INDIAN JOURNAL OF HISTORY OF SCIENCE

Vol.38 No. 3 ISSN 0019-5235 September 2003

SUPPLEMENT

Brāhmasphuṭasiddhānta (ch.21) of Brahmagupta with Commentary of Pṛthudaka, critically ed. with Eng. tr. and Notes

(to be continued)

Setsuro Ikeyama

INDIAN NATIONAL SCIENCE ACADEMY
NEW DELHI
2003

Published for:
Indian National Commission for History of Science
by
Indian National Science Academy
Bahadur Shah Zafar Marg
New Delhi 110 002

Printed at Nirmal Vijay Printers, B-62/8, Naraina Indl. Area, Phase-II,
New Delhi-110 028. Tel.: 25704549, 51418526, (M) 9811053617

TABLE OF CONTENTS

ABBREVIATIONS

Sources:

ABh the *Āryabhaṭ iya* of Āryabhaṭa
BS the *Bṛhatsaṃhitā* of Varāhamihira
BSS the *Brāhmaphuṭasiddhānta*
KhKh the *Khaṇḍakhādyaka* of Brahmagupta
PS the *Pañcasiddhāntikā* of Varāhamihira
SDV the *Śiṣyadhivṛddhida Tantra* of Lalla
SS the *Sūryasiddhānta*
SSS the *Siddhāntaśekhara* of *Śrīpati*

References :

CESS Census of the Exact Sciences in Sanskrit
DSB Dictionary of Scientific Biography

Institutes:

BORI Bhandarkar Oriental Research Institute
VVRI Viṣveśvarānanda Vedic Research Institute

Translation

I, 1–3; XXI, 1–16

Cosmological and Geographical Discussions

Mahādeva is victorious, whose feet are strewn with the splendor of the gems on the heads of bowing gods and asuras, and who causes the creation, endurance, and destruction of the world! I, 1

The calculation of the planets mentioned by Brahmā, which became flaccid after a long time, is explained truthfully by Brahmagupta the son of Jiṣṇu. I, 2

The wheel of stars which is bound to the polar star was created rotating clockwise at the beginning by Brahmā together with the planets which were between Pauṣṇa (Revatī) and Aśvinī. I, 3

The rotation of the planets and nakṣatras is not the same everywhere for the residents of the earth.

Because the knowledge about that is from the sphere, therefore I will explain the sphere. XXI, 1

Everything is lacking evidence such as that in the śāstra of the Jina the nakṣatras and the planets are in one hemisphere in existence, and because of this the beginning of the day and of the night for the gods as they stand higher up and lower down is when ⟨the sun is at⟩ the beginning of Capricorn and Cancer (respectively); and the covering (eclipse) of the sun and moon is caused by rāhu; and the seas and continents stand farther and farther out on the earth (which is like) the belly of a mirror; the stars are doubled; (and) at a side of

rectangular Meru of great size are two suns, two moons, and fifty-four nakṣatras.

Since he objects to the manifestation of false knowledge on the part of those whose last resort is the destruction of the hemisphere, this āryā, the sūtra of an initial assertion, is appropriate at the beginning of the entire golādhyāya because of his intention to tell the usefulness of the sphere.

That is as follows. **The rotation of the planets and nakṣatras is not the same.** Let this declaration stand at every application.

Everywhere. A planet which is above those standing at Laṅkā is on the southern horizon for those standing at Meru. Whatever (planet) is above Meru that is on the northern horizon at Laṅkā. In the same manner, whatever planet is above Laṅkā to the east, that is on the western horizon at Yamakoṭī and on the eastern horizon for the residents of Romaka. The declaration is made that such examples are known only from the sphere, not from anything else.

Secondly, because the earth has a spherical shape the cage of the constellations is also spherical because it stands surrounding the earth.

Their **rotation is not the same everywhere for the residents of the earth. Not the same** means not equal; **everywhere** means in all places; **for the residents of the earth** means for observers standing on the earth.

Knowledge about that is from the sphere (means) the understanding of that is from the sphere; because (means) from that; **therefore I will explain the sphere** (means) therefore I will tell about the sphere. This is the meaning of the sūtra.

Whether the rotation of the planets and nakṣatras is the same or not, do observers who stand in their own places perceive it as it is seen everywhere?

It is said that **the knowledge about that is from the sphere.** So a dissimilar rotation is made similar with the knowledge of the sphere.

This also cannot be said because the movement of the planets is constant; otherwise it wouldn't be given in numbers. Therefore it is objected that this āryā verse, whose main object is to tell the purpose of beginning with the sphere, is as it were meaningless.

There is not a fault. This is the purpose of the learned Brahmagupta. The teacher said **"the knowledge about that is from the sphere"** by reflecting on the coming treatise with such (words) as "there is opposition as those who are ignorant of the globe of the earth and have completely opposite opinions explain everything in a different way with such statements as the earth is large and has the shape of the belly of a mirror; Meru is also large and in the shape of a corpse; the polar star is fixed on its ⟨top⟩; the planets and nakṣatras are hanging down from the lower part of Meru but are perceived by us as if up above; by means of that the daily rising and setting of the sun and so on are the same for all observers wherever they stand; the northern half of the ecliptic beginning with Capricorn is the beginning of a divine day." Therefore what is excellently spoken is brilliant. We will explain in detail in a sūtra here and there. Let it be enough here since the subject has been discussed too diffusely.

Now he says an āryā for explaining the character of the sphere.

> **The sphere of the earth which is surrounded by the spheres of the moon, Mercury, Venus, the sun, Mars, Jupiter, and Saturn, and which is inside the sphere of the stars, is shaped by the good and bad activities of creatures. XXI, 2**

(The compound) beginning with "the moon, Mercury" is a dvandva.

The spheres of the moon, Mercury, Venus, the sun, Mars, Jupiter, and Saturn are the spheres of these. By the word sphere is meant here the circle of the place where a mean planet rotates. **Surrounded** by these means the sphere of the earth in the middle of these.

This is the meaning: Having put the center of the sphere of the earth at the center, whatever circle is produced with the hypotenuse (geocentric distance) of its own yojanas (as the radius), that is the circle of the sphere (of a planet). That (the sphere) of the moon is outside the sphere of the earth; then (comes the sphere) of Mercury, and then the spheres of Venus, the sun, Mars, Jupiter, and Saturn in order—seven (in all). This sphere of the earth surrounded by these is **within the sphere of stars**; the sphere of whatever stars there are.

A sphere is a type of figure which is to be understood by all mathematics since it is a figure. As grammarians obtain the correctness of a word by means of bases, affixes, (their) augments, elision, sound changes, and (sound) augments, etc., ritualists (perform) sacrifices by means of a ladle etc., and the best physicians also treat wounds on the head etc. by means of a lotus or fire etc., in a like manner here calendar-makers obtain the true nature of the sphere such as the rotation of the planets or the shape of the earth by means of sines, arcs, versed sines, arms, uprights, hypotenuses, perpendiculars, rods, circles, etc., and by means of various types of true geometry.

The roundness is determined by those who have destroyed contrary opinions just from the roundness by means of the characteristics which are to be found in a sphere; the absence of a support for the sphere ⟨of the earth⟩ and the sphere of the constellations in a later āryā; the size and shapelessness of Meru from its smallness; its attachment to the earth by means of the elevation of the orbits (of the stars); and the cage of the constellations from which the planets

and nakṣatras are suspended by the differences of (their) rotations produced in different places. Risings and settings by means of Meru are abandoned because of the explanation of various sunrises. Rāhu is abandoned because of the words beginning: *the covering agent of the moon is large.*Because there is no great circumference other than the circumference of the ocean of the sphere of the earth, the greatness of remaining oceans is contradicted. There is to be known a circle, a circle of rising, a six-o'clock circle, etc.; the meaning is learned.

One should make three circles having any measurement with light woods of the same length or rods of bamboo. Having placed one of them in the east-west direction and the second in the north-south direction, one should place crossings (svastika) produced above and below. Then (having placed) the third (circle) outside of the two circles like a girdle, one should place crossings produced in four directions. Join the three circles so that the circle looks as if it were one at every joint. There whatever is the east-west circle that is marked with sixty ghaṭikās and the remaining two circles with 360 degrees. These are central circles (viṣuvanmaṇḍala).

Then fasten obliquely another circle, which has as much measure, is marked with 360 degrees, and touches outside at the eastern and western crossings, south from the top crossing on the north-south circle at twenty-four degrees. And also fasten that circle of zodiacal signs on the same circle obliquely from the bottom crossing at the same twenty-four degrees north. There the beginning of Aries is at the eastern crossing; the beginning of Libra is at the western crossing; the beginning of Capricorn is at twenty-four degrees by the (circle) sinking to the south from the top crossing; and the beginning of Cancer is (at the point) by the (circle) elevated as much to the north from the bottom crossing. Thus this is the ecliptic.

Then, beginning another circle having the measure of the ecliptic and marked with 360 (points) from the beginning of Aries, in whatever place the node of the moon happens to be, tying it there, then a second tying is to be made at a place distant by half a circle. One should place (this circle) obliquely so that the first half is north of the ecliptic (and) the second half south, so that between it and the ecliptic are the degrees of latitude, with the ninetieth degree at the two ties on either side. This is the inclined circle, (and) thus this is the sphere of the moon.

For Mercury and so forth the cage of the sphere of each is to be made in proportion to the measure of its hypotenuse (geocentric distance) in yojanas. These are outside of the cage of the moon--(that) of Mercury, then (that) of Venus, and so on up to the eighth, the cage of the constellations.

Having made two holes at the south and north crossings of all cages, one should cause a very long iron rod, which splits in halves the crossings of all the cages, whose tip protrudes on both sides, which is broadened to the south and north, and which is capable of bearing the cages, to enter those two (holes). The ecliptics of all (the spheres) are as previously, and from the ecliptic the (planetary) orbits as in the case of the moon (are to be made). But this is the difference: one should place (them) in such a way that the degrees of latitude specified for each are between the ecliptic and the orbit at the ninetieth degree from the two tyings. The rest is the same.

There is no inclined circle in the sphere of the sun since the motions of all the planets to the south and the north are imagined in harmony with its motion; its motion is on the ecliptic. There is a different latitude for each nakṣatra in the sphere of the constellations; because there are no nodes there, no (inclined circle) is indicated. The day-circles are to be indicated for (the zodiacal signs) beginning with

Aries and for the planets at the end of (their) declinations. Then the sphere-shaped earth is to be represented by mud or something else on the iron rod in the middle of all the spheres. In this way this sphere of the earth is within the sphere of the constellations surrounded by spheres.

Then having tied one end of a string at the eastern crossing, one should tie the second end passing through the earth at the western crossing. Then one should tie another string piercing the earth at the upper and lower crossings. Now wherever the break is made by the string at the upper side of the sphere of the earth, Laṅkā is at that place on earth. Wherever (the break) is in the lower Siddhapura is there. Wherever is the break in the east Yamakoṭī is there. Wherever is the string-break in the west Romaka is there. Wherever is the break (made by) the iron rod to the north on the sphere of the earth Meru is there. Wherever (is the break made by the iron rod) to the south Vaḍavāmukha is there. The two pole stars are at the two tips of the rod as it comes out. Whatever is the circumference of the earth passing through Laṅkā, Yamakoṭī, Siddhapura, and Romaka, that is the whole region without latitude and one should make marks everywhere. Thus this globe stays level at Laṅkā.

But only this cage of the constellations is seen since all the different planets, even though the nakṣatras and the planets move on different spheres, are perceived there as if moving on one sphere. Therefore only one (sphere) is made.

We also make an explanation of the matter. The equator is (also) the prime vertical (going) from east (point) to west (point) for an observer standing at Laṅkā. Whatever is the second (circle) that is the (one from) north to south. Whatever is the third that is the horizon standing like a girdle; that is also the six-o'clock circle. Thus

it is in places without latitude; everywhere else it is particular.

Here the circles of the heavenly sphere which have not been commented on are explained for the sake of (these) other places. This is as follows. Having made four large circles, one should place one circle from east to west; that is the prime vertical. The second one going from south to north as before with crossings produced at the top and bottom is the one from north to south. The third like a girdle is the horizon. The fourth is attached at the eastern and western crossings, and also at extreme latitudes to the south and the north; the meaning is that it is attached to the circle from north to south above the northern crossing of the sphere of heaven at a distance equal to the degrees of the locality's latitude; and also attached to the circle below the southern crossing at the same distance. That is the six-o'clock circle.

Then having put the sphere of the earth with the spheres (of the planets) in this sphere of heaven and having made two holes at the two intersections of the six-o'clock circle and the north-south circle, one should cause the tips of the iron rod to enter them. That is the situation of the sphere at every locality on the line north of (the region of) Laṅkā up to Meru. This kind of construction of the spheres is mentioned by us for the sake of convenience. We will mention the rest here and there.

Now for explaining the high and low positions of the two poles for those who stay at Meru and Vaḍavāmukha on the sphere of the earth, he says an āryā.

The sphere of the earth is in heaven. The gods stay on its upper part on Meru, the daityas on the lower part. In heaven, above and below them, two poles stay at the tips of the axis of

the constellations. XXI, 3

In heaven, in space, there is the sphere of the earth. **The gods stay on its upper part on Meru**, that is, Meru is in the upper part on the sphere of the earth and the gods stay there. **The daityas on the lower part**, that is, the daityas who live in Vaḍavāmukha are in the lower part of the sphere of the earth. **In heaven (the two) stay at the tips of the axis of the constellations**, that is, in heaven, in space, the two stay at the tips on either side of the axis of the constellations. **Above and below them, two poles stay at the tips of the axis of the constellations**, one above and the other below. **Of them**, i.e., of the gods and daityas. This means that whichever (pole) is above the gods that is below the daityas and whichever is above the daityas that is below the gods.

This very strange thing is said: "**the sphere of the earth is in heaven**". Even a tiny solid thing is not seen staying in space. How (is it possible) for the earth which is very large and (fulled with) many wonders such as mountains, cities, oceans, islands, elephants, horses, chariots, etc.?

This is not to be objected. It is because of its nature; just as fire has the nature of burning, wind has the nature of moving, and water has the nature of moistening, and there is no promoter for them with regard to their own spheres of activity, in like manner this earth which has the nature of supporting is not supported. Therefore it is immovable in space and supports all.

If (you say) "let the earth remain falling ⟨like⟩ a stone", it is also impossible because lumps of earth etc. which are thrown up by children are seen approaching the earth.

If (you assert) "the earth falls slowly", it is impossible because the earth is very heavy.

Or else if (you say) "certainly it falls, but where? Let it fall down," how is this? "Down" is certainly dependent; the word "down" presupposes something. As the earth is under various creatures such as ourselves and space is above, what is "under" that earth which is under everything? If (one says) "heaven", then it results that (the earth) falls in all directions at once; in this matter falling upwards and sideways do not occur because it contradicts observation and "down" has its restricted (meanings) removed.

It is also not possible to say this with the intention that some sort of support from falling is imagined; there is the occurrence of infinite regress because another (support is required) because of that one's materiality, (and) another because of that one's. So it is said that (the support) stands by its own power. Why is that power not assumed for the earth since it has priority? It is known that a power is certainly assumed for the earth. Otherwise because of mutual underness in all directions there would be no stability of creatures as also of oceans and so on. Therefore it is shown that the sphere of the earth, which has no material support, but does have a particular power, stays in heaven.

So some immaterial support is assumed. The stability of the wind and so on, whose feasibility is accomplished by this and is connected to dharma and adharma because it is mentioned by the teacher in the previous verse, is understood by us. We have no skill in the part on the pramāṇas (means of certain knowledge) since we have become examples for grammarians of a karmadhāraya compound because of (our title) "caturveda". Only examine with the vision of Jyotiḥśāstra whether the āryā sūtra is explained reasonably or unreasonably.

Here the teacher Lāṭa (says):

The sphere of the earth, symmetrically round, stands in

> heaven without support on all sides; it is created by all the good and bad activities of creatures.

So also in the *Vasiṣṭhasiddhanta*:

> The earth, consisting the mahābhūtas (great elements) stands in the middle of the space of the cosmic egg for the existence of all creatures; it stands like a round sphere.

Now this is explained by the demonstration of a sphere. That is as follows. Having pulled the tip of an iron rod to the north from the tip of the latitude of one's own place, one should make it enter in the hole at the crossing above the sphere of heaven. In the same manner one should place the second tip (of the rod) in the hole at the crossing of the sphere of heaven below the southern tip. When the sphere is prepared in this way, the meaning of the āryā is known by itself. Wherever above the sphere of the earth is the break of this iron rod, there is Meru, the abode of the gods. Wherever below it (another break of the rod is), there is Vaḍavāmukha, the abode of the daityas. One pole is at the tip of the rod above Meru and the second is at the tip of the rod above Vaḍavāmukha. The gods and the demons think that they are below each other.

Here also Āryabhaṭa (says):

> Meru is at the center of the land. Below that at the center of the water is Vaḍavāmukha. The gods and demons think that they always stand below each other.

So also in the *Pancasiddhāntikā*:

> The whole (sphere) is covered by trees, mountains, cities, men, gods, rivers, oceans, etc.; at its center is Sumeru, the residence of the gods; the daityas stand below. As

> the shadow of those sitting on the bank of a river is seen facing downward, so is the motion of demons; they also think the gods are below (them).

In like manner is the existence of Laṅkā and Siddhapura, or of Yamakoṭī and Romaka below one another. Thus an under-part is assumed everywhere. It is impossible to say (which is) the upper or lower part of the earth with the highest truth because creatures live everywhere since the sphere of the earth is covered by men, animals, insects, seas, mountains, cities, trees, rivers, etc. like the protuberance of a Kadamba flower with filaments.

Here also Āryabhaṭa (says):

> As the protuberance of a Kadamba flower is covered all around by flowers so is the sphere of the earth by all creatures born on land and born in water.

The teacher Lāṭadeva (says) the same:

> (The earth) is covered by mountains, rivers, and seas, by towns, kingdoms, trees, animals, horses, etc., like the protuberance of a Kadamba flower all around by flowers.

Whatever was mentioned by the teacher beginning with **"its upper part"**, that also with respect to the indication of dharma and adharma is explained, (the fact that) the earth is always below everything and space above. Whoever says that Meru is at the center in order to refute a material support they do not know the stability of the oceans with respect to watery and dry parts.

Whatever was indicated as the equator marked by sixty ghaṭikās above the region of no latitude, that is the horizon for those who stand on Meru. Whatever is the six-o'clock circle, that is the prime vertical

(for them) because their east and west (points) and the two "axes" of rising amplitude are attached to the horizon. It is exactly the same for those living at Vaḍavāmukha. The sea also is like a girdle for both (of them). For the gods the half of the ecliptic beginning with Aries is always to be seen standing above their horizon. In the same manner, the half (of the ecliptic) beginning with Libra (appears) for the daityas. The primacy of Aries and Libra is for the sake of implying the equator.

Laṅkā is to the south for those living directly north of Laṅkā and Meru is to the north. Yamakoṭī is to the south for those standing directly north of Yamakoṭī and Meru is to the north. Siddhapura is to the south for those standing directly north of Siddhapura and Meru is to the north. Meru is to the north for those standing directly north of Romaka and Romaka is to the south.

For those standing on Meru, the directions are always east because the decision about direction is based on the sun. Wherever the sun rises that is the east and wherever it sets that is the west. But it is not (like that) there. So it is said that wherever (the sun) is first seen at the beginning of the day is east; wherever at noon is south, wherever (the sun) disappears, being invisible, is west, and wherever it is at mid-night that is north. But for those standing on Meru (the sun) on the equator, having risen once is seen repeatedly rotating in all directions. Therefore the assumption of the separation of directions does not occur there. So it is said that wherever at the beginning of a day (the sun) is first seen that is east.

This also is not (true) because the beginning of a day of a corrected solar and a civil (day) do not happen simultaneously. If (you say) it happens occasionally, even so not at a fixed point.

Thus the situation of the sphere for those staying on the (prime-meridian) line (connecting) Meru and Vaḍavāmukha is explained.

For those standing in between them eastness and westness are distinguished by the calculation of the longitudinal difference. In order to demonstrate, one should rotate the sphere of the earth by a distance equal to the distance travelled. If the place is east, then one should rotate it to the west; if (one's) place is west, then to the east (because the two holes) entered by the tips of the rod are the same as in the arrangement of the sphere at a given locality.

Thus this measure of direction is explained. The equivalent in time is to be inferred by one's own intelligence.

Having thus explained the situation of the two poles for those standing at Meru and Vaḍavāmukha, now he says (an āryā) in order to explain the rotation of constellations and so on.

> **The circle of the constellations, bound to the two poles and staying on the horizon, rotates clockwise for the gods and counterclockwise for the demons, being driven by the pravaha wind. XXI, 4**

Bound to the two poles (means) fastened to the two pole stars. It goes clockwise; **going clockwise** means going to the right. **Of the gods** (means) of those standing on Meru. **Staying on the horizon**; Whatever circle is produced because of a peculiarity of the earth, that is the horizon; where space is perceived to be as one with the earth all around, staying there, attached to that. **The circle of constellations** means the circle of nakṣatras, the equator.

Counterclockwise for the demons; that circle of constellations attached to the horizon rotates to the left for the daityas. Even for a moment it is not still, **being driven by pravaha wind**; always impelled by the pravaha wind which goes to the west.

Whatever is this circle of constellations for the gods who stand at the upper (part) of the sphere of the earth it is attached to the horizon because the circle of constellations is the equator and that is said to be the horizon for those standing on Meru. What stays there is rotated by the pravaha wind. That is observed by the gods as going clockwise and by the daityas as going counterclockwise because they are below one another.

Clockwise and counterclockwise is determined in this way: when someone sits down doing something on his right hand, that is seen on the left hand of his mirror image. All of this one should demonstrate on the sphere causing the two tips of the rod to enter the crossings on the upper and lower (parts) of the sphere of heaven.

The phrase **bound to the two poles** is in order to demonstrate universality. The universality of the circle of constellations consisting of the twelve zodiacal signs is up to the two poles and up to the center of the earth as if in the middle of the wide bands. This is the intention. The meaning is that there is no region in the cage of the constellations together with the spheres and the earth that is beyond the twelve zodiacal signs.

Other people, however, explain it in a different manner. (They say): the sphere of the earth rotates to the east; the cage of the constellations with the circle of constellations is immovable. In this way also the attainment of being clockwise and counterclockwise is exactly the same.

Not so. If the sphere of the earth rotates, birds etc. would never come back again to their own nest from heaven. Also rain-clouds would not release a lot of water in one place because (they would) go ahead of each place. The polar stars etc. would also always go westward. Trees, mountains, etc. would be shattered, blown by the wind caused by the speed of the sphere of the earth.

Here Varāhamihira (says):

> If it were so, eagles etc. would never approach their nest again from heaven.

and so on. Therefore the earth does not rotate. The circle of constellations rotates being blown by the pravaha wind.

Varāhamihira (says) the same thing:

> Straight above Meru in space is the axis; another pole is below staying in space. The throng of the stars bound there is rotated by the pravaha wind.

In like manner Āryabhaṭa (says):

> As the cause of (their own) rising and setting, the cage of the constellations with the planets rotates, blown constantly by the pravaha wind, going directly west from Laṅkā.

The meaning of Āryabhaṭa's sūtra is (that the cage of the constellations) is attached to the horizon for those standing at Meru and Vaḍavāmukha.

Thus (is it said) in the *Pauliśasiddhānta*:

> Above it (Meru) is the (north) pole in heaven; the circle (of constellations), which is bound to it by chords of wind, driven by the wind, revolves to their risings and settings.

Thus (is it also said) in the *Vasiṣṭhasiddhānta*:

> There at the tip, the sphere of the stars, which is covered with planets, nakṣatras, and constellations, constantly revolves to the right in space.

In these (verses), there is a reference to Meru by means of a pronoun.

Having thus explained the situation and rotation of the poles and the circle of constellations for those who live at Meru or Vaḍavāmukha, he says (an āryā) for the sake of the remaining places.

> **Elsewhere in all directions the cage of the constellations rises up and the pole descends; at Laṅkā the circle of the constellations goes from the eastpoint to the westpoint and the two poles are on the horizon. XXI, 5**

Elsewhere (means) at another place than Meru and Vaḍavāmukha, **in all directions**, in all directions, **the cage of the constellations rises up** means that it is drawn away from the horizon. The cage of the constellations is the cage of the constellations; the circle of nakṣatras is the equator. **The pole descends** (means) the pole is oblique from midheaven in the northern dry part other than Meru and in watery part other than Vaḍavāmukha.

At Laṅkā the circle of the constellations goes from the eastpoint to the westpoint; the use of the word "Laṅkā" is in order to imply places without latitude. There the circle of the constellations goes from the eastpoint to the westpoint, that is, goes up and down. **The two poles are on the horizon**; the meaning is that there for an observer at a place without latitude the two poles, the northern and the southern, are seen attached to the horizon.

This is the intention. The earth is spherical in shape and stands at the center of the cage of the constellations. For the sake of explaining that, having tied one end of a string to a pole star, one should tie the second (end of the string) which passes through the earth,

Meru, and Vaḍavāmukha to the second pole star. In the same manner, one should tie (another string) passing from the eastern crossing through Yamakoṭī, the earth, and Romaka to the western crossing. In the same manner, one should tie (another string) passing from the southern crossing through Laṅkā, the earth, and Siddhapura to the northern crossing.

Then the similarity of the two similar spheres of (the constellations and of the earth is proved by imagining (their) divisions. Short zodiacal signs, degrees, etc. on the small circle and long ones on the big one are proper and are imaged as similar on the spheres of the constellations and of the earth. Therefore as much as the equator is indicated to be from the pole (on the sphere of the constellations), so much is the region without latitude from Meru (on the sphere of the earth). It is to be applied in the same way on other (spheres).

All the arguments of the equations are mutually in each quadrant of the spheres of the constellations and of the earth. The quadrants are ninety degrees of the degrees of the circle of constellations. Wherever an observer stands, he is above the sphere of the earth and that observer sees half of the sphere of the earth. He doesn't see the second half which is hidden by the earth. Therefore by as many degrees of the sphere of the earth as someone is south of Meru, by so many degrees of the sphere of the earth does the pole sink down in the north. So does (the other pole) from Vaḍavāmukha by exactly as many degrees. The two crossings of the equator are above the region without latitude. One should demonstrate this after having placed the sphere with the six-o'clock circle and the poles at the tip of the latitude of his own place.

As long as it is a place without latitude, the circle of constellations runs from the eastpoint to the westpoint and the two poles are on the horizon. The gods do not see the half of the sphere of the earth which

is south from the region without latitude. Neither do the daityas see the half of the earth to the north. In the same way those who stand to the north of the (place) without latitude do not see the south pole and those standing to the south (do not see) the north (pole).

But if the earth were flat, then the half of the ecliptic beginning with Aries would be always visible for those standing to the north of the ocean and the pole would not be attached to the earth. This will not happen because it contradicts perception. If the cage of the constellations were assumed to be in the shape of a parasol, the sun though staying among the twelve zodiacal signs would be always visible to us because Meru's covering it was refuted by us previously. If the earth were spherical in shape, then that having the shape of a parasol would always be visible for those on Meru and always invisible for those living at Vaḍavāmukha because of the earth's covering it since the whole ecliptic is the parasol. So because there would be no finding out the ascendent and so on and the Sine of the co-latitude, the Sine of the latitude, and so on, this faction is worse. Therefore the earth is spherical in shape and so is the cage of the constellations. Because of this, the Sine of the co-latitude is the radius at the equator, the region without latitude and the Sine of the co-latitude does not exist at Meru and Vaḍavāmukha, nor does the Sine of the latitude at (the place) without latitude. The co-latitude (at the equator) is ninety degrees because latitude is the elevation of the pole. In the same way it is to be applied at intermediate (places).

Here the teacher Lāṭa (says):

> As the circle of constellations rises up from that region in all directions, so the pole leaves midheaven.
> A cloud standing on an eminent place rises as if splitting the surface of the earth, in the same way it (the pole star)

stands above the others in the crowd of stars.

Having thus explained the differences in the rotation and the appearance of the circle of constellations from the difference of place, now he says (an āryā) explaining those of the blessed sun.

> **The gods see the sun on the equator going to the right on the horizon and the demons (see it) going to the left. Men who stand in a region without latitude (see the sun) on the equator going due west. XXI, 6**

The word "**they see**" is to be used everywhere. **The gods** who live on Meru (see it) **going to the right**, going clockwise. **The demons** who live at Vaḍavāmukha (see it) **going to the left**, going counterclockwise. (Thinking) what? he says "**the sun**". Where? **On the horizon**, on the circle of the horizon, that is, attached to the earth. When? **On the equator**, standing on the equator; the meaning is on the equinoctial day. **Men who stand in a region without latitude (see the sun) going due west**; observers standing at a place without latitude such as Laṅkā and so on see the sun on the equator going straight up and down. That is the meaning of the sūtra.

One should demonstrate this on a sphere having inserted the two tips of an iron rod into the two holes of the upper and lower crossings of the sphere of heaven. One should rotate the sphere of the constellations after having made a mark representing the sun on the equator crossing, but in explaining the gods and the demons and in explaining the places without latitude having put the two tips of the rod into the two crossings at the north and south of the sphere of heaven. The rest is the same.

Here the teacher Lāṭa also (says):

> The gods see the sun on the equator proceeding to the right on their own horizon of visibility. The daityas (see it) going to the left. Those who are wearied by fighting (see it) staying on the line of the prime vertical.

His (Lāṭa's) (word): "those who are wearied by fighting" (means) those who dwell in the region without latitude.

So Varāhamihira (says):

> For the gods, the sun, rising at the beginning of Aries and going on the earth-circle, revolves to the right;
> (for those) at Laṅkā (it revolves) overhead; for the foes of the gods in the opposite direction.

Now he says (an āryā) for the sake of explaining the two days of the gods and of the demons according to the appearance and non-appearance of half the ecliptic.

> **The gods always see the northern half of the ecliptic beginning with Aries going to the right. The daityas see the southern half beginning with Libra going to the left. XXI, 7**

The northern (means) northern. **Half of the ecliptic**, a semi-circle. **Beginning with Aries**, beginning with Aries. **Going to the right**, going to the right. **The gods always**, those who live on Meru always. **See**, behold. **The southern half beginning with Libra** going to the left **the daityas** who live at Vaḍavāmukha always see; that is the remainder of the statement.

Here Āryabhaṭa (says):

The gods who stand on Meru see the northern half of the sphere of the constellations (moving) to the right; the ghosts at Vaḍavāmukha to the south (see) the half going to the left.

Here the use of two words "beginning with Aries" and "beginning with Libra" is for the sake of indicating the equator. Therefore, having placed two tips of a rod at the two crossings at the top and bottom of the sphere of heaven one should demonstrate everything. There, wherever is the intersection of the ecliptic and the equator, the sun is there at the beginning of Aries on the equator. (The sun) staying there with its disk half concealed is observed by those staying at Meru and also by those living at Vaḍavāmukha revolving all around for one day like a bull (around) a post. Then it is observed further north every day because of its motion on the ecliptic by an interval equal to the declination of that day up to the end of Gemini. Staying there it is seen by those dwelling on Meru to be removed from the horizon by twenty-four degrees and to be revolving all around. Then it is seen sinking down every day because of its motion on the ecliptic up to the intersection of the equator and the ecliptic at the beginning of Libra. There, again, it is observed by the gods and demons with its disk half concealed as if staying at the eastern crossing, and revolving all around. It is not seen below that by the gods because the equator itself is their horizon. Then (the sun) by its motion on the ecliptic is observed by the daityas rising up to the south up to the end of Sagittarius. Having risen up there by twenty-four degrees, it is again seen (proceeding) with the motion of descending up to the crossing at the beginning of Aries. Then it sets because of the horizon. Then sunrise for the gods (occurs) at the beginning of Aries and setting at the beginning of Libra; it is the opposite for the daityas. The sighting

of the moon and so on without latitude is to be applied in the same manner. The determination of ascent and descent because of latitude is to be applied by one's own intelligence.

Thus (the half of the ecliptic) consisting of the six zodiacal signs beginning with Aries has always risen for the gods. The sun staying there has always risen, too. It gives one hundred and eighty-three rotations less a bit because of the circle of constellations. In the same manner, (the half of the ecliptic) consisting of the six zodiacal signs beginning with Libra has always risen for the daityas. The sun staying there has always risen, too. It gives another one hundred and eighty rotations plus three less a bit because of the circle of the constellations. Therefore, when the sun stays (in the half of the ecliptic) consisting of the six zodiacal signs beginning with Aries, it is a divine day; when the sun stays (in the half of the ecliptic) consisting of the six zodiacal signs beginning with Libra, it is their night. (The day and night) of the daityas are the opposite.

Whoever desire that a divine day (begin) when (the sun) stands at the beginning of Capricorn and a night when (the sun is) at the beginning of Cancer, the gods do not as a rule stand on Meru for them. If (the gods) stood on Meru, how do they see the three zodiacal signs beginning with Capricorn and how do they not see the three zodiacal signs beginning with Cancer? You, even though very wise, can not say that the standing and rotation of the sun is elsewhere than on the ecliptic.

Here Varāhamihira (says):

> Reverence be to those who say: "For the wise (gods) who dwell on Meru it is day when the Sun is in Aries, Taurus, and Gemini, night when it is in Cancer and so on." In whatever places beginning with the beginning of Aries

(the sun) is to the north and also returns (from Cancer), how is it visible there and again not visible while it is there?

One should demonstrate all this on the sphere.

Now he says (a half āryā) making clear the exact meaning.

The gods and daityas see the sun which has risen once for half a solar year. XXI, 8ab

"A year of the sun" means the sun's passing through its circle. For half of that the gods and the daityas see **the sun which has risen** just **once**. This means that the gods see (the sun) moving in the six zodiacal signs beginning with Aries for six solar months; the daityas see (the sun moving) in the six zodiacal signs beginning with Libra for the other six months.

Here a demonstration is explained in the previous āryā. One should demonstrate everything on the sphere arranged in this way.

Here Varāhamihira says:

For those who dwell on the top of Meru the sun, having risen once, is visible for six months moving in the six (zodiacal signs) beginning with Aries; afterwards it is visible for the daityas.

Here the teacher Lāṭa (says):

For half a year the sun, having risen once, is seen by the gods.

So Āryabhaṭa:

The gods see the sun which has risen for half a solar year; so do the ghosts.

The divine measure which is mentioned by the teacher in (the chapter on) mean motion with the words *"the revolutions of the sun are divine days"* has been explained here in one and a half āryās.

Now with the second half of the āryā *"lunar months are the days of the ancestors"* he says the explanation of the day of the ancestors.

> **The ancestors who have gone to the moon (see the sun which has risen) for half a lunar month, men standing on the earth for their own day. XXI, 8cd**

Those who have gone to the moon are **those who have gone to the moon**; the meaning is, those who performed (sacrifices) who are called ancestors. A month of the moon is a lunar month which consists of thirty tithis. A half of that is fifteen tithis. From half of the eighth (tithi) of the dark half till half of the eighth (tithi) of the light half the ancestors see: **"the sun which has risen once"** is supplied. That is a day of the ancestors.

"Men standing on the earth for their own day" means: men standing on earth, such as ourselves, for their own day, for a day of their own. Because (the explanation is) completed by just the word "dina (a day)", the use of "sva (their own)" serves as an explanation of the difference of a day at each location. It means that men see the sun which has risen once from their own sunrise until their own sunset. It is not a nychthemeron because of its conflict with observation and because of its similarity in all aspects to the measures of the divine and ancestral (days).

Here is the reasoning for an ancestors' day: when the moon has no latitude, at the beginning of the first tithi of a light half, whatever string is drawn from the center of the earth to the center of the sphere of the sun also splits the center of the sphere of the moon because (the

longitude) of those two is identical. Wherever is the breakthrough of the string on the upper (surface) of the moon, the ancestors are there. It is noon for them at that time. They see the upper half of the sphere of the moon, we see the lower half by imagining the center of the breakthrough of the string on the lower (surface of the) sphere of the moon because (the luminaries) cover each other up. Therefore, at that time, we do not see the sphere of the moon at all because the brightness of the moon is caused by the falling of sunbeams (on it). It is said in the Veda that *(the moon) is very gracious, having the rays of the sun.*

360 degrees are to be arranged on the sphere of the moon as on the sphere of the earth. Then twelve parts are (produced) by division by thirty. So much is the motion in a tithi. The center of the sun slips westward from the center of the upper (surface of the) moon by twelve degrees each tithi. By this motion, there is a falling of rays of the sun on the half (of the moon) that is visible to us until the whiteness on the moon is apprehended by us. In this way as soon as (the sun) sets for the ancestors by ninety degrees, for us however (the moon) is half white. That is at the middle of the eighth (tithi) of the light half. After this is the night for the ancestors, (but) an increase of whiteness for us. At the end of the full-moon it is midnight for the ancestors, (but) maximum increase of whiteness for us; the distance (between the sun and the moon) is half a circle. Then, by the motion in later nights it is sunrise for them at the middle of the eighth (tithi) of the dark half, it is again half white for us because of that decrease in whiteness in nine zodiacal signs. Then by its motion in the previous days, the end of the moonless (night) is their midday, but for us the invisibility of the whiteness.

Thus the moon is not perceived for twelve time-degrees on both sides from the end of the moonless (night), (but) it is full (for twelve

time-degrees on both sides) from the end of full-moon because of its remoteness from contact with the sun.

Whoever want the beginning of the first (tithi) to be the beginning of the day of the ancestors, not everything happens for them in this way. Therefore the use of (the word) "month" is for implying thirty tithis. When someone says "I came from the village in a month", there is no counting of the month beginning with the first (tithi). Like this in that case, so here also. This explanation for the ancestors who are at the center of the upper (surface) of the moon is to be demonstrated. But if the ancestors were everywhere on the sphere of the moon like the filaments on the protuberance of Kadamba flowers, the depression and elevation (of the sun) must be used for them as for those who stand on the sphere of the earth. They would also see a shorter or longer half-month like the day and night of men. One should demonstrate all of this as it is on the sphere. By this explanation, the determination of the elevation of the cusp of the moon and the reason for the day of the ancestors (occur). Sometimes one day consists of thirty ghaṭikās, at other times of sixty ghaṭikās, and at other times there is no day; then one day is up to six months of ours. We will explain all this elsewhere in the explanation of the decrease and increase of day and night in the positioning of the six-o'clock circle.

Here Āryabhaṭa (says):

> The ancestors who have gone to the moon (consider) half a lunar month (to be what) men here (consider) half an earth day.

Since whoever say that the increase and decrease are caused by Dakṣa's curse and that the moon is above the sun are refuted by a very clear explanation of the sphere that begins with the words

"of the moon which is always below (the sun)" and that was said by Varāhamihira, (that) explanation of the sphere by means of a pair of spheres of the sun and of the moon is to be demonstrated here.

Now he says (an āryā), demonstrating where on the earth Laṅkā, the poles, and Ujjayinī are.

> **Laṅkā is at a quarter of the circumference of the earth from the top of the earth and the bottom of the earth. Avantī is to the north of Laṅkā at a fifteenth part of the circumference of the earth. XXI, 9**

By the words **"the top of the earth"** is meant Meru and by the words **"the bottom of the earth"** Vaḍavāmukha. The use (of the word) **"Laṅkā"** is in order to indicate the region without latitude. This is, therefore, the meaning: the region without latitude is round about (the earth) at a quarter (of the circumference) of the earth from Meru and Vaḍavāmukha. Laṅkā, Yamakoṭī, Siddhapura, Romaka, and so on, which are included in it, are right there. This was already explained by us.

Avantī again is directly north of Laṅkā. By the word **"Avantī"** is meant Ujjayinī; there the terrestrial latitude is twenty-four degrees. Twenty-four degrees are a fifteenth part of 360. It was correctly said: **Avantī is to the north of Laṅkā at a fifteenth part of the circumference of the earth.**

The circumference of the earth is 5,000 (yojanas). Its fourth part is 1,250. Laṅkā is to the south of Meru by so many yojanas. Again, the circumference of the earth is 5,000; its fifteenth part is $333\frac{1}{3}$; Ujjayinī is directly north of Laṅkā by so many yojanas. After subtracting these from 1,250, the yojanas of a fourth part of the circumference of the earth, $916\frac{2}{3}$ remain. Meru (can be reached) by going to the

north along a circumference (of the earth) by so many yojanas from Ujjayinī. One should demonstrate everything on the sphere.

At the tip of its maximum declination to the north the sun makes midday above there (Ujjayinī); also the other (planets) beginning with the moon when they have no latitude.

The use (of the word) "Ujjayinī" also is in order to indicate (other) places whose latitude is twenty-four degrees. Therefore that place where there are twenty-four degrees of latitude is everywhere at a fifteenth part of the circumference of the earth from the region without latitude. It is to be applied in the same manner to (places) south from the region without latitude. The (distance in) yojanas and so on should be used with regard to the second pole star and Vaḍavāmukha and so on.

Having thus set forth the knowledge of the distance between Ujjayinī and the region without latitude, now he tells the knowledge of the distance between any given place and the region without latitude.

The equator (is distant) by the yojanas of the product of the degrees of latitude and the circumference of the earth divided by the fractional part of a circle. XXI, 10ab

The product of the circumference of the earth and the degrees of latitude is **the product of the degrees of latitude and the circumference of the earth**; the meaning is the mutual multiplication of the degrees of latitude of one's own place and the circumference of the earth. (The result) from that multiplication is divided by the degrees of a circle—the result is divided by 360. Whatever is obtained, those are yojanas. The equator is (distant) by so many yojanas from that place, above which place is the equator; the meaning is that the region without latitude is (distant) by so many yojanas.

This is as follows. The degrees of latitude at Kānyakubja are 26;35. When the circumference of the earth, 5000, is multiplied by those, the result is $132916\frac{2}{3}$. Then when this is divided by 360, the result is the yojanas of the distance between Kānyakubja and the region without latitude, $369\frac{2}{9}$. Having subtracted the resulting yojanas from the yojanas of a fourth part of the circumference of the earth, the remainder is $880\frac{7}{9}$. Meru is (distant) from Kānyakubja by so many yojanas by going along a circumference. It is to be applied in the same way to any other places. The explanation of the proportion should be demonstrated on a proper sphere.

Now he says (an āryā), announcing this meaning:

> **In this way the sun is above (that place) by the yojanas (corresponding to) its degrees of depression (from the zenith). Any other (distance) is (found) by proportion. XXI, 10cd**

Whatever are the degrees of depression at a given midday (which are obtained from the rule:) *"the sum or the difference of the degrees of declination of the sun at midday and the terrestrial latitude as the directions (of the declination and latitude) are the same or different (respectively),"* these are taken here. With those **degrees of depression** (calculate the yojanas) as mentioned in the previous half āryā. This is meant. Having multiplied the circumference of the earth by the degrees of depression at noon at a given place one should divide (the product) by 360. The result is yojanas. **The sun is above (that place) by those yojanas**; the meaning is: whatever place is directly north or south (of the equator) by so many yojanas, the sun is above that place at noon of that day. It is also to be used in the case of the moon and so on by means of their degrees of depression. If the degrees of depression are to the north, the place is northerly or

(if) to the south, southerly. To the north of a place whose latitude is 24 degrees northern depressions of the sun never happen.

Any other (distance) is (found) by proportion (means) another distance (is found) by proportion; the meaning is: the yojanas from the distance between two given (places) which are directly south or north of (each other are calculated) in this way.

This is as follows. At Kānyakubja at the beginning of the southern ayana (at the summer solstice), the degrees of depression are 2;35. Having multiplied the circumference of the earth by this and divided (the product) by the degrees of a circle, the result is 36. Whatever place is south of Kānyakubja by so many yojanas, there the time of midday is shadowless.

Between two given places as follows. The degrees of latitude at Sthāṇvīśvara are 30;12, at Ujjayinī 24. Their difference is 6;12. The circumference of the earth multiplied by this and divided by the degrees of a circle is $86\frac{1}{9}$. So many yojanas are the distance between those two (places). (Compute) in the same way for any other (places). As here the calculation of the yojanas is by means of the degrees of latitude and the degrees of depression, in the same way the computation of the degrees of latitude is accomplished by the reverse operation. Here the explanation of the proportion should be demonstrated as before. Exactly in the same way is (the method) to be used south of the region without latitude up to Vaḍavāmukha.

Now he tells the calculation of the sphere of space.

The rotations of the moon multiplied by 324,000 is the circumference in yojanas of space. XXI, 11ab

The circumference consisting of yojanas is the circumference in yojanas; the circumference in yojanas of space is **the circumference in yojanas of space**. He says: "how (is it computed)?" **The**

rotations of the moon are 57,753,300,000. "Being of what sort?" **Multiplied by 324,000**; the meaning is that the rotations of the moon (in a kalpa) multiplied by 324,000 are the yojanas in the sphere of space.

This is as follows. The rotations of the moon, 57,753,300,000, are multiplied by 324,000. There are produced 18,712,069,200,000,000. So many are the yojanas in the circumference of the sphere of heaven and also the yojanas of the motion of the planets in a kalpa. He will say: *"of each (planet), but not moving for an infinite time."*

How is a restricted circumference talked about? Some (say): this is the circumference of heaven whose darkness is destroyed by the multitude of the rays of the sun; beyond (that) the darkness is dense and is perceived by us as blue.

Others (say): this is the circumference of the egg in the middle of which stands the sphere of the earth itself together with the spheres (of the planets). Its membrane appears to us as if blue.

In both cases there is not any contradiction of nature since the movement of the planets above the sphere of the constellations is checked.

Here is our understanding of the meaning: 1,871,202,692 multiplied by 10,000,000 and by 10 is the circumference of the illuminated heaven; it is also the journey in yojanas of the planets in a kalpa.

What was also mentioned in the *Vasiṣṭhasiddhānta*:

> The earth, consisting of the great elements, stands at the middle of heaven in the cosmic egg,

and so on occurs in his (Brahmagupta's) acceptance of the egg and is explained by the pupils of Āryabhaṭa. The measure of the sky reached by the rays of the sun is indicated by the teacher by means of an indication of the circumference of heaven.

Since it was established by such (an authority) that: "the circumference of heaven is the product of the rotations and the sphere of a given planet," does not the difficulty of the line beginning **the rotations of the moon** appear to us to be absurd? There is no fault since the measure of the sphere of the moon is 324,000.

Because one will calculate the spheres of the planets just from the circumference of heaven, because of one's ignorance of that how can the product of that and the rotations be calculated? Oh, it is the same also in the case of knowing the sphere of the moon.

No, it is not the same. Here it is said the accomplishment of the calculation of the spheres of the sun and the moon is by another method.

Let it be said: what is this method?

This is as follows. One should get the mean disk of the moon in minutes from the conjunction of the moon and the earth at the rising or the setting (of the moon). One should find out by one's own intelligence how many vināḍis and breaths there are (for the rising or setting) of the disk (of the moon) every day until (the moon) completes a rotation. (From) the rising of all the days put together and divided by so many days is the mean time for the disk to rise in one day. The (result) converted into breaths is the mean (number of) minutes in the measure of the moon. They are thirty-two. The calculation of the mean measure of the sun (is executed) in exactly the same way for an observer standing at the same place. He will mention the measure of the moon in yojanas as *"480"*. Then, when the measure in yojanas is divided by the measure in minutes, the result is fifteen. So many yojanas are the measure of each minute in the region of the sphere of the moon.

The sphere of every planet is 21,600 minutes. Because he will say: *"on a small (sphere) the zodiacal signs and degrees are short,"*

therefore, multiply 21,600 by 15. There is produced the measure of the sphere of the moon in yojanas, 324,000.

In the same way for the sun because it is said in the chapter on shadows: *"by means of the part of a ghaṭikā from the rising of a star (the measures) of the sun and the moon (are computed) from the rising (time) of their measures."* Then the teacher began to tell correctly the calculation of the orbits of all (the planets) with the lunar sphere as the base. Because there was no remainder from the computation in that case, the measures in yojanas of the rest of the planets are not mentioned. Therefore the teacher began here to say everything by means of the circumference of heaven. Let it be such.

We also admit that the measure of the moon in yojanas was established by that performance. There is no discrepancy because it is similar to the agreement on its sphere. The truth is thus. Even though the calculation of the measure of the moon was said by us to be from the conjunction of the moon and the earth at the time of setting or rising, yet it is impossible to grasp the time beginning with vighaṭikās by human measurement. How (is it possible to measure) the parts of a breath? We considered the matter zealously; everything is impossible because of the dips and rises of the earth and because of the intervention of big mountains and forests.

However the traditional doctrine is our criterion in such matters as the hypotenuse (geocentric distance) in yojanas, the measure of the sphere, the circumference, and the rotations at Meru, Laṅkā, and Vaḍavāmukha because they are unattainable. Because Vasiṣṭha, Garga, and so on are mentioned as those whose minds are pure because of the power of their asceticism, we, knowing small parts from the books composed by them, say: "this is right, this is wrong" like students at dinners in other persons' houses.

Having thus mentioned the measure of the sphere of heaven, now he tells the calculation of the spheres of all the planets from it.

> **By the rotations of whatever (planet the circumference of heaven) is divided, (the result) is its sphere. (The circumference of the sphere of) the sun is a sixtieth part of (that of) the constellations. XXI, 11cd**

(The word) "the circumference of heaven" is supplied. **By the rotations of whatever** planet the circumference of heaven is divided, (the circumference of) its **sphere** expressed in yojanas is obtained.

This is as follows. The circumference of heaven is 18,712,069,200,000,000. When it is divided by the rotations of the sun in a kalpa, (which are) 4,320,000,000, (the result is) the measure of the sphere of the sun, $4,331,497\frac{1}{2}$. In the same manner, (when divided) by the lunar rotations, (we get) the yojanas of the lunar sphere, 324,000. In this way the calculation of the spheres of all (the planets) is to be explained by us. When computed they are written down with their own verses:

> The measure of the sphere of the sun is indicated here as $4,331,497\frac{1}{2}$; of the moon 324,000; Mars' is 8,146,916; Mercury's is measured by 1,043,211; the measure of the sphere of Jupiter is 51,374,822; Venus' is 2,664,630; Saturn's is said to be 127,668,787. All are mentioned without their fractional parts. The sphere of the constellations is said to be measured by 260,000,000 diminished by 110,150 (i.e., 259,889,850 yojanas).

(It is stated: **"the circumference of the sphere of) the sun is a sixtieth part of (that of) the constellations"**, The constellations are the nakṣatras. Whatever is a sixtieth part (of) their

(sphere), the sun is on that. This is meant: at whatever distance the sun is from the center of the earth, the nakṣatras are at a distance of sixty times as much from the center of the earth.

If (one should ask:) "what is the use of this in this sūtra whose purpose is to explain the spheres?", its use is to know the sphere of the nakṣatras. Whatever was said (with the words) "the nakṣatras (are at a distance) sixty times (that) of the sun," it was meant that the sphere of the sun multiplied by sixty is the sphere of the nakṣatras. This is on the assumption of the coincidence of the circle on the sphere of the sun and (its) deferent circle. Otherwise, the sun is on its deferent circle. How can the sixtieth part of (the sphere) of the constellations be mentioned since the center of the deferent circle is not at the center of the earth? You will know this in the demonstration of true motion. It is as follows. The sphere of the sun is $4{,}331{,}497\frac{1}{2}$. That multiplied by sixty produces the sphere of the nakṣatras, that is, 259,889,850. This was mentioned before by us.

The śīghra and manda (apogees) and the nodes rotate on an "ecliptic" whose measure is (that of) the sphere of each planet; therefore it is not (arranged) separately for them (the apogees and nodes). We will demonstrate this later in the explanation of true motion.

Now he mentions the equality of the motion in yojanas of the planets:

> **The planets on their own spheres travel an equal (number of) yojanas— equal to (the yojanas) in the circumference (of the sphere) of the constellations in sixty (solar years), equal to (the yojanas) in the circumference (of the sphere) of heaven in the solar years of a kalpa. XXI, 12**

The spheres of their own are **their own spheres**; on these (the planets travel) **an equal (number of yojanas)**. The meaning is that the travel in yojanas of all the planets is the same. This is as follows. The yojanas of the sphere of the constellations — the sphere of the constellations was said to be equal to 260,000,000 diminished by 110,150 — are 259,889,850. Each planet on its own sphere goes so many yojanas to the east in sixty solar years; the planet moves in a solar year yojanas equal to the sphere of the sun. The yojanas of the circumference of heaven are 18,712,069,200,000,000. Each planet travels on its own sphere so many yojanas in a kalpa.

Here the daily motion (is calculated) by proportion with (the number of) civil days in a kalpa; if the yojanas of the circumference of heaven are (obtained) by the civil days in a kalpa, then how many (are obtained) by one civil day? The result is the daily motion consisting of yojanas, 11,858 and $\frac{1135935900000}{1577915450000}$ as fractional parts of a yojana.

There are two proportions with this daily motion in yojanas. The mean longitudes of the planets are calculated with the (daily) motion. This is as follows. If one rotation is obtained by the yojanas of its own sphere, then what (is obtained) by the yojanas of its daily motion?; (this is) the first proportion whose result is the daily motion. Then the second proportion: if the daily motion of the planet (is obtained) by one day, then what (is obtained) by the accumulated days (ahargaṇa)? In the first proportion the multiplier is one; (it is) the divisor in the second. When the two (ones) are removed because of their equality, the multiplier of the accumulated days is the daily motion consisting of yojanas (and) the divisor is the yojanas of its sphere; the result is (the mean longitude of) the given planet.

Also Āryabhaṭa (says) the same:

The planets complete the circumference (of the sphere) of

the constellations in sixty solar years and the circumference of heaven in a kalpa, revolving equally on their own spheres.

All the planets surely have the same motion with respect to (their) motion in yojanas, but why are different motions perceived by us? Having raised this doubt he tells two āryās for demonstration.

Under (the sphere of) the constellations, the spheres of Saturn, Jupiter, Mars, the Sun, Venus, Mercury, and the Moon in order beginning with (that of) Saturn (move) faster with respect to the motion in minutes. Short zodiacal signs and degrees are on a small (circle), long (ones) are on a large (circle). The Moon completes a small circle in a short time; Saturn (completes) a large (circle) in a long time. XXI, 13–14

A troop of stars is **a constellation**, the meaning is the cage of nakṣatras. Under that, **the spheres of Saturn, Jupiter, Mars, the Sun, Venus, Mercury, and the Moon in order**; the meaning is explained in detail by us in the explanation of the nature of the sphere of the earth. **(The spheres) beginning with (that of) Saturn (move) faster**; according to the order of the spheres, Jupiter (moves faster) than Saturn, Mars (faster) than Jupiter, and so on up to the Moon when the planets moves suitably to the east. Or **(the planets) beginning with Saturn (move) faster** (means) Saturn is the fastest; Jupiter is slower than it; Mars (is slower) than Jupiter, and so on in this order; the slowest is the Moon if the planets always move to the west. This determination of quickness and slowness is **with respect to the motion in minutes**; the meaning is: with

respect to the motion consisting of minutes. On the other hand, (the planets) move equally with respect to the motion in yojanas.

The motion (means) the daily motion. It can occur in two ways. According to the first opinion, the planet moves to the east from a nakṣatra by a velocity equal to the minutes of its daily motion. According to the second opinion, the planet with so much (velocity) slips to the west from a nakṣatra, and slipping, it also goes to the east.

Thus the planet which is above has the same (motion in) yojanas as the planet which is below. Because the sphere of (the planet) which is above is large and (that) of (the planet) which is below is small, the zodiacal signs and parts of zodiacal signs of the large sphere are large. Therefore it is said: "**short zodiacal signs and degrees are on a small (circle)**"; the parts of zodiacal signs are short, small, on a small circle and are large on a large circle. Because of that the moon completes a small circle in a short time; Saturn, on the other hand, completes a large circle. Because the moon and Saturn share the same motion (in yojanas), the difference of their motions (in minutes) is caused by the difference of their spheres. On the lunar sphere fifteen yojanas are the measure of a minute; on the sphere of Saturn, on the other hand, a minute (corresponds) to 5,910 yojanas. This is the meaning of the second āryā.

The meaning mentioned in the first verse: "**(the planets) beginning with Saturn (move) faster with respect to the motion in minutes**", is considered the same in both opinions concerning motions.

Here the first opinion. The earth is immovable. But the cage of the constellations together with the planets rotates westward impelled by the wind at every moment; the daily rising and setting of all the

planets and nakṣatras (occur) by means of that (rotation). There, the planets in the cage of the constellations also go to the east; they complete their own spheres, by passing through the constellations, going day after day (a distance) equal to their own daily motions in minutes like worms on potter's disks or persons who fall into the stream of a great river and try to go against it. Therefore, (the planets) are perceived by us as moving to the east after the correction of longitudinal difference.

The second opinion: the earth is immovable. All the nakṣatras and the planets are perceived directly by us as going westward constantly. The nakṣatras are much faster than the planets because they are farther from the sphere of the earth and the impetus produced by the pravaha wind is always stronger for them. Saturn is under them. It is slower with respect to them in the motion to the west. The impetus of the wind is weaker because it is closer to the earth. In the same way, after that ever lower (planets are) in order of slowness the moon is the slowest; but hawks etc. are slower than it; and clearly we are the slowest who touch on the earth. In this way a nakṣatra goes to the west passing Saturn. Therefore the residents of the earth say "Saturn as it traverses the distance between the nakṣatras moves to the east". In the same way (the argument) is to be used for all the planets. Otherwise, all the nakṣatras and the planets are heavenly objects, and there it is possible that two motions would occur for the planets at once. Because the motion is indeed a function of the wind, it is impossible to say that two opposing winds from the east and the west cause two impetuses simultaneously for one heavenly object. Whichever is stronger draws (it) in its own direction because an object like a planet and so on does not have a material support. (If one says:) "there is a material support, standing on which that (planet) goes," that is not so. If it were, it (the support) would hide

(the object). But luminous objects are seen. Therefore, whatever is mentioned in the first opinion, such as "(the planets move) like worms on potter's disks" or "like persons who fall into the stream of a great river" is mentioned because of the lack of supports. The correction of longitudinal difference (which is mentioned) by the previous (opinion) is also employed in our opinion.

Not so. There also are faults in your opinion. One is because retrograde motion is impossible because whatever planet stands lower than the nakṣatras has motion just to the east, and when slipping from the nakṣatras and moving to the west, it has retrograde motion. How is it in both these (motions)? You may say: "when the planet is above the nakṣatras, the nakṣatras are passed over by that; the passed over (nakṣatras) slip to the east and it (the planet) is perceived (retrograding) to the west." This is also impossible to say because a planet never stands higher than the nakṣatras since the position of the rotation of the planet is fixed. Moreover, when retrograding, the planet is very close to the earth; its hypotenuse (geocentric distance) in yojanas is very short and the measure of its disk is large. Therefore, the planet in its maximum retrogression is closer to the earth than at any other time. It can not be explained thus in this always-going-to-the-west opinion. Therefore, this is also an erroneous opinion. There is also another fault. Here the station of the planet is obtained by you just by its being higher or lower . Then how does the same slipping measured in yojanas (occur) from a difference of position? A difference of the wind is mentioned by you, but it (the difference of position) is not here. Whatever is the difference of the slipping, that is measured in minutes because the planets make rotations by a circular motion and the circle is large for the one far away and is small for the one near by. Thus the second fault is also unremovable. Other faults are also to be joined with this indication.

Of these two opinions the first opinion is correct because six kinds of motion of the planets are perceived by us. Six divisions of the motion are produced—upwards, down, and to the east, west, south, and north. Among these six divisions of motion the application of the slipping of the planets is perceived only in the eastward motion and not in the rest. All those (remaining) motions are unable to make their motion with a difference. Therefore the eastward motion is (also) the self-produced motion of the planets. It is a difficult question whether the westward (motion) is caused by the pravaha wind or the rotation of the earth because the cage of the constellations has an even form. The teacher will tell in the explanation for the true motion that the motion of a planet becomes true because of the manda and śīghra epicycles with the phrase beginning with: *"the center of the circle or the sphere is at the center of the earth."* We will explain in detail there, too.

Now he speaks of the calculation of the radius for a circumference in minutes of a rotation.

> **Whatever is the square root of the square of the minutes of a circle divided by ten, that is the diameter (of the circle). Its half is the radius for the sake of the measure of the hypotenuse (geocentric distance) in yojanas. XXI, 15**

The minutes of a circle means the minutes in a rotation, 21,600. Their **square**, square; of that square. Of what sort? **divided by ten. Whatever is the square root, that is the diameter.** Whatever is **a half of it**, of the diameter, that is **the radius** for the circumference of a rotation. **For the sake of the measure of** its own **hypotenuse (geocentric distance) in yojanas**; the meaning is a proportion (is used) with that in the calculation of the true hypotenuse in yojanas.

This is as follows. The minutes of a circle are 21,600. Their square divided by 10 is 46,656,000. Its square root is $6,830\frac{7100}{13660}$; this is the diameter. Its half is $3,415\frac{3550}{13660}$; this is the radius for the sake of the correct computation of the hypotenuse in yojanas.

He removes the doubt: why was the radius of the minutes of a circle said to be *3,270* before?

> **Because the radius of the minutes of a rotation are (expressed) in minutes but not with seconds, the Sines are also not accurate. Therefore another radius was made (by me). XXI, 16**

Whatever is the radius (calculated) from the minutes of a rotation, it includes fractions, and Sines produced from it are also not considered to be accurate. Therefore being afraid of the loss of the results, another radius was imagined by me. The result in the form of an arc is identical. The hypotenuses in yojanas are calculated by assuming their rotations on their own spheres, and are connected to the radius of a rotation. The meaning is that the remaining calculation is not lost because of the given radius since the arcs are also proportional to it.

The section on the spheres in general (is concluded).

XXI, 17–23 Production of Sines

Now the explanation of the true motion is demonstrated.

There at first, he tells two āryās for the sake of demonstrating chords.

> **Having given numbers to eighths of zodiacal signs in direct and reverse order from the joints of the quadrants one should bind strings to each pair (of numbers). (These are) chords. Their halves are sines. Their differences are the (first-order) differences of the sines. These are placed in reverse order from the last. Or the arrow is the versed sine. An arc is from those two. XXI, 17–18**

Eighths of zodiacal signs; the meaning is the 96th parts of the circle of constellations. **Having given numbers** there **in direct and reverse order**; **from the joints of the quadrants**; the joints of the quadrants, from those, the meaning is from every three zodiacal signs. Then **one should bind strings to each pair (of numbers). Chords** are produced in this way.

This is said. One should draw a circle on level ground with a compass opened to 3270 digits. There one should make two equal straight lines, from east to west and from south to north, so that it is divided into four parts. Those are the four quadrants. Then having set three zodiacal signs on each quadrant, one should make marks; then make eight marks on each sign. Thus there are 96 marks on the whole circle. Then, one should draw a line on the eastern portion (of the circle) having stretched a string from the east-west line to the two marks on either side. It is the first chord; the meaning is that it is

214 multiplied by 2. In the same manner, having stretched a string to the two marks on either side next to them, one should draw lines until twenty-four (lines are drawn); 3270 multiplied by 2 is on the 24th string. Then in reversed order, having stretched strings to the next two marks on either side, one should draw lines up to the west. Thus 48 chords are produced.

Their halves are sines; halves of these chords such as *"sines: 214, 427"* are produced by means of 47 lines since the middle one is common. Thus 96 sines are produced on the whole circle.

Their differences are the (first-order) differences of the sines; for each one of those sines, whatever is produced when the difference between each (sine) and the next sine is made, that is the (first-order) difference of the sines. In this way 96 (first-order) differences of the sines are produced for all the sines in the four quadrants in direct and reverse order. ⟨This is⟩ as follows. The first sine is 214; the second is 427; their difference is 213; this is the (first) difference of the sines. In this way everywhere up to the last difference of the sines, 7.

These are placed in reverse order from the last. Or the arrow is the versed sine. These differences of the sines are placed in reverse order from the last sine; whatever is the difference of the sines related to the 24th sine, it is the first (difference) in the calculation of the versed sines. In the same manner, the 23rd is the second, and so on up to the first as the last. Or, in order, whatever is the arrow corresponding to the first chord, it is the first sine in the calculation of the versed sines. Thus (the arrow) of the second (chord) is the second; of the third is the third, and so on up to the radius.

An arc is from those two. The arc of a sine and of a versed sine (corresponding to the sine) is the same.

Since in the same way there are 48 chords (drawn) from the north-

ern portion to the southern portion (and) their halves are sines, their differences are the differences of the sines. (They are) to be used in regular order and in reverse order in the same way on the earth and on the observational sphere. On the ecliptic, on the other hand, the sine (is used) in regular order in (the quadrants beginning with) the beginning of Aries and Libra and in reverse order in (the quadrants beginning with) the beginning of Cancer and Capricorn.

Having thus demonstrated the nature of the chords, now he says (a verse), demonstrating the production of the eighth, twelfth, and sixteenth sines.

> **The square roots from the fourths of the square of the radius separately multiplied by one, two, and three are the differences of the eighth, twelfth, and sixteenth (sines). Other (differences of sines) are from them. XXI, 19**

Whatever is the square root from a fourth of the square of the radius multiplied by one, that is the difference of the eighth sine in order. In like manner, the square root of a fourth of the square of the radius multiplied by two is the difference of the twelfth sine. The sixteenth sine is produced by this calculation from (the square root of the square of the radius) multiplied by three.

In this matter this is the explanation. The eighth is the chord of two zodiacal signs and is equal to the radius because, since six equilateral triangles are made on a circle, the chord equal to the radius is in two zodiacal signs. One should demonstrate all this on the circle drawn as explained. He (Brahmagupta) will also mention *the sines of (arcs which are) a sixth, a fourth, and a third of the circumference of a circle.*

Here it is said that "*the chord of a sixth of the circumference (of a circle) is equal to the radius*". Whatever is the square of the radius, that is the square of the eighth chord. And in the calculation of sines it was demonstrated that the square root of a fourth of the square is taken because the square is an equal quadrilateral. It is also said that *(the area of) an equal quadrilateral is a square. The product of two equal (amounts) is also a square.*

Then once the eighth (sine) is obtained, the upright is the sixteenth sine because it stands perpendicularly on the arm equal to the radius. Its size is known by this: "*the perpendicular is the square root of the square of an arm diminished by the square of its own base.*" The hypotenuse is the radius. Having subtracted the square of the upright from the square of the hypotenuse, the square root is the arm, the eighth sine, because the figure is a rectangle.

The twelfth is the chord of three zodiacal signs. It is also the chord of a fourth of the circumference. An equal rectangle is constructed with it. There the hypotenuse is equal to the diameter. Both the upright and arm are equal to the twelfth chord. The sum of the square of those two is the square of the hypotenuse equal to the square of the diameter. It was also said: "*whatever is the square of the upright increased by the square of the arm that is the square of the hypotenuse.*" When the square of the upright is subtracted from the square of the hypotenuse, the half of the square of the diameter, which is twice the square of the radius, is left. So much is the square of the arm; and also so much is the square of the twelfth chord. In the calculation of a sine, it was demonstrated that the square root of a fourth of the square of the chord is taken.

For the sixteenth chord as the arm the hypotenuse is the diameter and the upright is the eighth chord. This is a rectangle in the same manner. The eighth chord is the upright equal to the radius. When

its square is subtracted from the square of the diameter, three times the square of the radius is left. It is the square of the hypotenuse diminished by a fourth and also is the square of the sixteenth chord. From this, the square root of (its) fourth is the sixteenth sine. In the same way one should demonstrate every sine by lines on the circle drawn before. **Other (differences of the sines) are from them**; by this mentioned method, the remaining other sine-pieces (tabular sines) should be demonstrated by arranging the arm, upright, and hypotenuse.

In case (one should ask) how he tells three āryās to explain it.

> **Having subtracted a fourth of the sum of the squares of the even-numbered (tabular) sine-pieces in direct and reverse order (i.e., sine and versine) of the same (arc), which (is left) unerased, from the square of the radius, (take) two square roots of them. The first (produces the sine-)pieces of half of those (arcs). The second (produces the tabular sines whose numbers) are equal to twenty-four diminished by that. (It is used) for the production of the fourth, second, first, tenth, eleventh, fifth, seventh, sixth, third, ninth, and so on. Thus sine-pieces, small or large, (are obtained). The first pieces are the sines of the sixth, fourth, and third parts of the circumference of a circle. XXI, 20–22**

Whatever are the two even-numbered sine-pieces in direct and reverse order (i.e., sine and versine), such as the second or fourth (tabular sine), of the same arc, the two squares of these are the two

squares of the even-numbered sine-pieces in direct and reverse order of the same (arc). (Whatever is) the sum of those two squares, a fourth of that sum is **a fourth of the sum of the squares of the even-numbered sine-pieces in direct and reverse order of the same (arc). Having subtracted** that **which (is left) unerased.** From what? He says: **"from the square of the radius." The two square roots of them;** the two square roots of those two are the two square roots of them. The first is the root from the number which was left unerased. The second is the root from the square of the radius diminished by the unerased (number). **The first (produces the sine-)pieces of half of those (arcs);** the meaning is: whatever is the root of the unerased (number), that, however much is the number of the tabular sine from which the computation was made, is the tabular sine having half that number. If the calculation is made with the twelfth sine, it is the sixth (sine). Thus (the rule) is to be applied always in the calculation of even-numbered (sine-)pieces. Therefore in the word **"(the sine-)pieces of half of those (arcs)"** there is an indication of the plural.

Whatever is **the second** root, (it produces) **(the tabular sines whose numbers) are equal to twenty-four diminished by that.** The meaning is: whatever (sine-)piece is produced from the first root, having subtracted its number from twenty-four, a (sine-)piece having the number of the remainder is produced. Thus the production of the second is to be known always by the first when it is produced. The word **"(the tabular sines whose numbers) are equal to twenty-four diminished by that"** (is in that form) because it is plural.

The second root is to be used **for the production of the fourth, second, first, tenth, eleventh, fifth, seventh, sixth, third, ninth, and so on.** The meaning is more clearly explained. This

is as follows. Let the eighth sine be an arm and (the eighth) versine-piece an upright which is also the "arrow" of the eighth chord. From that, the square root of a fourth of the sum of the squares of the arm and upright is half of the hypotenuse. It indeed is the sine-piece of fifteen degrees, that is, the fourth sine. Moreover, subtract a fourth of the sum of the squares of that arm and upright from the square of the radius. The square root of the remainder is the twentieth sine-piece, (whose number is) equal to twenty-four diminished by that. Because the fourth sine-piece is the arm, from this it was demonstrated (that) the twentieth (sine-piece) is the upright, and the radius is the hypotenuse.

In the same manner, as the fourth and the twentieth are calculated when the eighth is known, so are the second and the twenty-second from the fourth, the first and the twenty-third from the second, the tenth and the fourteenth from the twentieth, the eleventh and the thirteenth from the twenty-second, the fifth and the nineteenth from the tenth, and the seventh and the seventeenth from the fourteenth. Thus fourteen sine-pieces (are derived) from the eighth. So the sixth and the eighteenth (are derived) from the twelfth, the ninth and the fifteenth from the eighteenth, the third and the twenty-first from the sixth. These are six sine-pieces (derived) from the twelfth. Thus twenty (sine-pieces are produced). Three—the eighth, the twelfth, and the sixteenth—were derived previously as well as the radius; thus twenty-four sine-pieces are shown. Therefore, it is said: **"of the fourth, second, first, tenth, eleventh, fifth, seventh, third, sixth, and ninth and so on.** "kṛtā" (means) 4, "yamalau" 2, "ekaḥ" 1, "dik" 10, "īśāḥ" 11, "iṣavaḥ" 5, "sapta" 7, "guṇāḥ" 3, "rasāḥ" 6, and "nava" 9. What was demonstrated by the teacher: "when the fourth is obtained, the second (is obtained); when the second, the first. In the same manner, when the fourth (is obtained),

the twentieth (is obtained), and from that the tenth. When the second (is obtained), the twenty-second (is obtained), and from that the eleventh" and so on was explained by us in detail.

Thus sine-pieces (are obtained); by this procedure sine-pieces **small or large** are produced. Or by this explanation of a circular figure, twenty-four (sine-)pieces small or large are to be produced.

Whatever are the beginning (sine-)pieces, which are said to be the eighth, twelfth, and sixteenth, those are **the sines of a sixth, a fourth, and a third parts of the circumference of a circle.** The sine of a sixth part of a circumference is the eighth (sine-)piece, of a fourth part is the twelfth sine, and of a third part is the sixteenth sine. This was already explained by us.

An example. The square of the radius, 10,692,900, multiplied by one, two, and three is 10,692,900, 21,385,800, and 32,078,700 respectively. The square roots of their fourth parts are 1,635, 2,312, and 2,832; these are the eighth, twelfth, and sixteenth (sine-)pieces. As for the rest, (calculate) as follows. The eighth sine-piece is 1,635 in direct order and 438 in reverse order. A fourth of the sum of the squares of these two, called "the unerased", is 716,267. Its root is 846. This is the fourth sine-piece. After subtracting the unerased from the square of the radius, the remainder is 9,976,633. Its root is 3,159. This is the twentieth sine-piece. (Compute) everywhere in the same manner.

Now demonstrating the calculation of the remaining (sine-pieces) by another method with the established eighth, twelfth, and sixteenth (sine-)pieces he said an āryā.

> **Or else whatever (results) from a fourth part from the diameter multiplied by an even-numbered versed (sine-)piece after calculating the men-**

tioned (sine-)pieces (are the remaining sine-pieces). No (other) calculation of the sines is more concise than this. XXI, 23

A versed (sine-)piece which has an even number is **an even-numbered versed (sine-)piece.** Having multiplied the diameter, 6,540, by that, its fourth part should be taken. The meaning is that it is equal to *a fourth of the sum of the squares of the even-numbered sine-pieces in direct and reverse order of the same (arc).*

This is meant. The eighth versed (sine-)piece, which has the same number as the eighth sine-piece, 1,635, is 438. The diameter, 6,540, multiplied by this is 2,864,520. Its fourth part is 716,130. (When one has) subtracted this, which is "the unerased", from the square of the radius, 9,976,770 remains. The square-root of the first, 846, is the fourth sine-piece. The root of the second, 3,159, is the twentieth sine-piece. This calculation should be made after calculating eighth, twelfth, and sixteenth (sine-)pieces. Because the calculation such as division by four and so on is to be done in the same way as what was mentioned before, it is said, "**or else**" by this method is the calculation of sines. (The calculation of sines) from the twelfth and the sixteenth is to be known (as being done) in the same way.

No (other) calculation of the sines is more concise than this; the meaning is that, though a lot of methods for the calculation of sines are mentioned by a lot of teachers, no other is more concise than this.

Here this is the explanation. When the diameter diminished by a versed (sine-)piece is multiplied by it, (the result is) the square of the sine-piece having the same (serial number) as the versed (sine-)piece. Having added the square of the versed (sine-)piece to it, a fourth part (of the sum) is enjoined to be taken by the teacher. The

whole diameter multiplied by the versed (sine-)piece is (used instead of) the sum of the squares of the sine-piece and the versed sine-piece because (generally) when a diameter diminished by whatever amount is multiplied by that (amount), (the result) is the square of the radius diminished by the square of the radius decreased by the multiplier. (Having considered) that the square of the multiplier is the square of the versed sine-piece here and that it will be additive, computing from the square of the sine, the whole diameter is multiplied by the versed (sine-)piece. Then by its fourth part everything is produced as before.

If (one asks) how the diameter ⟨diminished and⟩ multiplied by the versed sine-piece is the square of the sine-piece, in this case this is the confutation (of the doubt): the rule of (taking a) square (beginning with) **"the product of a number increased and decreased by a given number."** One should demonstrate all this on the circle as drawn.

Thus the production of the sines.

XXI, 24–30 Manda and Śīghra Equations

Now demonstrating the reason for that correction of all the planets (to) equality with observation by means of applying the manda and śīghra equations which is demonstrated in (the chapter on) the corrected motion he tells with three āryās.

> **The center of the circle of the orbit is at the center of the earth. The mean (planet) rotates on its own orbit in succession (of the signs) from the manda apogee and contrariwise from the śīghra apogee. The center of the epicycles is at (the longitude of) the mean (planet) and it rotates with its mean velocity. (The planet) rotates on its (the epicycle's) circumference from its apogee; contrariwise from the manda apogee and in succession from the śīghra apogee. Because (an observer) on the earth doesn't see the true (planet) as equal to the mean (planet) on the orbit, therefore, the difference between that and the mean (planet) is subtracted from or added to the mean (planet). XXI, 24–26**

The circle of the orbit is **the circle of the orbit.** Or else the orbit as a circle is **the circle of the orbit.** Its center is **at the center of the earth. The mean (planet rotates) on its orbit**; whatever is the circle of the orbit mentioned just before, on that is the mean (planet). **In succession from the manda apogee**; it rotates in succession from the end of the degree of the manda apogee; the meaning is that the planet goes on after passing the manda apogee. **It rotates contrariwise from the śīghra apogee**;

It rotates contrariwise from the end of the degree of the śīghra apogee; the meaning is that it slips westward from the śīghra apogee.

The center of the epicycles is at (the longitude of) the mean (planet) and it rotates. (the word) "having perigee and apogee" (is from) "perigee" and "apogee." On whatever circle occur these two pertaining to the planet, that is **the circle having perigee and apogee (epicycle).** The meaning is: whatever circle is drawn with a radius equal to the difference between the circle of the orbit and the deferent, that is (the epicycle). The first one is the manda epicycle and the second is the śīghra epicycle. The center of these two is **the center of the epicycles** and **it rotates at (the longitude of) the mean (planet)**; the meaning is that the center of the epicycles, which is mentioned in the preceding āryā (with the words): **"the mean (planet) rotates on its own orbit in succession from the manda apogee,"** rotates at (the longitude of) the mean (planet), but the planet does not.

The planet, on the other hand, (rotates) **on its (epicycle's) circumference from its apogee; contrariwise from the manda apogee**; it rotates from the end of its apogee on the circumference of its manda epicycle contrariwise, in reversed direction. As much as the manda epicycle with its center on the circle of the orbit is moved, from the region of the apogee of its deferent in the direction of the (mean) planet, the planet, which stands on the circumference of the manda epicycle, is seen slipping in the reverse direction by so much.

On the contrary, **in succession from the śīghra apogee**; whatever is said (to rotate) on the circle of the orbit contrariwise from the śīghra apogee, that is the center of the śīghra epicycle, not the planet. The planet, on the other hand, rotates in succession on its circumference. As much as the śīghra epicycle having its center on the circle of the orbit is moved, from the region of the apogee of its deferent

in the direction of the (mean) planet contrariwise, by so much is the planet which stands on its circumference seen (moving) in succession.

Because an observer **on the earth doesn't see the true** planet **as equal to the mean (planet) on** its own **orbit**, therefore, ⟨**the difference between that and the mean (planet)**⟩ is **added to or subtracted from the mean** planet.

This meaning: "therefore the real planet does not rotate on the circle of the orbit" is explained in detail by me. There one should draw a circle on flat ground with a compass set to the radius; that is the circle of the orbit. Its center is the center of the sphere of the earth. One should draw two lines from east to west and from south to north passing through the center of the circle. When this is done, four quadrants are produced. Then having made three zodiacal signs in each quadrant, one should make marks. Having made thirty degrees in each zodiacal sign, one should make marks everywhere. The zodiacal signs beginning with Aries in order from the east on the circle of the orbit prepared with quadrants, zodiacal signs, and degrees.

Then at whatever zodiacal sign, degree, and minute beginning from the first point of Aries is its manda apogee, having made a mark there and having stretched a string reaching the center of the earth from that mark, one should draw a line. Then on that line from the center of the earth in the opposite direction one should put a string which is measured by the sine of its maximum manda equation transformed into the radius of the orbit, because so much is the radius of its manda epicycle. Wherever the string touches, having marked it as the center one should draw a circle with radius equal to (that of) the circle of the orbit. That is the manda deferent because it is said by the teacher in the supplementary chapter on the true motion that *the center of the deferent, which is equal to the circle of the orbit,*

(shifts) in the sky from the center of the earth towards the apogee on the radius of its own epicycle. Then having stretched a string equal to the radius from the center of the manda deferent along the previously given line, one should cause it to reach the circumference of the deferent. There at that point is the maximum apogee of the deferent; it is named the apogee. Whatever point on the circle of the orbit is touched by this line, previously marked as the apogee, at that point is the center of the manda epicycle. Having made that circle with a bamboo stick with the radius equal to the distance between the mark on that (circle of the orbit) and that deferent, one should place it so that the center of that circle is on the circle of the orbit. One should make it to be joined by a long bamboo stick which reaches the apogee of the deferent and the center of the earth. Thus the formation of the manda deferent and epicycle.

Then at whatever point from the beginning of Aries in zodiacal signs, degrees, etc. on the circle of the orbit is its śīghra (apogee), having made a mark there, stretching from that mark a string reaching the center of the earth, drawing a line, then having stretched in the opposite direction a string along the same line from the center of the earth equal to the sine of its maximum śīghra equation and transformed for its circle, one should make a mark at its end; that is the center of the śīghra deferent. Having made its center, one should draw a circle with the radius of the circle of the orbit; that is the śīghra deferent. Then one should extend a string equal to the radius following the previously given line to the circumference of the deferent from the center of the śīghra deferent. The maximum apogee of the śīghra deferent is there. The distance between that and the circle of the orbit is the sine of the maximum śīghra equation. One should make the śīghra epicycle with a bamboo stick with a radius equal to it. One should cause it to be joined with a long bamboo stick which

reaches the center of the earth and the apogee of the deferent. Then its center is at the point where the mark of the śīghra apogee was previously (noted) on the circle of the orbit.

One should make five circles which are marked with 360 (degrees) and with direction lines. The circle of the orbit, the manda deferent, and the śīghra deferent are made either by bamboo sticks or by drawing; but the two epicycles certainly need to be made with bamboo sticks and joined with a long stick because the equations are known from their motion.

After establishing thus, he says two āryās to demonstrate the production of equations.

> **Because the koṭiphala is above (the end of) the radius in the first and last quadrants and below in the second and third, therefore (the radius) increased or decreased by that is the "upright". The hypotenuse is the square root of the sum of its (the "upright"'s) square and the square of the bhujaphala. The radius is divided by that and multiplied by the bhujaphala. The arc of the result (is the equation). The equation is (obtained) in this way by calculation in the case of the śīghra. XXI, 27–28**

By the word **the koṭiphala** the koṭi of the epicycle which is calculated by proportion is referred to. The koṭiphala is what is described (by the word): *the two sines (of the bhuja and the koṭi) multiplied by it (the circumference of the epicycle) and divided by 360.* **From the radius**, from the circle of the orbit. ⟨**Because**⟩ **(it is) above in the first and last quadrants and** ⟨**below in the second and third**⟩; because it stands above the circle of the orbit in the first

and fourth quadrants, and enters within the circle of the orbit in the second and third quadrants, therefore the "upright" is greater than the radius by the "upright" of the epicycle in the first and last (quadrants); the "upright" is less than the radius by that (amount) in the second and third (quadrants); the meaning is that it is the "upright" of the hypotenuse of the deferent. It also means that **therefore** the radius **increased or decreased by that is the "upright"** in the calculations of the manda and śīghra (equations).

The hypotenuse of the deferent is, on the other hand, **the square root of the sum of its (the "upright"'s) square and the square of the bhujaphala. "It"** indicates the corrected "upright"; its **square.** By the word **the bhujaphala** the sine of the bhuja of the epicycle is referred to; also its **square. The square root of the sum** of these two (squares) is the hypotenuse; the meaning is the distance between the center of the earth and the real planet.

The radius is divided by that and multiplied by the bhujaphala; here is the explanation of the proportion: if so much of an "arm" pertains to the true hypotenuse, what (pertains) to the radius? The meaning is that it is the sine of the equation of the planet at its place on the circle of the orbit. **The arc of the result**, the arc of the result is to be computed; that is the śīghra equation.

The equation is (obtained) in this way by calculation in the case of the śīghra; the meaning is that the equation told by me in this way is set forth ⟨not⟩ only by the explanation but also by calculation.

Surely whatever is the explanation in the case of the computation of the śīghra (equation) is the same in the case of the computation of the manda equation; then why it is said: **"the equation is (obtained) in this way by calculation in the case of the śīghra"**?

In the manda calculation also the computation of the equation by means of the true hypotenuse is proper, but it is not mentioned by the teacher here. He tells an āryā for removing this doubt.

> **The multiplier of the bhuja and the koṭi is the hypotenuse multiplied by the circumference (of the epicycle) and divided by the radius. (Calculated) repeatedly in the manda (calculation), the result becomes the same as the first. Because of this the hypotenuse is not (used) here. XXI, 29**

What is **divided by the radius**? It is **the hypotenuse**. Of what sort? He says **multiplied by the circumference (of the epicycle)**. What is it? **The multiplier of the bhuja and the koṭi**; the meaning is that it is the true circumference (of the epicycle) at a point on the manda deferent. **(Calculated) repeatedly in the manda (calculation)**, in the manda calculation, **the result is the same as the first**; equal to the result obtained with the mean circumference.

Two proportions are (used) here. (The first proportion): if this (mean) circumference of the manda (epicycle) pertains to the circle of the radius, what is (the circumference pertaining) to the circle of the true hypotenuse? The result from this is the true circumference (of the epicycle), and an equation is derived with it. Then the second (proportion): if so much of an equation is at the point of the true hypotenuse, how much (is the equation) at the point of the circle of the orbit? Here, the radius is the divisor in the first proportion and the multiplier in the second. The true hypotenuse is the multiplier in the first and the divisor in the second. Thus when all are removed, only the mean circumference stands as the multiplier of the sines of

the bhuja and the koṭi. **Because of this** reason, the hypotenuse is not calculated by me in the manda calculation.

This is as follows. Having arranged the circle of the orbit, the manda deferent, then the śīghra deferent as mentioned before, and having placed two epicycles at their own positions on the circle of the orbit, the explanation of the correction of the planets is to be demonstrated. At whatever zodiacal sign, degree, and minute from the beginning of Aries the (mean) planet stays, a mark is to be made there. Then one should move the manda epicycle toward the (mean) planet from the position of the manda apogee; one should move it so that the center of its circle after moving along the circumference of the circle of the orbit stands at the point marked as the (mean) planet. The line of the anomaly of the epicycle standing there is above that (point). Wherever is the intersection of its (the epicycle's) circumference and the circumference of the deferent in front of the apogee, the planet corrected by the manda equation is there. And also the zodiacal signs, degrees, etc. of the circumference of the epicycle and those of the deferent are equal there; the difference is produced by whether (they are measured) regularly or in reverse. Just the distance between the planet and the apogee is the anomaly on the deferent.

Then in the first quadrant (the sine) of the passed over (degrees) is the sine of the bhuja and (the sine) of the (degrees) to be passed over is the sine of the koṭi because with regard to the apogee of the deferent, the sine of the bhuja is always (drawn) in the north-south direction and the sine of the koṭi is east-west in the drawing. It is reversed in the second quadrant. In the third (quadrant) it is as in the first. In the fourth as in the second because (the sine) of the remaining degrees from half a circle is the sine of the bhuja in the second and fourth (quadrants).

The two sines of the bhuja and the koṭi in the deferent effected in this way are transformed (into sines) in the epicycle by proportion: if the sines of the bhuja and the koṭi pertain to a circumference of 360, then how much are they in the manda epicycle whose measure is a given (number of) degrees? The meaning is that the bhujaphala and the koṭiphala are obtained separately in a given circle. When it stands thus, the true hypotenuse is obtained by (the procedure) beginning with (the words): *"the radius increased by the koṭiphala in (the first and last) quadrants"*.

Though the calculation of the (manda) equation should be achieved by this as is the śīghra calculation, the bhujaphala is called by the teacher the equation of the planet. This āryā is presented by the teacher in order to remove this deficiency. Because whatever is the circumference (of the manda epicycle) in conformity to the deferent, the sine of the bhuja in the deferent ought to be multiplied by it, and that circumference is transformed repeatedly by the proportion with the true hypotenuse. The equation obtained by that is again transformed by the proportion with the radius. From this, whatever arc of the bhujaphala (is obtained) with the circumference of the circle of the orbit, that is the manda equation of the planet.

The sun and moon rectified only by the manda (equation) are true and agree with observation. But for (the planets) beginning with Mars the center of the epicycle is referred to by the words "the planet rectified by the manda (equation)." Otherwise, the planet would exist at two places at the same time. The same is in the case in the manda calculation of all the planets. Then having moved the śīghra epicycle from its (original) position so that its center goes backwards along the circumference of the circle of the orbit, one should set its center at the point on the circle of the orbit determined by the manda equation. When it stands thus, one should demonstrate everything on the śīghra

epicycle. Wherever is the intersection of the śīghra epicycle positioned thus and the deferent, to the west of the position of its own apogee and to the east of the apogee of the epicycle, there is the true planet. Because the planet rotates on the deferent—it is stated by the teacher in the supplementary chapter on the true motion: *"the true planet rotates with its mean velocity on the circumference of the deferent"*—therefore, having computed the sines of the bhuja and the koṭi on the deferent, transforming them in the epicycle by proportion, then deriving the true hypotenuse with the explanation of (the phrase) *"the radius increased by the koṭiphala,"* the (śīghra) equation of the planet on the circle of the orbit is calculated as was mentioned. When this was been done, the planet agrees with observation.

As for the production of the negativity or the positivity (of the equation) of the sun, whatever point on the circle of the orbit is touched by the stick (ending at the center) of the manda epicycle, the mean planet is there; and wherever the circumference of the deferent is touched by that circle (the epicycle), the true sun is there because, at whatever point the circle of the orbit is touched by the line to the apogee of the manda deferent, beginning from that (point) up to the point touched by the stick (ending at the center) of the epicycle as many degrees etc. as there are, so many are there from the place of the apogee of the deferent up to the intersection of its circumference and the circumference of the epicycle. Therefore, whatever string is stretched from the true planet to touch the center of the earth, it goes to the west of the mean planet in the first quadrant. An observer standing on the earth sees the sun standing there. Therefore that distance is subtracted in the first quadrant of the anomaly on the deferent because the deferent is above the circle of the orbit in that situation. But it is reversed in the third (quadrant) because the circle of the orbit is above it. In the second quadrant the explana-

tion is as in the first, where the sines of the bhuja and the koṭi are (calculated) after subtracting (the anomaly) from a half-circle. In the fourth quadrant it is as in the third, where the sines of the bhuja and the koṭi pertain to the remainder of a half-circle.

In the same manner one should demonstrate everything relating to the moon with circles as stated before.

As for (the planets) beginning with Mars, again whatever point is determined on the circle of the orbit by the manda computation, having put the center of the śīghra epicycle there one should demonstrate the rest. In this case (the equation) is positive in the first quadrant of the deferent because the apogee of the deferent stands to the east (of the planet). Then (the equation) should be applied in reverse in the remaining quadrants.

The explanation of the manda is said with application only in the manda calculation. When it is calculated accurately, the anomaly pertaining to the circle of the orbit reaches the maximum equation at three zodiacal signs on it. It is not produced with application in the case of the śīghra equation because the explanation is not the same. It is said that it is done so because the difference is very small.

The śīghra equation, however, is determined in the quadrant of the circle of the orbit (and) its positivity or negativity is obtained in the quadrant of the deferent as long as it is increasing on its own because of the fact that the differences (of the equations) are unequal when the sines of the bhuja and the koṭi of the deferent are shown so that everything that is to be considered is understood by itself on the circles set down.

In the manda calculation the planet completes the circumference of the manda epicycle in reverse by passing through one rotation of the anomaly. In the śīghra calculation the planet completes the circumference of the śīghra epicycle by passing through one rotation

of the anomaly. From this all (phenomena) such as (heliacal) rising and setting, retrogression, and direct motion are to be demonstrated.

When the planet stands on the line coinciding with the sun, then its entering into and leaving the maximum setting point are determined by its own time-degrees.

The planet distant (from the sun) by a half-circle is at its maximum retrograde motion because, (while) it moves in its normal direction at the lowest part on the circumference of its śīghra epicycle, it looks as if it were moving westward. An explanation for eastern motion is also possible there because the planet is found moving eastward at a velocity equal to its daily motion. Even so, if a negative equation relating to one day resulting from the string which reaches both the planet on the deferent and the center of the earth is greater than its own daily motion on the circle of the orbit, then the difference between its daily motion and the difference of seeing it on the deferent and on the circle of the orbit is the velocity of its retrograde motion. Everything should be demonstrated on the śīghra epicycle.

On the manda epicycle, however, the planet rotates in reverse. Even if it is at the lowest part there, it is found to be moving eastward. That is the reason why the sun and moon have no retrograde motion.

In like manner, the explanation of such things as positivity and negativity on the two epicycles should be established by one's own intelligence by means of the string which reaches both the planet staying on it and the center of the earth.

Whatever was said in this matter (with the words) beginning: **"the hypotenuse divided by the radius"** is a different explanation in the manda calculation because the bhujaphala on the deferent and on the circle of the orbit has a small difference. This is, however, otherwise in the śīghra calculation. If it is not done in this way there, then a big difference in degrees would result. Therefore, because of

the small difference the hypotenuse is not to be calculated in the manda calculation.

Surely the planet is only one. What are these manda apogees, śīghra apogees, and nodes? If (they) are real (objects), why are they not perceived as the planet is? If unreal, what is the assumed reasoning for them? He tells the removal (of the doubt) here.

> **The apogee is assumed for the explanation of the motion of the planets. So is the node. The increase and decrease of the daily motion and the size are by means of the hypotenuse (geocentric distance). XXI, 30**

The apogee is assumed for the explanation of the motion of the planets; the assumption of the epicycle (nīcocca) and so on is for the pupils' understanding of the real motion of the planets because by means of that the eastern and western motion (of the planet) is caused to be seen. **So is the node**; the node is assumed just for the explanation (of that) by means of which the northern and southern motion (of the planet) is determined. Therefore the meaning is that just the planet alone has reality.

In case (it is asked:) how did he imagine (them)? he says: "**the increase and decrease of the daily motion and the size are by means of the hypotenuse.** This is the intention. Having determined (the longitude of) the planet at a certain time on a certain day by an instrument such as a stick, one should determine (it again) at the same time on the following day. Having observed the difference there one should determine the true daily motion by means of its circumference. When one has divided (the minutes of) the circumference (of the circle) of the constellations, 21600, by the solar civil days of its rotation, the mean daily motion is produced. Then, if the (true)

daily motion is less than its mean daily motion, the planet is above the circle of the orbit. If greater, then it is below it. The hypotenuse of the sun and moon, because it is the distance between the planet and the center of the earth, is also known by means of that. In the same way one should observe the minimum and the maximum daily motions up to passing through one rotation. Its passing through a rotation occurs with its mean motion. From this it is known (that) even though the planet is not on the circle of the orbit because of the difference of its daily motions, the planet still rotates on a circle (whose size is) equal to that (circle of the orbit) because its passings through a rotation are the same. The manda hypotenuse is very long when the motion is very slow and is very short when the motion is very fast. The difference between the maximum manda hypotenuse and the radius is (the sine of) the maximum equation, (and) that is the radius of the manda epicycle. By means of that the various equations (are produced) in the interval of a half (of a rotation). From that the imagining of an apogee in the case of the sun and the moon is employed.

The difference between the true and mean daily motions of (the planets) beginning with Mars is, however, distinguished by means of the manda and śīghra hypotenuses. There the fixing of the difference by means of the manda hypotenuse is as in the case of the sun and is small. The fixing of the difference by means of the śīghra hypotenuse is large and is to be applied in reverse. Then the difference between the śīghra apogee and the planet corrected by the manda (equation) is to be effected on the circle of the orbit. It is small when the hypotenuse is long and large when it is short. Then (prescriptions) such as: "the difference in this case is greater than the mean daily motion because it is perceived to be retrograde, etc." should be applied according to one's own intelligence.

The daily motion of the anomaly is known from its production of the equation. That combined with the daily motion of the planet is the daily motion of the śīghra apogee. The difference between the daily motion of the manda anomaly and the daily motion of the planet is the daily motion of the manda apogee. The meaning is that the śīghra and manda epicycles do not really exist.

In the same way, the inclined circle is fixed at the south and north at the beginning and end of the maximum latitude. Then, because the maximum latitude is observed differently at different places, the motion of the nodes is calculated.

As the daily motion is calculated by means of the hypotenuse, in the same way is (the daily motion calculated) by means of the (apparent) diameter. The determination of the apogee etc. in the same way is also suitable here because the explanation is the same.

It seems to us that as for the manda and śīghra hypotenuses not only are the apogees etc. imagined but also the rotation of the cage of the constellations because it is impossible for the planet to have two motions simultaneously. But when the rotation of the earth is imagined, then, even though the cage of the constellations stands still, the daily rising and setting can occur. The planets moving on the ecliptic or on the inclined orbit move eastward. In the same way the southern and northern motion is effected by the inclined orbit. The earth rotating a rotation in a nychthemeron meets with the sun on the horizon. The planet goes inclined forward. The explanation for the retrograde motion etc. is the same.

The teacher Āryabhaṭa also agrees to the rotation of the earth because it is said in (the chapter of) the ten gītikās: *"the earth moves a minute of arc in a breath."* So (it is said) in the 108 āryās:

As (a man) who rides on a boat and is moving forward

sees a motionless (object) moving backward, so immovable stars move exactly to the west at Laṇkā.

This āryā (though) honored is explained otherwise by Bhāskara and others because of their fear of the people. It is impossible for us and others to know the truth here; but because his intention is accomplished by inference the explanation of the true motion in this way is demonstrated in its proper place. We will demonstrate every āryā-sūtra there.

Thus the explanation of the true motion.

XXI, 31–48 Lunar and Solar Eclipses

Now the explanation of solar and lunar eclipses is demonstrated. There in the calculation of their diameters two hypotenuses in yojanas are used; in connection with that he tells the calculation of the hypotenuses in yojanas of all the planets.

The orbit (in yojanas) multiplied by the radius and divided by the minutes in a circle is the hypotenuse (geocentric distance). XXI, 31ab

The word **"orbit"**, because (it is said) with regard to a species, is singular. By the use (of the word) **"radius"**, the radius (measured) in the minutes of a rotation, 3,270, is understood because the hypotenuse (measured) in the minutes of a rotation is mentioned for the sake of measuring the hypotenuse in yojanas. Therefore **the orbit** of a given planet is **multiplied by the radius**; (and) **the minutes in a circle**, the minutes of a rotation—it is **divided by** these. What is the hypotenuse? The meaning is that it is the radius in yojanas between the center of the earth and the circle of its orbit.

Here is a proportion. If a radius equal to 3,416 is in the circumference equal to the minutes of a rotation, then what will it be in (a circumference) equal to the orbit of the given planet? The result is the hypotenuse in yojanas. The calculation of the hypotenuses of the orbit of all (the planets) is in this way. The hypotenuses in yojanas determined by me are written in a pair of verses. That is as follows.

The hypotenuse of the sun is 685,018 (yojanas). The moon's is 51,240. Mars' is equal to 1,288,420. Mercury's is 164,982. Jupiter's is 8,124,833. Venus' is 421,406. Saturn's is 20,190,582. Of the constellations it is sixty times the sun's.

These are mean hypotenuses in yojanas of the planets and nakṣatras.

Now for the rectification of the (mean) hypotenuses in yojanas he tells the second half of the āryā.

> **The (mean) hypotenuse (in yojanas) multiplied by its own hypotenuse in minutes and divided by the radius is the true (hypotenuse in yojanas). XXI, 31cd**

"Its own" combined with "hypotenuse in minutes" is **its own hypotenuse in minutes; multiplied** by that. What is that hypotenuse? The meaning is the mean hypotenuse in yojanas mentioned just before. **Divided by the radius is the true (hypotenuse)**; the meaning is that what is divided by the radius is the true hypotenuse in yojanas which reaches both the planet on its deferent and the center of the earth. Thus (is effected) the rectification of the hypotenuses of all (the planets).

The explanation: a proportion is also mentioned here.

Now he says an āryā for demonstrating the measure of the disks in yojanas of the earth, the sun, and the moon and explaining their natures.

> **The diameters of the earth, the sun, and the moon consisting of soil, fire, and water (respectively) are 1,581, 6,522, and 480 in yojanas. XXI, 32**

Of the earth, the sun, and the moon consisting of soil, fire, and water respectively; the earth is the earth, the sun is the sun, and the moon is the moon; the earth, the sun, and the moon, a dvandva compound. Their **diameters in yojanas.** It is perceived

visibly by us that the earth consists of soil. The sun consists of fire; it is also perceived by us as such. The moon consists of water; it is also known by the fact that it has cold rays because the rays of the moon (exist) because it reflects the rays of the sun. Just as the sun's rays falling elsewhere (than on the moon) on water cause that water to have rays, so do they also in this case (of the watery moon). The diameters of the earth, the sun, and the moon are in yojanas having so great numbers. The diameter of the sphere of the earth is 1,581; the meaning is 1500 increased by 81. The diameter of the sphere of the sun is 6,522; the meaning is 6000 and 500 increased by 22. The diameter of the sphere of the moon is 480; the meaning is 400 increased by 80.

Now he says an āryā for calculating the diameter of the shadow of the earth in yojanas at the moon's distance.

> **Having subtracted the true hypotenuse of the moon multiplied by the difference of the diameters of the earth and of the sun and divided by the hypotenuse of the sun from (the diameter of) the earth, the diameter of the shadow of the earth at the lunar orbit (is obtained). XXI, 33**

The earth and the sun (form the dvandva) earth-sun; the two diameters of those two are **the two diameters of the earth and of the sun**; write down **the difference** of the diameters; the meaning is the difference of the diameters in yojanas of the sun and the earth. **The true hypotenuse of the moon multiplied** by that; the meaning is the true hypotenuse in yojanas of the moon. Divided by the hypotenuse of the sun (forms the tatpuruṣa) **divided by the hypotenuse of the sun**; the meaning is divided by the true hypotenuse in yojanas of the sun. Then **having subtracted the**

true hypotenuse of the moon multiplied by the difference of the diameters of the earth and of the sun and divided by the hypotenuse of the sun: (thinking) from what? he says: **from the earth,** from the diameter of the earth. **The diameter of the shadow of the earth at the lunar orbit.** By the use (of the word) "the lunar orbit" he indicates the place where the moon stands. Therefore, the meaning is that the diameter of the shadow of the earth in yojanas is at a place on the deferent of the moon. The meaning is that because of that the diameter of the shadow of the earth is large when it is below that (orbit) and is small when above. That is as follows. The diameter of the earth is 1,581; the diameter of the sun is 6,522; The difference of those two is 4,941. The meaning is that multiplying the true hypotenuse in yojanas of the moon on a given day by this and dividing by the hypotenuse in yojanas of the sun, having subtracted the result from the diameter of the earth numbered 1,581 the diameter of the shadow of the earth at a place on the lunar orbit, i.e., on its deferent, is produced.

Here the explanation of the calculation of the shadow (cast) by a lamp is to be explained by the radius of the sun. That is as follows. If the koṭi of the shadow equal to the difference between the radius of the earth and the radius of the sun is obtained from the shadow equal to the hypotenuse of the sun in yojanas, then how much is (obtained as the koṭi) from (the shadow) equal to the true hypotenuse in yojanas of the moon? The result is the difference between the radius of the earth and the radius of the shadow of the earth produced at the place of the moon. The meaning is: when one has subtracted this from the radius of the earth and multiplied it by two, the diameter of the shadow of the earth at a place on the lunar orbit is produced. The teacher described the radius of the sun decreased by the radius of the earth after multiplying it by two. It is also described that whatever is

the result from that, the whole of that is subtracted from the diameter of the earth. Therefore the meaning of the sūtra is accomplished in this explanation.

Here in this āryā the use (of the word) "lunar orbit" implies the manda deferent of the moon because the calculation is demonstrated with the true hypotenuse in yojanas.

Now he says the conversion into minutes of the diameter of the shadow of the earth in yojanas.

The radius multiplied by that and divided by the (mean) hypotenuse of the moon is the minutes in the measure of the shadow. XXI, 34ab

(The word) **"that"** refers to the diameter of the shadow of the earth which was mentioned just before; multiplied by it (forms the tatpuruṣa) **multiplied by it**. The meaning is that **the radius** numbered 3,416 is the radius of the minutes (21,600) in a rotation. **Divided by the (mean) hypotenuse of the moon**; divided by the mean hypotenuse in yojanas of the moon. **The minutes in the measure of the shadow** are produced.

Here are two proportions. If the minutes equal to the radius pertain to the mean hypotenuse in yojanas of the moon, how many (minutes) pertain to the true hypotenuse in yojanas of the moon? The result is the true hypotenuse in minutes. Then if the minutes equal to the true hypotenuse in minutes of the moon pertain to the true hypotenuse in yojanas of the moon, how many (minutes) pertain to this true diameter of the shadow of the earth? The true hypotenuse in yojanas is the multiplier in the first (proportion) and the divisor in the second. When those two are eliminated because of their equality, the radius is the multiplier and the mean hypotenuse in yojanas is the divisor of the diameter of the shadow of the earth. The result

in minutes is the measure of the shadow at a place on the manda deferent of the moon.

Now he says the calculation of the minutes of the measures of the sun and moon in yojanas.

> **In the same way, the radius multiplied by the diameters of the sun and the moon and divided by their own hypotenuses (are the diameters in minutes of those two). XXI, 34cd**

The radius multiplied by the diameters of the sun and the moon; multiplied by the mean measure in yojanas of the sun in one case and by the measure in yojanas of the moon in the other case. **Divided by their own hypotenuses**; the meaning is that they are to be divided separately by their own true hypotenuses in yojanas. The meaning is that thus the two measures of the sun and the moon in minutes are found.

Here are two proportions. If the minutes equal to the radius are obtained by the yojanas equal to the mean hypotenuse in yojanas, then how many are (obtained) by the yojanas of the mean measure? The mean measure in minutes is obtained. Then the second (proportion): if these (minutes of the measure) are (obtained) in the case of the mean hypotenuse in yojanas, then how much is the measure (obtained) in the case of the true hypotenuse in yojanas? The second is an inverse proportion because the measure is small when the true hypotenuse is large and is large when (the true hypotenuse) is small. In this case, the mean hypotenuse in yojanas is the divisor in the first (proportion) and the multiplier in the second. When those two are eliminated because of their equality, the radius is the multiplier and the true hypotenuse in yojanas is the divisor of the mean diameter

in yojanas. The result is the minutes of the true measures of the sun and the moon.

Having thus explained the measures, now he says an āryā for explaining the nature of solar and lunar eclipses.

> **The shadow of the earth covers the moon, the moon covers the sun, if the latitude (of the moon) is less than half of the sum of the measures at the end of the fifteenth (tithi) of the light half and of the other (half). XXI, 35**

(The word) " **covers**" is connected to both (eclipses). Therefore this is the meaning. **The shadow of the earth** covers **the moon** and **the moon** covers **the sun** respectively **at the end of the fifteenth (tithi) of the light half and of the other (half).** (Thinking:) why is there not always (an eclipse)? he says: **if the latitude (of the moon) is less than half of the sum of the measures.** If the latitude (of the moon) at a given moment is less than half of the sum of the measures in minutes of the one which is eclipsed and of the one which eclipses, then there is the possibility of an eclipse.

In this matter this is the explanation. The shadow of the earth rotates always at the distance of half (the circle of) constellations from the sun. The moon is also at the distance of half (the circle of) constellations (from the sun) at the end of full moon (tithi). If the (moon) staying there is not excessively removed from the ecliptic, then it enters into the shadow of the earth because the length (of the cone) of the shadow of the earth is greater than the hypotenuse of the moon. The shadow of the earth rotates on the ecliptic, but the moon staying on its inclined orbit close to its node enters into the shadow of the earth because of the largeness of the shadow of the earth.

The sun, on the other hand, is covered by the moon because the sun is above and the moon is below. It (the moon), moving faster, goes eastward after covering (the sun) like a piece of cloud. Those two are on one line at the end of the moonless (tithi). At that time, if the moon is not excessively removed (from the ecliptic) by means of its true latitude, then a solar eclipse (occurs) because the distance between the centers of the two disks is the (moon's) true latitude. If it is greater than half ⟨of the sum⟩ of the measures, then contact of the two disks does not occur. If it is less, then they enter each other. In this way one should demonstrate everything on the sphere.

Surely the eclipse of the sun or the moon is caused by rāhu; why is it said, **"the shadow of the earth covers the moon, the moon covers the sun"**?

Doubting this (the teacher) who wishes to reject (the idea that) the eclipse is caused by rāhu says at first (the following verse) in order to explain the two eclipsers.

> **The obscurer of the moon is large because the half-covered (moon) has blunt horns. The half-covered sun has sharp horns. Therefore its (obscurer) is small. XXI, 36**

This āryā is clear in its meaning.

The sun is covered by one thing and the moon by another. Because the half-covered moon has blunt horns it is known that its eclipser is something large. Because of the sharpness of the horns of the half-covered sun it is known that its eclipser is small. It is impossible to assume any other two suitable (eclipses) except the shadow of the earth and the moon.

He says two other defects of the (idea that) the eclipses are caused by rāhu.

If rāhu eclipses the moon from the eastern part, why doesn't it (eclipse) the sun in the same manner? Why is the half duration (of the eclipse) of the sun not as long as that of the moon? XXI, 37

This is also clear in its meaning.

As the moon is observed being eclipsed from the eastern part of its disk why is the sun not observed in the same way? The sun is eclipsed on the western part of its disk. How can this happen if rāhu is one? Again the moon, moving eastward, causes its position to be in the shadow of the earth at the eastern part of its disk. Therefore its eclipse-beginning is produced in the eastern direction. On the other hand, the moon moves faster than the sun. Going eastward it causes the (solar) eclipse-beginning on the western part of (the sun's) disk. The meaning is that therefore an eclipse is not caused by rāhu.

Another (thing): if the eclipser is the same, why is the half duration in a solar eclipse not as long as that in a lunar eclipse? The shadow of the earth is large; therefore a long half duration is produced. On the other hand, the moon is rather small; therefore a small half duration is produced for it. Therefore an eclipse is not caused by rāhu.

He mentions another defect of (the idea that) an eclipse is caused by rāhu.

Are the sun and rāhu different for each region because of the difference of the magnitude in the solar eclipse? Therefore solar and lunar eclipses are not caused by rāhu. XXI, 38

The meaning of this āryā is easily understood.

The sun is just one and the same everywhere. Also rāhu. Then why is it that a total eclipse (occurs) at one place, a partial eclipse at another, and no eclipse at a third in the case of a solar eclipse? The magnitude is the same everywhere in the case of a lunar eclipse. How can this happen if svarbhānu (rāhu) is one? But when the moon covers the sun, because of its smallness the difference of magnitude is produced by parallax, latitude, terrestrial latitude, longitudinal difference, etc. As the sun covered by a piece of cloud is not seen at one place, is seen at another, and is seen half-covered at a third, so it is also here. The coverer of the moon is the shadow of the earth. It has one form everywhere like a column of smoke; the moon entering there is covered on all sides. Therefore a lunar eclipse which is the same for all is produced. From this he concludes that **therefore solar and lunar eclipses are not caused by rāhu.**

The explanation is to be demonstrated on a proper sphere.

(The following is) the explanation of the shadow of the earth pertaining to the just mentioned āryās. Having tied two strings at the degree(s) on the earth that are the ninetieth on either side of the degree on the earth that is equal to that at which the sun stands on the ecliptic, and bringing their two ends together at the degree on the ecliptic that is half a circle distant from the degree on the ecliptic rotated by the sun, one should tie (them there). When it is arranged thus, the shadow of the earth is between the two strings. We will present the explanation in detail for each verse of the two chapters on eclipses.

Here the teacher Varāhamihira (says):

> If the moon facing eastward in the seventh zodiacal sign from the sun does not go too far to the north or south, then it enters the earth's shadow. The moon which

is below (the sun), conjoining (with it) from the west, covers the sun as a cloud. Therefore a solar eclipse is different according to the view in each place.

He tells two āryās in order to explain that Varāhamihira and others by whom it was said that "the shadow of the earth covers the moon (and) the moon the sun" are outside of the two śāstras of dharma (smṛti) and results (śruti).

What was said in this way by Varāhamihira, Śrīṣena, Āryabhaṭa, Viṣṇucandra and so on is outside of the Vedas, of the smṛtis, and the saṃhitās (and) is opposed by the people. If this is so, the result(s) of the eclipses which (are) mentioned by Garga and so on in their saṃhitās do not exist, nor do the result(s) of the sacrifices, muttering prayers, and ritual bathings. XXI, 39–40

The meaning is understood.

He states that an eclipse is caused by rāhu.

The two (solar and lunar) eclipses are caused by rāhu; this is established by (people including) cowherds, women, and so on. It is also established that it has many results; here (in the same circumstances are) the result(s) of muttering prayers, sacrifices, and ritual bathings. XXI, 41

This āryā has a manifest meaning.

He expresses approval of this meaning by means of the statements in the smṛtis.

> **Bathing is not mentioned in the smṛtis since rāhu is seen elsewhere at night. When the sun is swallowed by rāhu, all water is equivalent to the Gaṅgā. XXI, 42**

(This verse) is clear in its meaning.

Here he explains a statement in the Vedas in this meaning.

> **This is a statement of the Veda: the female asura, Svarbhānu, pierced the sun with darkness. XXI, 43ab**

This half verse is clear in its meaning. How is it that it was agreed by him, after abandoning what he previously clearly said, that "surely the moon enters the shadow of the earth in its own eclipse, (and) the moon covers the sun in a solar eclipse," that an eclipse is caused by rāhu as is made clear by the people?

Because public opinion appears to be false in such (statements) as: "the month is finished at the end of the white half in many places," "the moon is above the sun," and "its wanning and waxing is by Dakṣa's curse," and so on, that which was doubtful is impossible. Whatever is the statement in the smṛtis, that was intended for the prohibition of night bathing. And whatever is the Vedic statement, that is recited there with the explanation of its meaning that it is intended for the cooking of the milk of a non-white (cow) with a white calf as oblations to Soma and Rudra. The statements in the itihāsas are intended to explain the results of eclipses. Whoever causes the defectiveness of the sun and moon, that is rāhu for them. The

statements in the saṃhitās of Garga and so on explain auspiciousness and inauspiciousness by means of signs such as the direction, color, and deflection (of eclipses). Whoever swallows (a luminary), that is also rāhu for them.

However, while the true eclipser is not known, how can the time, magnitude, measure, duration, deflection etc. which agree with observation be known? Therefore whatever was mentioned before together with its explanation is correct.

(Considering) that what is repugnant to the people should not be mentioned, he says half an āryā for this purpose.

> **As there is unanimity of the Vedas, saṃhitās, and smṛtis, therefore there is mention of this. XXI, 43cd**

This half verse is clear in its meaning.

Now he demonstrates their unanimity with two āryās.

> **Whatever (part) of the moon enters into the darkness of the shadow of the earth at the end of the fifteenth (tithi) of the light half, rāhu covers that because of the gift of a boon by Brahmā. Whatever (part) of the sun consisting of fire the moon consisting of water and standing below (the sun), having moderated heat, covers at the end of the month, rāhu covers that (part) of Savitṛ. XXI, 44–45**

(These two verses are) clear in their meaning because of the authority of the āgamas.

The meaning of: "let it stand below there" is: "let its situation be below rāhu." But (thinking:) "since it (rāhu) is one, how is it that

the quantity of the measures is different in solar and lunar eclipses?", he adds a statement of (its) oneness.

> **In a lunar eclipse rāhu standing at the orbit of the moon, being equal (in size) to the diameter of the earth's shadow, covers the moon; in a solar eclipse, being equal (in size) to the moon, it (covers) the sun. XXI, 46**

This āryā is clear in its meaning.

Someone speaks in this way: "rāhu has a great size. How is it that which is larger than the sizes of the shadow of the earth and the moon is not perceived?" He answers him.

> **Whatever is the excess the fact that the diameter of rāhu consisting of darkness is seen by the sun destroys that. Therefore rāhu is equal (in size) to the diameters of the shadow of the earth and the moon. XXI, 47**

This (verse) which is without application is clear in its meaning.

Now he summarizes the theory of the statement of its oneness.

> **Therefore the shadow of the earth does not cover the moon in an eclipse nor does the moon (cover) the sun; rāhu which stands there and is equal (in size) to their diameters covers the sun and the moon. XXI, 48**

This āryā is clear in its meaning.

The explanation of the nature of eclipses (ends).