852/3 AD by the Muslim mathematician al-Qâ'inî, who discusses the Jewish permanent calendar and its relation to the Christian holidays. We present the translation of the full text of the document along with its analysis emphasizing its major contribution to our understanding of the development of the Jewish calendar in respect to the Christian calendar. In particular it is shown that the version of the Jewish calendar described by al-Qâ'inî had been different from the present permanent calendar the details of which were described a century later.

The document, found in the Khuda Bakhsh Oriental Public Library in India, is shown to be the most important source describing the development of the present permanent Jewish calendar and its relation to the Christian holidays. Parts of the discussion in the document related to the correspondence of the Jewish calendar to the Christian holidays of 'Palm Sunday', 'Good Friday', Easter, and the 47 days of the Lent fast, were used in a modified version by al-Bîrûnî in his historical reviews 150 years later without mentioning al-Qâ'inî's contribution.

Key words: Islamic astronomers, Jewish calendar, Jewish and Christian Holidays, al-Qâ'inî.

1. IBN BÂMSHÂDH AL-QÂ'INÎ AND HISTORICAL BACKGROUND

The development of science in general and astronomy in particular can be studied through its direct influence on the daily and seasonal religious

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and commercial practices which are dependent on the calendars. This is especially true when it comes to the determination of the dates of religious Holidays which are derived from the celestial longitudes of the sun and the moon.

Religious constraints imposed on the interrelations between astronomical events and religious Holidays could not be satisfied without the confidence that they were not contradicting astronomical data. As a consequence, any knowledge regarding the methods used to determine the dates of the Holidays provides invaluable information to our understanding of the history of religion along with the history of science and astronomy. Such methods were described by several renowned Muslim astronomers in the 8th through the 11th centuries, such as al-Khwârizmî, al-Bathanî, or al-Bîrûnî, whose contributions to the science of Astronomy and Mathematics played a leading role in the scientific world, and are considered as some of the most important milestones in the history of science (Kennedy, 1956, Sezgin 1967-, and Gingrich, 1986). Several original treatises authored by the above mentioned scientists and others can be found in the Khuda Bakhsh Oriental Public Library in Patna, India, and our present work presents such a contribution of the relatively lesser known astronomer al-Qâ'inî (Sezgin, 1967 -, Hasan Khan,Razaullah Ansari & Khan Ghori, 1998). It is our belief that after studying the details of his contributions, his name should also be included in the list of astronomers whose scientific works are equally valuable.

In 852 AD, al-Qâ'inî wrote a treatise on the Jewish luni-solar calendar which also includes several remarks regarding the Christian Holidays that were dependent on a luni-solar calendar.

Al-Qâ'inî, whose full name was Abu'l Hasan 'Alî bin 'Abdallâh bin Muhammad bin Bâmshâdh al-Qâ'inî, was a distinguished astronomer who flourished in the 9th century AD in Persia. He is the author of another preserved article in astronomy entitled: "*Al Maqâlatu fî Istikhrâj Sâ'ât mâ baina tulû' al Fajr wa Ash Shams Kulla Yaum min Ayyâm as Sanati bi Madînati Qâ'in*" which is a treatise describing "the calculation of time based on astronomical principles from the appearance of the morning star to sunrise and from sunset to the disappearance of twilight".

Al-Qâ'inî made the calculations for what had probably been his home town Qâ'inî located in central Persia, in Khurasan, between the cities of Mashâdh and Iṣfahan, and was cited by al-Bîrûnî for his contributions in Mathematics though his presently discussed treatise is not mentioned.

Nevertheless, the fact that al-Qâ'inî's name was mentioned by al-Bîrûnî, led Davidian and Kennedy, the authors of the manuscript entitled: "Al-Qâ'inî on the duration of Dawn and Twilight" [*JNES*, 20.3 (1961) 145-153], to believe that he might have been a contemporary of al-Bîrûnî (late 10th to early 11th century).

It is only by the analysis of our present work that we can suggest that mid 9th century (which included the years 850-855 AD referred to in his treatise, see below), better fit the years in which al-Qâ'inî was active.

Nevertheless, al-Bîrûnî's work written 150 years later is our main source to evaluate several concepts and terms used in al-Qâ'inî treatise:

In his 10-11th century's manuscript discussing the traditions of several ancient nations (al-Bîrûnî, pages 69-70, and 282), al-Bîrûnî emphasized that the Nestorian Christians in Khurasan as well as in Syria and Iraq, combined Jewish and Greek months, the latter termed by al-Qâ'inî as "the Assyrian (اسر نائنه) months": They used the months of the Greeks (the Julian calendar), but have adopted the 1st of the Greek October as the beginning of their year, to make it closer to the Jewish new-year. However, the month of the Jewish Tishrei always preceded that date (This is not true anymore in the Gregorian calendar, and, for example, in 2005 AD the 1st of Tishrei was in the 3rd of October).

The Nestorians called their months by Assyrian names, some of which (bolded) are identical to the Jewish names used by the Jews in the luni-solar calendar (but, with different number of days, except for the month of Nisan):

Tishrîn Kedim of 31 days (al-Qâ'inî uses the name *Tishrîn al-Awwal*);

Tishrîn Herai of 30 days (in other sources - *Tishrîn Uth-thânî*);

Kanun Kedim of 31 days; Kanun Herai of 31 days;

Shebbath of 28 days (or, once in 4 years - 29) – in Hebrew - *Shevat*;

Adhar of 31 days; **Nisan** of 30 days; **Iyar** of 31 days; Haziran of 30 days;

Tammuz of 31 days; **Abb** of 31 days – in Hebrew - *Av*;

and **Ilul** of 30 days – in Hebrew - *Elul*.

As al-Bîrûnî pointed out, these months have even been adopted by the Muslims who used them for dating events that are not related to the religion (al-Qâ'inî makes it clear in his statement VIII – see below)[1].

Al-Qâ'inî was not the only Islamic astronomer in the 9th century AD who devoted a treatise to the subject of the Jewish calendar. In fact, his treatise was written 28-30 years after al-Khwârizmî's work on the same topic (al- Khwârizmî. 824; see also Kennedy, 1956 and Cohen, 2004).

Similar to al-Qâ'inî's approach (to be further discussed below), al-Khwârizmî's treatise on the Jewish calendar contained 2 different aspects of the calendar – the structure of the calendar and its relation to the astronomical coordinates of the sun and the moon. As an astronomer al-Khwârizmî found it important to include, in addition to the list of regulations, several celestial longitudes of the planets corresponding to specific dates in the Jewish calendar. In particular, al-Khwârizmî included the celestial longitudes of the mean sun and the mean moon for the date of "Adam's new moon" (the start of the 2nd year from creation in the *Present Permanent Jewish Calendar* - *PPJC*). As a consequence, al-Khwârizmî's work provides an important insight into the believed to be the correspondence between the beginning of the spring season (the origin of the celestial coordinate system) and the mean new moon of Nisan in the "year of creation"[2].

Al-Qâ'inî, as an astronomer and a mathematician, did not mention the celestial coordinates of the sun and the moon but, instead, provided us with a unique document that discusses the dates of Passover in the months of March and April (the Assyrian solar months of Adhar and Nisan) relative to the Julian date of the Vernal Equinox, thus providing a direct comparison between the solar and the luni-solar years.

But, no less important is its unique objective contribution to our understanding of the strong association that existed between the Jewish Passover and the Christian Holidays of Easter and Palm Sunday even as late as the 9th century AD when the treatise was composed. Even-though in the early 11th century historical review, al-Bîrûnî stresses the tension that had built up between the Christians and the Jews regarding the dates of Passover and Easter[3], al-Qâ'inî treatise, describing the state of the art of the calendar in the mid 9th century, shows no signs of such hostility between the two parties.

The Christians were divided to various sects[4], but, in the 1st Millennium AD they were all counting the years from creation according to the Septuagint version of the Bible.

The year of creation played an important role in the regulations of the luni-solar calendar, since it was used as the 1st year of the 19 years cycle which determined the intercalated year. The length of the solar year was believed by the Christians to be the Julian year (365.25 days)[5]. 19 such years do not sum up to exactly 235 mean synodic months as assumed in the *PPJC*. Nevertheless, even with that length of the year, within a period of a few hundred years [483 or 502 years as discussed by us elsewhere – see Cohen, 2004 and Cohen 2007 (2005)] the same intercalation sequence in each cycle of 19 years can satisfy a requirement such as the requirement that the full moon of Nisan be within 1 month after the Vernal Equinox. Therefore, in order to determine if a year is intercalated the Christians used their *Era Mundi* modulo 19.

On the other hand, the Jews counted their *Era Mundi* based on the Masoretic Text of the Bible, which is according to the *PPJC* the year 3761 BC.

Al-Qâ'inî's treatise provides us with an important clue as to what had been believed to be the year of creation adopted by the Nestorians. According to al-Qâ'inî the cycle of 19 used by the Christians started 1 year earlier than the Jewish cycle. This would suggest that the *Era Mundi* used by the Nestorian Christians was either 5510 BC (the Greek Orthodox Church value for *Era Adami* is 5508 BC) or 5491 BC, 1 year earlier than the year suggested by al-Bîrûnî[6] based on *Khâlid b. Yazîd b. Mu'âwiya b. 'Abî-Sufyân*.

But, above all such details, the importance of al-Qâ'inî's treatise also lies in the fact that the Christians and the Jews in his region were interrelated in their calculations aimed to determine the dates of Easter relative to Passover. For example, as will be emphasized below, Easter, as claimed by al-Qâ'inî, was always within 1 week after the first day of Passover. This important relation between the two religions is even reinforced by the fact that it is described by an Islamic astronomer, suggesting that all different calendars were known to the cultural elite in the region which included Khurazan, the northern parts of India, and Persia.

2. Conclusions derived from the Treatise

Before we present the translation of the treatise and our analysis of its statements, we find it important to emphasize that with the several direct and indirect statements that al-Qâ'inî's work contains, it can be considered as one of the most important and unique sources regarding a former version of the permanent Jewish calendar that had been in use from the 4th century AD till the 9th century AD. Such a former version can now be shown to have been different in several aspects from the present Jewish calendar.

In particular, it is different in so far as the length of the tropical year used by the Jews before adopting the 19th part of 235 mean synodic months value, and the intercalation sequences are concerned. One of the main clues to the fact that the document is not describing the *PPJC* can be found in Statement II (See Translation below) from which it can be concluded that all Passovers must start after the Julian date of March 21st, whereas the use of the *PPJC* for a cycle of 19 years, which includes the year 852 AD, shows that Passover would have taken place on the Julian date of the 17th of March as well (see **Table 1**)[7].

This is just one of the several main conclusions that can be derived from the treatise, which we summarize as follows (see explanations in the translation chapter):

1. In 852 AD the Jews and the Christians in Khurasan, Iraq, and Syria (and, probably, in other countries as well) were still basing their calendars on the Julian 20th-21st of March as the date of the Vernal Equinox,[8] even though, as became known in later decades, the real Vernal Equinox had already shifted 4 days backwards to an earlier Julian date compared to its 4th century AD date, and all their intercalation sequences had to be off in the middle of the 9th century AD by 3-4 days. We note that Maimonides (12th century AD) emphasizes that[9]

   > 1-9. As to the solar year, there are certain Jewish Sages who have held that it consists of 365 days and one quarter of a day, which is six hours, while some have held that the fraction is less than a quarter of a day. And the

Table 1. The dates of Passover in the 9th century AD according to the PPJC# and according to al-Qâ'inî.

A	B	C	D	E	F	G	H	I
Year From Molad Tohu (*PPJC#*)	Year in the *PPJC* cycle	Year AD	Year SE III	Julian Date of the 1st day of Passover (*PPJC#*) "guach adzat"**	INTERC.	"Gregorian" Date of the 1st day of Passover (*PPJC#*) "guach adzat"	Julian Date of the 1st day of Passover (Qā'ni) "gehach adut"	INTERC.
4598	19	837/8	1148	13 April		1704 (Sa)	13 April	
4599	1	838/9	1149	3 April		704	3 April	
4600	2	839/40	1150	23 March		2703	23 March	
4601	3	840/1	1151	10 April	*	1404	10 April	*
4602	4	841/2	1152	31 March		304	31 March	
4603	5	842/3	1153	20 March		2403 (Tu)	19 April	*
4604	6	843/4	1154	6 April	*	1004	6 April	
4605	7	844/5	1155	26 March		3003	26 March	
4606	8	845/6	1156	15 April	*	1904	15 April	*
4607	9	846/7	1157	5 April		904	5 April	
4608	10	847/8	1158	24 March		2803	24 March	
4609	11	848/9	1159	11 April	*	1504	11 April	*
4610	12	849/50	1160	1 April		504	1 April	
4611	13	850/1	1161	21 March		2503	21 March	
4612	14	851/2	1162	9 April	*	1304	9 April	*
4613	15	852/3	1163	28 March		104	28 March	
4614	16	853/4	1164	17 March		2103 (Sa)	15 April	*
4615	17	854/5	1165	6 April	*	1004	6 April	
4616	18	855/6	1166	26 March		3003	26 March	
4617	19	856/7	1167	13 April	*	1704 (Tu)	13 April	*

#*PPJC = Present Permanent Jewish calendar*.

** See **Appendix I**.

A. Year 1 starts in the autumn of 3761 BC; **B.** The values in this column are equal to the years in A mod(19); **C.** The corresponding Christian years are from autumn to autumn; **D.** Year 1 of SE III starts in the autumn of 311 BC; **E.** The dates of Passover calculated based on the regulations of the *PPJC#* the intercalation sequence of which is 3,3,2,3,3,3,2 (column **F**). This intercalation sequence is known as the "Guach Adzat" sequence (גו"ח אדז"ט); **G**. The dates of F in "Gregorian" dates which would have differed in the 9th century AD from the Julian calendar by 4 days. We note that in the Gregorian calendar the 21st of March corresponds to the Vernal Equinox and all Passover's dates would have been within 1 lunar month from the VE; **H**. According to al-Qâ'inî year 1 in the cycle is 4599 from Molad Tohu. The only intercalation sequence that brings all 1st days of Passovers to be within 1 lunar months after the Julian date of the 20-21 of March (satisfying statement II), is the intercalation sequence 3,2,3,3,3,2,3 (خخخب خخخب) (column I). This intercalation sequence is denoted by al-Qâ'inî as 3,2,9,2,3 (خبطبخ) and is known as the "*Gehach Adut*" sequence (גה"ח אדו"ט) – See **Appendix I**.

> same disagreement may be found also among the Greek and Persian scholars."
>
> 2-9. According to the opinion which holds that the year consists of 365¼ days, there remains in each cycle of 19 solar years a balance of 1 hour and 485 parts."

Although Maimonides mentioned that such a disagreement had existed in the times of the "Sanhedrin" (till the 4^{th} century AD), it is implied that the Jews referred to in al-Qâ'inî's work were still using March 21^{st} as the beginning of spring. Consequently, they were among the Jewish Sages who in the 9^{th} century used Y1=365.25 days. But, a disagreement described by Maimonides that would have started in the 3^{rd} – 4^{th} century, namely, when other Jewish Sages were using Y2=365.2468222 days for the length of the tropical year, should have led after a few centuries to disagreements in the determination of the new moon of Nisan twice in each cycle of 19 years. Assuming that all Jews celebrated Passover in the same month this conclusion suggests that based on al-Qâ'inî's work, all Jews might have used Y1 (=365.25 days) until the 9^{th} century AD as the length of their solar year.

2. **The epoch of the Seleucid Era used by the Jews and by the Christians in this part of the world was the autumn of 311 BC. This Epoch is denoted by Frank (1952) as SE III** (this is further discussed in the translation below).
3. Easter was called *al-Fiṭr* and according to a statement made by al-Qâ'inî, had always been kept by the Christians to be within a week from Passover. If the Jews were off in celebrating Passover more than a lunar month after the Vernal Equinox, so were the Christians regarding the date of Easter.
4. The Christian Holidays of "Palm Sunday" (called *Sa'ânîn*) and "Good Friday" were also observed with strong linkage to the Jewish luni-solar calendar.
5. The ancient Babylonian intercalation requirement, intended to ensure that the new moon of Nisanu (Parker and Dubberstein, 1956) would always occur after the Vernal Equinox, was used by the Christians in the 9^{th} century with a modification introduced based on the assumed changes in the celestial longitudes of the lunar Nisanu relative to the Vernal

Equinox. This is further discussed in Cohen, 2007 with the translation in Hebrew..

6. The cycles of 19 years used by the Jews for the intercalation sequence are shown by al-Qâ'inî to have started in the 9^{th} century 1 year after the corresponding Christians cycles.
7. A Seleucid calendar, used by the Arabs, counted the number of the purely lunar years from the Seleucid Epoch. This consequence is based on our interpretation of the text according to which the lunar year number in the Seleucid (solar) year 1162 (see **translation** below) was 36 years larger.
8. Although the basic intercalation sequence :3,3,2,3,3,3,2, used in the *PPJC* is mentioned in the text, it corresponds to the intercalation sequence used by the Christians, starting, as mentioned above, 1 year before the sequence in the *PPJC*. However, since it is claimed that Easter had always been within a week from the first day of Passover, it is possible they the Christians started to count the 19 years cycle 3 years before the Jews.
9. The intercalation sequence used by the Jews till the 9^{th} century was 3,2,3,3,3,2,3 and was only later (in the 10^{th} century) changed to the present sequence 3,3,2,3,3,3,2 from the same epoch. This statement is in contradiction to the claim made by other researchers (starting from al-Bîrûnî till the present days) that all different intercalation sequences should be explained by the use of a different starting year for the Metonic cycle, but do not reflect an actual difference in the intercalated years.[10]

3. The translation and interpretation of al-Qâ'inî's treatise

Below we present the full text of al-Qâ'inî's treatise as translated and interpreted by us from the Arabic language:

"A book on the Jewish calendar written by

Abu'l Hasan 'Alî bin 'Abdallâh bin Muhammad bin Bâmshâdh al-Qâ'inî

علئ بن عبدالله بن محمد بن با مشاذ القائنئ ابو الحسن

You should know that according to the Jews a new 19 years cycle starts in the year 1148 Alexander's [Epoch (SE III)].[11]

And, if you want to know in what year of the cycle of 19 you are, take the number of years that had passed according to the Jews since the creation of the world till [for example] the year 1163 SE III [namely] 4613, and then subtract 19, 19 and the remainder would be the year in the cycle.[12]

And the reason for the subtraction of 19, 19, is that the lunar year is not equal to the solar year during the cycle of 19 which includes intercalation.

A solar year is 11 days longer than a lunar year and the total difference in 19 years amounts to 7 months, and by adding the 7 months to the 19 lunar years the 2 cycles are balanced.[13]

In relation to the year 1161 [SE III], the cycle of 19, will be completed [relative to the solar year] in the year 1167 SE.

[But] by adding 12 we get 1179, and, if we, [now] subtract 19, 19 we get 1.[14]

And the reason for that result, is related to the fact that the Christians start their cycle of intercalation 1 year earlier, after adding the 12 [years], and [most of] the years in the cycle starts 11 days earlier.[15]

If we multiply the number of the intercalated years, 7, by the number of the days, 19, by which they exceed the solar year, the addition amounts to 133 days; whereas multiplying the 12 deficient years by 11 days the total deficiency is 132 days which is 1 day less than 133. Then we add 1 day and the cycle is balanced.

And, now, if you subtract 11 days from the solar year, being 365 days long, the time gap between 2 consecutive Passovers will be equal to 1 lunar year, namely, 354 days, and if you add 19 days, then the time gap becomes 384 days.[16]

[I.] And if you want to know when is the start of the solar year based on your knowledge of the Assyrian date of the 14th of the lunar month of Nisan in [the Assyrian month (السهر بالسرنانيه) of] Adhar or Nisan[17],

then:

1. If the 14th is in the [Assyrian] month of Adhar, add 4 to its [Assyrian] date and take 7 and 7 off the result. The remainder [being less than 7] is determined as the “origin” of the year.

2. If the 14th is during the [Assyrian] month of Nisan then take its date without any addition and subtract 7 and 7 with the remainder being then the "origin" of the [solar] year.[18]

[II.] And if you want to know the date of the 14th [of the Jewish Nisan] in the [Assyrian] months of Adhar or Nisan in the present year (based on its date of the 14th in the preceding year), then:

1. If it was in Adhar, deduct 12 from its date and the resulting value will be its date in Nisan.
2. If it was in Nisan then you should subtract 11 to get its date in Nisan of the current year. However, if the preceding date was 11 or less [in the month of Nisan], then you should add 20 and the resulting value will be the date of the 14th in the current Adhar.[19]

[III.] And if you want to know the dates of "[Good] Friday" (termed by al-Qâ'inî in Arabic as *al-Juma't al-Sirkâd* الجمعه السركاد)[20], knowing the Assyrian date of the lunar 14th, then, if it is in Nisan add to it the "source" of the year, and if the resulting sum is greater than 7, subtract 7 and use the result to determine the dates of [Good] Friday with the help of God.[21]

[IV.] And if you know "Good Friday" in respect to the [Jewish] 14th, you can now date Sunday in which *al-Fiṭr* ("Easter Sunday") is celebrated, since Passover always falls between *Sa'ânîn* ("Palm Sunday"), and *al-Fiṭr* (الشعانين و الفطر).[22]

[V.] And if you know the Assyrian date of *al-Fiṭr* then:

1. If the date is in Nisan, add 11 and the resulting value would be the date of the start of the [Lent] feast in the Assyrian month of Shebbath.
2. If the date of *al-Fiṭr* is in Adhar, add 11 and then subtract 31 and the resulting value would be the date of the start of the fast in Shebbath.[23]

[VI.] And if you want to know the date in the lunar calendar when you know the Assyrian date in a given month, take the Assyrian day of the month of the first new-moon of the year (القمر السركاد) and add 29.5 days for each Assyrian month starting from Tishrîn al-Awwal until you reach the Assyrian month of interest. If the sum is larger than 29.5 then subtract 29.5 and the difference between your result and the Assyrian date of interest is its date in the lunar calendar.[24]

[VII.] And if you want to know to what year in the cycle of 19 the first new moon (of Tishrîn al-Awwal) corresponds, take the years of Alexander and add the 12 years of Adam. Then, subtract 19, 19 and the remainder is the year number ("*hçleq*" = determined here as Modulo 19) in the cycle.

And the way the **Jews** intercalate is (جبطبج) j,b,t,b,j [which, with the assignation of numeric values to letters, means 3,2,9,2,3] and as to the symbols [shown below in the text] j, j, and b - "*jim*" represents 3 years and "*bah*" - 2 years.[25]

If we take the year one thousand, one hundred and [two and sixty] [without] the 36 years [as counted] by the Arabs, to which 12 years are added, we get 1174.[26]

By subtracting 19, 19, the remainder would be 15 which is greater by 1 compared to the result obtained by the Jews.

And according to the Christians the intercalated years are (ججب ججججب) j, j, b, j, j, j, b.

And the Jews intercalate from the creation of the world till the above mentioned year being 4612. And if you subtract 19, 19, the remainder is 14.

And that is the difference between the 2 methods of calculation [used by the Jews and by the Christians] resulting from the way they count the years from the beginning to the end.[27, 28].

[VIII.] And if you want to know when did the High Holidays of the sons of Israel[29], take place and if the year is full, normal, or, deficient, and if it is an intercalated or a regular year, then:

Take from the Arabs the Assyrian dates of the present and previous Passovers, and count the days between the 2 Passovers. If the number of days is

1. 353, the year is deficient and regular;
2. 354, - normal and regular;
3. 355, - full and regular;
4. 383, - deficient and intercalated;
5. 384, - normal and intercalated;
6. 385, - full and intercalated.

Then, subtract from the number of days between the 2 Passovers 15 days for the [Jewish] Nisan then if the year is intercalated subtract 30 for the [Jewish] Adhar I and 29 for Adhar II, and if the year is regular just subtract 29 for Adhar I.[30]

Then, if the year is full subtract 30 days for each month of Mar-Heshwan and Kislev, or 29 days for each when it is deficient, or, 29 for Mar-Heshwan and 30 for Kislev when it is normal.[31]

[IX.] And if you know that Passover of the present year is on a Sunday, you know that "*Atzeret*" was on a Monday, and "*Rosh Hashanah*" was on Tuesday.[32]

And Passover cannot be on a Monday, Wednesday or Friday, and similarly, *"Rosh-Hashanah*" cannot be on a Sunday, Wednesday, or, Friday. And that is the rule known as "not b-d-v (see Appendix I) for Passover and not a-d-v for *"Rosh Hashana*"".[33]

4. The Intercalation Sequence

We shall now show that the Calendar described by al-Qâ'inî is different from the *PPJC*:

When we apply the *PPJC* regulations for the years 4599 - 4617 (the 243rd cycle in the Jewish calendar) which correspond to the years starting from the autumn of 838 AD and to the years 1149 SE III - 1166 SE III, we get the following Julian dates for the first day of Passover:

4599: April 3rd, and 4614: March 17th (See Table 1).

An example of the calculation of the Julian date of Passover is given below:

1. As mentioned above, according to the *PPJC* Adam's new moon occurred on a Friday at 8 am in the year 3760 BC on the 26th of September, 12 months after the epoch of the Jewish calendar.
2. The new moon of the Jewish month of Tishrei in the year 4615, which is the beginning of the 17th year of the cycle, takes place 242 cycles of 19 years and 185 months after Adam's new-moon, or a total of 57055 months.
3. In 57055 mean Jewish months there are 1,684,868 days, 1 hour and 175/1080 parts of an hour, and the new-year's new-moon of year 4615

will be on a Monday at 9 am and 175 parts. Therefore, according to the PPJC, *Rosh-Hashanah* will be on that Monday 1,684,868 days after Adam's new moon, corresponding to Monday, August 27th, 835 AD

4. Following the *A"T Ba"Sh* rule (Endnote 33), the first day of Passover of the preceding year 4614, takes place on a Saturday, 163 days before *"Rosh-Hashanah*", on the 17th of March 835 AD.

Therefore, the permanent Jewish calendar described by al-Qâ'inî is different since the earliest Julian date for the first day of Passover is required in al-Qâ'inî's statement II to be the Julian 21st of March.

Indeed, when al-Bîrûnî, 150 years later, was already aware of the new regulations of the *PPJC*, he mentioned the 17th of March as the earliest possible date for Passover, in contradiction to al-Qâ'inî's statement II.

The discrepancy does not exist only in the 9th century AD, but also in the 4th century when it is widely accepted that a permanent calendar replaced the observation dependent Jewish calendar. If we calculate the Julian date of Passover, using the *PPJC* regulations in the 4th century AD we get: For example, the new moon of Tishrei of the Jewish year 4140 MTE, in the Julian year of 360 AD (50710 months after "Adam's new moon"), took place on a Monday at 10 am (16 hours after the beginning of the Jewish day), and 905/1080 parts of an hour. Therefore, the 1st of Tishrei was on Monday, August 28th (Julian).

It follows that the 1st day of Passover was on a Saturday, on the **18th day of March**.

We, therefore, conclude that the version of the permanent Jewish calendar as described by al-Qâ'inî, is a different version which preceded the *PPJC*, and is based on the same length of the solar year as used by the Christians, namely, Y1 = 365.25 days. Only with that agreement between the lengths of the Christian and the Jewish solar years the Julian date of the 21st of March would have the same role in both calendars for, at least, 5 centuries (since the Nicaea Council - 325 AD -, on one hand, or 359 AD, the year believed to be the start of the first version of the permanent Jewish calendar, on the other hand) bringing the spring related Holidays of Passover and Easter to be observed always within 1 week from each other.

Table 2. The dates of the 1st day of the month of Nisanu in the Babylonian luni-solar calendar after Parker et al.

A.	The Julian year BC from autumn to autumn.
B.	The intercalated years are denote by a star (*). In the years 313 BC and 294 BC the month of Illul II was added instead of the month of Adhar II.
C.	The “Julian” date. We note that in the 4th century BC the “Julian” date of the real Vernal Equinox was, approximately, March 25th.
D.	The Seleucid year SE II
E.	The Seleucid year SE III

As can be seen, in all years Nisanu started within 1 lunar month after the Vernal Equinox.

A	B	C	D	E
320/19		1-Apr		
319/8	*	20-Apr		
318/7		8-Apr		
317/6		28-Mar		
316/5	*	16-Apr		
315/4		5-Apr		
314/3		25-Mar		
313/2	*	13-Apr		
312/1		3-Apr	1	
311/0	*	22-Apr	2	1
310/09		10-Apr	3	2
309/8		30-Mar	4	3
308/7	*	18-Apr	5	4
307/6		7 -Apr	6	5
306/5		26-Mar	7	6
305/4	*	*14*-Apr	8	7
304/3		4-Apr	9	8
303/2	*	23-Apr	10	9
302/1		12-Apr	11	10
301/0		1-Apr	12	11
300/299	*	20-Apr	13	12
299/8		9-Apr	14	13
298/7		28-Mar	15	14
297/6	*	16-Apr	16	15
296/5		5-Apr	17	16
295/4		26-Mar	18	17
294/3	*	13-Apr	19	18
293/2		3-Apr	20	19
292/1	*	22-Apr	21	20
291/0		1-Apr	22	21
290/89		30-Mar	23	22
289/8	*	17-Apr	24	23

In addition, we conclude that the intercalation sequence used by the Jews in the first version as described by al-Qâ'inî was 3,2,3,3,3,2,3 counting the cycles of 19 from the same year as in the *PPJC*. As shown in Table 1 only with the above intercalation sequence Passover could be observed within 1 lunar month after the Vernal Equinox in a 19 years cycle in which the new-moon of the lunar month of Nisan takes place in the same day as the beginning of spring (the Vernal Equinox) in the first year of the cycle.

The Christians used a different intercalating sequence, namely 3,3,2,3,3,3,2. But, as emphasized by al-Qâ'inî, they started their cycle 1 year earlier. In such a case all new moons of the Christians corresponding to the new years (or to the new moon of the preceding Nisan) in a cycle of 19, would take place 11 days later relative to the Vernal Equinox in comparison with the Jewish new years (except for the 6^{th} and the 17^{th} year in cycle, when they would occur 19 days earlier, due to the different intercalation sequences).

Following the Babylonian method (as well as the early Jewish method – Cohen, 2003), **Table 2**, the intercalation sequence used by the Christians was chosen to ensure that the beginning of the month of Nisan (14 days before Passover) would be within 1 lunar month from the Vernal Equinox. The use of the sequence 3,3,2,3,3,3,2 would have, thus, been the sequence required to achieve that constraint when used in the first centuries AD.

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Notes and References

1. Al-Bîrûnî included also a discussion on the Julian dates of Easter and the Lent fast in respect to the Julian dates of Passover (al-Bîrûnî, pages 299-303).

   However, as will be shown below, his data are different in several important aspects from al-Qâ'inî's statements which were made 150 years earlier. For example, al-Bîrûnî's interpretation regarding the intercalation method used by the Jews and by the Christians was influenced by the present permanent Jewish calendar *PPJC*'s (*Present Permanent Jewish Calendar*) regulations made public by Saadia Gaon in the 10^{th} century.

al-Qâ'inî's treatise, which was written in the 9th century, thus provides a unique understanding of the regulations of the first version of the permanent Jewish calendar.

2. This astronomical information about the celestial longitudes for "Adam's new moon" along with the added input provided by al-Khwârizmî regarding the celestial longitudes of the new moon corresponding to the "Building of the Temple" are shown by us [Cohen 2004 and 2007 (2005)] to have important implications on our understanding of the chronology of the Bible.

3. In al-Bîrûnî, p. 302, we find: " The followers of Christ wanted to know before-hand the Passover of the Jews, in order to derive thence the beginning of their Lent. So they consulted the Jews, and asked them regarding this subject, but the Jews, guided by the enmity which exists between the two parties, told them lies in order to lead them astray."

4. The main sects in addition to the Nestorians (named after Nestoriuos from the beginning of the 4th century AD) who were the majority of the Christians living in Syria, Iraq and Khurazan, were the Melkites (mainly in Greece and the neighboring countries) and the Jacobites who were living in Egypt.

5. The length of the solar year was declared in the Nicaea Council (323-325 AD) as Y1 = 365.25 days long. As we discuss in Cohen, 2004 and in Cohen 2007 [2005], the set of the celestial longitudes of the mean sun and the mean moon which are included in al-Khwârizmî's treatise, can be accurately explained only based on a solar year of 365.25 days and not, as erroneously explained by al-Khwârizmî, to have been based on the year mentioned by him to be 365.2468222 days long. The length of the year mentioned by al-Khwârizmî in 825 AD is the year used in the Present Permanent Jewish Calendar (*PPJC*) based on the 19th part of 235 mean lunar months of length M1, where M1= 29 days, 12 hours and 793 parts of an hour.

6. See al- Bîrûnî' p. 300. As discussed by us (Cohen, 2004), there are several other values for Era Mundi which are based on the Septuagint Text of the Bible. But, except for the value suggested by Eusebius, most of them are between the years 5490 and 5510 BC. We note that the value used by al-Bîrûnî contradict the difference of 1 year (modulo 19) as stated by al-Qâ'inî – See below.

   We note that after half a Millennium, year 1 of the 19 years cycle can be kept the same but the intercalation sequence must be altered, or, alternatively, the intercalation sequence can be kept constant and year 1 must be different. This will be discussed separately.

7. This conclusion was not discussed by al-Bîrûnî when he relates the dates of Passover to the Nestorian months.

8. The misplacement of the Vernal Equinox was changed by the Jews about half a century later mainly relying on accurate measurements made, for example, by al-Bathanî during the second half of the 9th century, and on the *Almagest* (Ptolemy, Claudius. 2nd century AD), that had been translated to Arabic during the 9th century. This conclusion can be reached based on the fact that the regulations of the *PPJC* based on a different length

of the year (from creation), namely, Y2 = 365.24682. days, was published by Saadia Gaon early in the 10th century AD. We note that although al-Khwârizmî discusses the use of the year Y2 already in 825 AD, we have shown (Cohen, 2004) that it resulted from his misinterpretation of the astronomical data corresponding the mean celestial longitudes on “Adam’s new moon” and “the building of the Temple new moon”.

We note that astronomical measurements performed early in the 4th century AD were used to verify that the dates of the 20th-21st of March corresponded to the beginning of spring. The difference between the exact length of the Tropical (also known as the Solar) year and Y1 amounts to 1 day in 127 years. Indeed, in 1582 AD, accurate astronomical measurements showed that the Vernal Equinox had shifted 10 days backwards relative to its Julian date in 325 AD. In the 9th century AD the Julian calendar’s spring date of March 21st was thus off by 4 days.

9. See Chapter 9 in Maimonides, 12th century treatise detailed in the Bibliography.

10. Several sources indicate that the Jews were using at least 2 intercalation sequences:

    1.: The 3rd, 6th, 8th, 11th, 14th, 17th and 19th years in a cycle of 19 years have 13 months (a sequence known as the *Guach Adzat* sequence – see **Appendix I**), and

    2.: The 3rd, 5th, 8th, 11th, 14th, 16th and 19th years in a cycle of 19 years have 13 months (a sequence known as the *Gehach Adut* sequence – see **Appendix I**).

    But, as claimed by al-Bîrûnî, and all other researchers to follow the use of the sequences were explained by pointing out that when sequence (2) is used when counting the years in its cycle from year 4 in sequence (1), the cycles are identical.

    We show that sequence (2) was used in the first version of the permanent Jewish calendar starting with the same epoch as the *PPJC* where sequence (1) is in use.

11. There are a number of Epochs used by different nations as the Epoch of their Seleucid calendar – See, for example, the discussion in Frank (1952, p. 30). In particular, it is widely accepted that the Jews, in the Land of Israel, were using the Seleucid calendar in which year 1 SE started in the autumn of 312 BC.

    However, as will be shown below, the Epoch of the Seleucid calendar discussed by al-Qâ’inî is the so-called Greek (or Alexander) era (لاسكندر) (adopting Frank’s notation SE III) which was mainly in use in Persia and which started in the autumn of 311 BC. It is also known as the Seleucid era of the Chaldeans.

    Year 1148 SE III, thus, starts in the autumn of 837AD.

    As can be derived from Table 1, in that year the lunar month of Nisan starts with the beginning of the month of April (= the Nestorian solar month of Nisan, see below) and therefore the year is described by al-Qâ’inî as the start of a 19 years cycle which is also the first year in the Christian intercalation sequence (see below). In the following year, the year 1149 SE III, the first of the Jewish Nisan coincides with the 21st of March (the solar month of Adhar) a date corresponding to the beginning of spring. It is that year that is used by the Jews as the first year of the intercalation sequence.

12. As mentioned above, the Masoretic Text Epoch (MTE) of the *PPJC* was in the autumn of 3761 BC with the new moon of Tishrei (MT = "Molad Tohu") on the night between Sunday and Monday at 11 pm and 204/1080 parts of an hour.

    "Molad-Adam" is the new moon of Tishrei at the start of year 2 of the PPJC and it took place in the year 3760 BC on a Friday, September 26th (Julian) at 8 am.

    It is clear from his own following discussions that al-Qâ'inî relates the year 1163 SE III to the year 4613 counted from "Molad Tohu" (noting that had al-Qâ'inî been referring to the 312 BC epoch, the difference between the two epochs would have to be 3449 years and not 3450).

    The year 4613 is the 15th year of a cycle of 19 counting from "Molad Tohu". On the other hand, al-Qâ'inî states below that to find the year in a 19 year cycle using the Alexander Epoch, one should add 12 , making the year 1163 the 16th year of the cycle.

    Consequently, as mentioned in the historical background above and as will be explained below, there are 2 different cycles discussed – The Jewish cycle of 19 years and the Christian cycle of 19 years starting 1 year earlier (based on the year of creation derived from the Septuagint Text of the Bible).

13. According to this statement made by al-Qâ'inî the solar cycle and the luni-solar cycle must be balanced in the sense that not only 1 cycle of 19 lunar years with the addition of 7 months have the same length compared to 1 cycle of 19 solar years, but, that they also have a common start. As explained by us the Jewish cycle has the Vernal Equinox as a common starting point with the luni-solar years whereas the Christian intercalation cycle starts when the 1st of the solar month of Nisan coincides with the 1st of the lunar month of Nisan (see al-Qâ'inî's paragraph denoted by us as Statement I).

14. As discussed above, year 1167 SE III is the year 4617 MTE. But,

    4617 mod(19) = 0, and not 1.

15. The intercalation sequence is dependent on the specific imposed requirement which must be achieved. We know of several different such requirements which controlled the intercalation. The best examples can be found in Chapters 9 and 10 in Maimonides' Laws of the Sanctification of the month (Maimonides, 12th century AD). In these chapters Maimonides describes in detail the methods used for the calculation of the dates of the beginning of spring as related to the number of days from the new moon of Nisan. In both chapters he made it clear that the new moons of Nisan had always been required to precede the Vernal Equinox. This is further discussed in Cohen, 2003.

    In the *PPJC* the imposed requirement had been made to relate the Vernal Equinox to Passover, 14 days later.

    As will be explained below, al-Qâ'inî's year is based on the length of the Assyrian (same as the Julian) calendar, namely 365.25 days and the requirement imposed by the Christians for the determination of the intercalation sequence was to ensure that all new moons of Nisan would occur after the Vernal Equinox. Such a requirement was used by the Babylonians the intercalation sequence of which is well recorded in the literature (Parker and Dubberstein 1956). By starting the intercalation cycle with the assumption that the first new moon of the cycle is close to the Vernal Equinox, in all other years the point of reference will differ by 11 days from an intercalation cycle that starts 1 year later provided that the same intercalation sequence is used. But, since, as will be described below, the Jews used the 3,2,3,3,3,2,3 sequence whereas the Christians used 3,3,2,3,3,3,2 only in 17 years out of the 19 years the Christian cycle starts 11 days earlier.

16. Al-Qâ'inî uses several "rules of thumb" for a general description of the Jewish calendar. A more detailed discussion regarding all 6 possible lengths of the Jewish year is presented by him at the end of his treatise.

17. As emphasized above, several months of the Julian calendar referred to by al-Qâ'inî used in the Middle East by the Christians and the Muslims have the same names as in the Jewish calendar. In particular, the Julian Adhar (=March) has 31 days and the Julian Nisan (=April) has 30 days.

    The beginning of the solar year as referred by al-Qâ'inî in this statement is relative to the 1st day of April. Therefore the origin of the year, which is the number of weekdays that separates Passover or the beginning of the lunar Nisan and their corresponding date in April (the Assyrian month of Nisan), is simply the date modulo 7.

18. It is useful to mention that adding 4 to the date in Adhar (Julian March) is not related to the fact that the Vernal Equinox was no longer occurring on the 21st of Adhar (Julian March) in the 9th century, but, as mentioned in Endnote 8, 4 days earlier, since in the following statement al-Qâ'inî leaves no doubts to the fact that the earliest Julian date for Passover was the 21st of Adhar (March).

19. Statement II is a regulation that was a valid statement when the Vernal Equinox was believed to have taken place in the Julian date of March 20th-21st.

    Consequently, this is an important statement regarding the fact that even in the 9th century the Julian date of the Vernal Equinox was still taken to be the same as had been determined five centuries earlier during the First Council of Nicaea (325 AD). At that year the Vernal Equinox had been accurately determined to coincide with March 20th and, for some years - March 21st.

    The Julian date of the Vernal Equinox was still taken by al-Qâ'inî to be the 20th or the 21st of Adhar (March), even though the real Vernal Equinox was shifted 4 days backwards to Adhar (March) 16-17, as a result of the difference of 1 day per 127 years between the Assyrian year and the real solar (Tropical) year.

    A few decades after al-Qâ'inî's treatise, changes that were documented early in the 10th century by Saadia Gaon show that statement II had to be modified with the regulations of the *PPJC* in its final version as formulated by the "Geonim". This is due to the fact that the basic length of the solar year was taken in the *PPJC* to be 365.2468222 days (the 19th part of 235 average lunar months of 29.530594 days each), and the date for the beginning of spring could no longer remain the same in the Julian calendar based on a solar year 365.25 days long.

The statement makes an important distinction between a Passover that takes place during the Assyrian month of Adhar (March) or during the Assyrian month of Nisan (April).

The reason for the different calculation for a Passover in March is related to the fact that between the 4^{th} century AD and close to the 9^{th} century AD, only 7 times in a cycle of 19 years, Passover would occur in the Julian month of March. All other 12 Passovers in the cycle would take place in April. The 7 Passovers in March would happen just before an intercalated Jewish luni-solar year: Since Passover has to coincide (as a rule of thumb) with the first full moon after the Vernal Equinox, its earliest date could be in the above mentioned centuries on March 21^{st}. Unless the year is intercalated, a Passover, one year after a preceding Passover that had taken place between March 21^{st} and March 31^{st}, would have to start **before** the Vernal Equinox.

It follows that as long as the Vernal Equinox was taken to be on the date of March 20^{th}-21^{st} all Passovers in March would be followed by a Passover in April, 13 lunar months later, with a Julian date which has a numerical value of the preceding Passover's date in March, minus 12, as specified in the statement.

For, **example**, if Passover started on March 23^{rd}, then in the following year it would start on April 11^{th}.

This is a rule of thumb for the average difference, and better accuracies can be found at the end of his treatise.

Certainly, when Passover took place in April then the following Passover would be, on the average, 11 days earlier.

Such a regulation would be valid in the century when al-Qâ'inî wrote his treatise only when the solar year is taken to be 365.25 days.

20. Based on the statement that follows (IV), it is likely that Juma't al-Sirkâd الجمعه السركاد refers to "Good Friday".

21. In present days programs made for the determination of the Julian or Gregorian dates of Easter, a term named "the Dominical Number" or the "Dominical Letter" is used. This term determine the weekday of the first day of spring based on the simple equations:

    D (Julian) $=\mathrm{A} + (y + (y\ /\ 4))$ **mod** 7, or

    D (Gregorian) $= (\mathrm{A} + 1) + [y + (y\ /\ 4) - (y\ /\ 100) + (y\ /\ 400)]$ **mod** 7,

    where y is the number of years since A was the week day of February 29.

    The Dominical Number for the 1^{st} of April of the Seleucid years is called by al-Ka'ini the "source" of the year and is assumed by him to be known.

By adding the "source" of the year to the solar date of the first day of Passover, we, thus, bring the sum (modulo 7) to be the weekday of the beginning of the solar year. Since, as assumed by al-Qâ'inî that that weekday is known when the length of the solar year is taken to be 365.25 days, "Good Friday" can be determined knowing the date of the Passover.

22. *Al-Fiṭr* is the name used by the Muslims for their most important Holiday of the year marking the end of the Ramadan feast.

    From al-Qâ'inî's 9th century treatise (and al-Bîrûnî in the 11th century) it can be seen that the same name had also been used by the Christians (at least by the Nestorian sect discussed by al-Qâ'inî and al-Bîrûnî) to mark the end of their Lent feast. *Al-Fiṭr* was thus the name used for Easter which took place on the first Sunday after the first full moon following the Vernal Equinox.

    One week earlier "Palm Sunday" known as Sa'ânîn (observed also presently by the Greek-Orthodox Church and also known as "*Oshana*") was known to be observed by the Nestorian sect. As mentioned in the historical background, the importance of this statement lies also in the fact that it describes strong links between the Jews and the Christians regarding the determination of the beginning of the spring with its first full-moon. Only with such strong ties can a statement be made relating "Palm-Sunday", Passover and Easter together in that order. This can be done only if the length of the solar year used by the Christians (365.25 days long) had also been used by the Jews. We note that al-Bîrûnî adds another restriction to the relation between Easter and Passover: al-Bîrûnî emphasizes the requirement that Easter must be on the first Sunday after the (mean) full moon whereas Passover can be on the day of the (mean) full moon. It follows that "Palm Sunday" can coincide with the first day of Passover whereas Easter, in that case, would take place 1 week after Passover, or, sometimes even more than 5 weeks after the beginning of Spring.

23. The Assyrian Shebbath (February) has 28 or 29 days. Therefore, the rule of thumb is valid only for the Assyrian years with 365 days.

    If, **for example**, *al-Fiṭr* is on March 30th, then the Fast would start on the 10th of Shebbath and the number days from the beginning of the feast till Easter would be 47.

    It is important to note that this statement proves again that the Vernal Equinox was taken to be on the 20-21st of March. Otherwise, adding 11 to 18 of March, for example, and then subtracting 31 would result in a date in the Assyrian Tebbeth (January), which is not an option in al-Qâ?inî's statement V.

    Since the earliest date of Passover is March 21st and, following al-Bîrûnî's requirement, the earliest date of al-Fitr is the 22nd , the earliest date of the beginning of the Lent feast is the 2nd of Shebbath.

24. This is, again a rule of thumb which will be followed at the end of the treatise by offering a better accuracy regarding the Jewish calendar.

The rule is as follows: If *al-Fiṭr* is, **for example**, on the 31^{st} of the Assyrian Adhar, and the first new moon of the year was on the 12^{th} of the Assyrian Illul, then we should add the sum of the 6 Assyrians months between Illul and Adhar = 30 + 31 + 30 + 31 +31 + 28 = 181 days, and subtract the result (modulo 29.5) from 12. As a consequence, we can now determine the start of the new moon in the Assyrian month of Nisan to be on the 8^{th} of the month. *Al-Fiṭr* would thus be on the 23^{rd} of the lunar month.

25. This statement emphasizes the fact that the Jews have their own sequence for intercalation. Figure 1. shows that the numerical combination of j,b,t,b,and j is separated from the rest of the text by an upper line used in Arabic texts to denote the use of a numerical value:

Fig. 1. "*Wa khasab al-Yahud* 3,2,9,2,3" – meaning that the intercalation sequence used by the Jews is 3, 2 ,9 (= 3,3,3), 2, 3. Picture taken from the original al- Qâ'inî's manuscript, p. 118[a].

26. The year discussed is 1162 SE III although the original text has a defect which makes it impossible to read. However, the rest of the sentence leaves no doubt about the 1162 value.

    Our straight forward interpretation of the 36 years mentioned in the text is that in 1162 solar years there are 36 years less than the number of lunar years as used by the Arabs counting from Alexander's epoch.

27. The most significant and clear information is that the intercalation sequence used by the Jews starts 1 year after the intercalation system used by the Christians. Since the year of creation of the Jews followed the Masoretic Text (as interpreted in the 9^{th} century and used by al-Qâ'inî) was taken to be 3761 BC, the year of creation used by the Christians based on the Septuagint Text was likely to be 5510 BC or 5491 BC as explained above in the historical background section.

    The number of years from creation as mentioned in al-Khwârizmî's treatise is different and is discussed by us separately (Cohen, 2004, and 2007 [2005]).

28. We note that the only intercalation sequence that would correspond to an earliest Passover in the 21^{st} of Adhar (Julian March) in a cycle of 19 years that starts with a cycle in which the new moon of Nisan takes place on the same day as the beginning of the spring is 3,2,9,2,3, where 9 corresponds to 3,3,3.

    Only when the new moon of the lunar month of Nissan of the first year of a cycle takes place 4 days after the beginning of spring the intercalation sequence of the *PPJC* (3,3,2,3,3,3,2) – see the Intercalation sequence section.

29. Such as "*Rosh Hashanah*" = the Jewish New-Year.

30. The following sentence should be added: 'Then, subtract 30 for Shebbath and 29 for Tebbeth.'

31. Another sentence to be added is: ‘Then, finally, subtract 30 for the month of the Jewish Tishrei and then you know the Assyrian date of “*Rosh Hashanah*” (see Endnote 29).’

32. This statement is presently known as the A”T Ba”Sh א"תב"ש (- for all Hebrew letters – see Appendix I) rule relating the weekdays of Passover to the weekdays of various Holidays in the Jewish calendar:

    Weekday 1 of Passover (א) = Weekday of (ת) “***T**ishah Beav*” (the 9th Ab) fast.

    Weekday 2 of Passover (ב) = Weekday of (ש) “***Sh**avuot*” (the festival of the giving of the Torah), or *Atzeret* preceding “*Simkhat Torah*” (“The Joy of the Torah” - which celebrates the completion of the annual Torah-reading cycle).

    Weekday 3 of Passover (ג) = Weekday of (ר) the beginning of “***R**osh Hashanah*”.

    Weekday 4 of Passover ( ) = Weekday of (ק) “***K**riat haTorah*” (The reading of the Bible in the Diaspora).

    Weekday 5 of Passover (ה) = Weekday of (צ) “***Z**om* of *Yom Kippur*” (the fast of Yom Kippur – the fast of the Jewish Day of Atonement)

    Weekday 6 of Passover (ו) = Weekday of (פ) “*Purim*” (Feast of Lots).

    Weekday 7 of Passover (ז) = Weekday of (ע) “*Atzmaut*” – Independence day.

    The A”T Ba”Sh rule reflects the fact that in the Jewish calendar the number of days between the Holiday of Purim and the Holiday of Succoth in the following year the number of days is constant for all 6 types of the Jewish years.

33. This regulation that “*Rosh Hashana*” cannot be on the weekdays of Sunday, Wednesday and Friday ([לא אד"ו ראש] [ולא בד"ו פסח] – See **Appendix I**) is the 1st of the 4 postponements (“*Dehiot*”) used for the determination of the first day of the Jewish month of Tishrei in relation to the calculated weekday and the time of the day of the mean new moon.

    Since the 3rd and the 4th postponements were made to ensure that there will not be more than 6 different possible lengths of the Jewish year, it can be assumed that all postponements were known to al-Qâ’inî as they were known to al-Khwârizmî.

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Appendix I

The use of the Hebrew Alphabet for the Determination of the Weekdays of the Jewish Holidays and the Intercalation Order

Figure 2 displays the 22 letters of the Hebrew alphabet with their numerical values 1 for the first letter (א) and 400 for the last letter (ת). (The additional 5 modified letters written differently when used at the end of a word are also displayed).

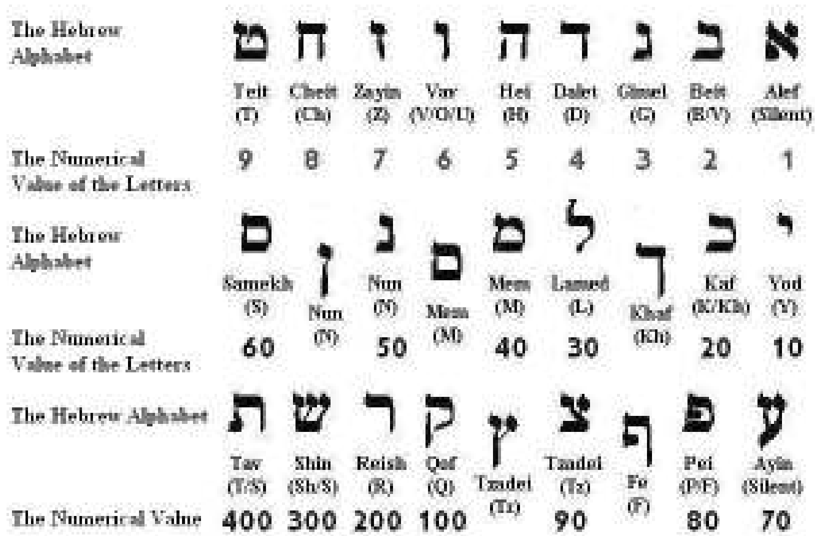

Fig. 2. The 22 letters of the Hebrew Alphabet and their corresponding numerical values. Also included are the 5 letters (ץ, ף, ן, ם, ך) written differently when used at the end of a word replacing the corresponding 5 letters צ, פ, נ, מ, כ.

The first 7 letters of the Hebrew alphabet are used to denote the 7 weekdays starting from א (*Aleph*) for Sunday. Consequently, when al- Qâ'inî discussed the days of the week in which Passover (*Pessach*) cannot start he used the Hebrew expression: לא בד"ו פסח ("*Lo Bad"u Pessach"*) which means that Passover cannot start on a Monday (ב), Wednesday (ד), nor on a Friday (ו). Similarly, the Hebrew expression לא אד"ו ראש (*"Lo Ad"u Rosh"*) is used for stating that "*Rosh Hashanah*" (the Jewish New Year) cannot start on a Sunday, on a Wednesday, and on a Friday as well.

The first letters of the Hebrew alphabet are also used to denote the weekdays of Passover which lasts for 7 days (in the Land of Israel) or 8 days (elsewhere). The rule א"ת ב"ש (*A"T Ba"Sh*) mentioned by al- al- Qâ'inî (see Endnote 32) had thus been used to relate between the weekdays of Passover and the weekdays of the main Jewish Holidays the Hebrew names of which started with the corresponding Hebrew letters starting from the last letter (ת) backwards. Consequently, the weekday of the first day of Passover (day א) would always be the weekday of a Holiday ***T**ishah BeAv* the Hebrew name of which starts with the last letter in the alphabet – ת. Likewise, the second day of Passover (day ב) would correspond to the weekday of a Holiday the name of which starts with the Hebrew letter ש ("***Sh**avuot*" - see Endnote 32).

In the case of the 19 years cycle the year number is indicated by the corresponding units' numeral. Consequently, for denoting the series of the 7 intercalated years in the cycle being the 3rd, 6th, 8th, 11th, 14th, 17th and the 19th years, the Hebrew letters אר ו"ס גן"ח (*Guach Adzat*) are being used. On the other hand, when the 7 intercalated years are the 3rd, 5th, 8th, 11th, 14th, 16th and the 19th years, the Hebrew letters used are אר ו"ס גה"ח (*Gehach Adut*).