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NEMICANDRA'S RULE FOR THE VOLUME OF A SPHERE

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Nemicandra (c. 981 AD) has given a very crude formula $V = 9/2\ (d/2)^3$ for the volume of a sphere. He might have got it from an old South Indian Jain work, which appears to have been influenced by some Chinese work.

Nemicandra's pupil Mādhavacandra Traividya has described sphere as an aggregate of six cuboids in his rationale to the formula and regards that the effective height (*vedha*) of the cuboid is equal to that of its hemispherical pit. Effort has also been made to analyse the background and perspective history of the origin of Nemicandra's formula.

Key words : Effective depth (*vedha*), *Gola*, Nemicandra, Volume of sphere, *Vyāsa*, *Trilokasāra*.

INTRODUCTION

Nemicandra (c. 981 AD) was a Jain sage and known for his knowledge of Jain scriptures[1] and mathematics[2]. He was present in the first consecration ceremony of the famous statue of Lord Bāhubalī, constructed by Sāmundarāya[3], his disciple at Śrāvaṇavelāgolā in Karnataka held on 13th March of 981 AD.

The *Trilokasāra*[4] of Nemicandra is the celebrated work in Prakrit on Karaṇānuyoga. His pupil[5] Mādhavacandra Traividya has written a commentary (*vāsanā*) on it with rationales in Sanskrit.

It contains several rules on mensuration. The rule for the volume of a sphere[6] is very crude.

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My main purpose in the paper is to discuss the rule with rationale, its application, and historicity.

He has not given any formula for the surface of a sphere and is silent on the topic.

NEMICANDRA'S RULE

The correct modern formula for the volume V of a sphere when its diameter d is known is :

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \tag{1}$$

$$\text{or} \quad V = 1.3\,\pi\left(\frac{d}{2}\right)^3$$

The famous Greek mathematician Archimedes (c. 225 BC) states that the volume of any sphere is four times greater than that of the cone with base equal to a great circle of the sphere and with height equal to the radius of the sphere. This statement justifies the formula [1].[7]

Bhāskara II[8] (c. 1114 AD to 1185 AD) is the first mathematician who has given the correct formula [1] in India in the following form :

$$V = \frac{1}{6}\,\pi\,d^3 \tag{2}$$

In the *Trilokasāra*, Nemicandra sets down a rule :

वासद्धघणं दलियं णवगुणियं गोलयस्स घनगणियं।

(TLS, Viśuddhamati, v. 19 first half, p 25)

The above rule rendered into Sanskrit[9] runs :

व्यासार्द्धघनः दलितः नवगुणितः गोलकस्य घनगणितम्।

"The cube of half the diameter (*vyāsa*) halved and then multiplied by nine gives the volume *(ghanagaṇita)* of a sphere (*gola*)".

This rule finds the formula :

$$V = \frac{9}{2}\left(\frac{d}{2}\right)^3. \tag{3}$$

The value for $\pi = 3$ has been adopted here, by Mādhavacandra[10] Therefore the above formula may be expressed as—

$$V = \frac{3}{2}\pi\left(\frac{d}{2}\right)^3$$

or

$$V = 1.5\,\pi\left(\frac{d}{2}\right)^3$$

This is far removed from the correct formula (1) and hence is a gross one.

Datta[11] inferred the formula (3) from the above rule.

In this context the paper, 'On the volume of a sphere in ancient India' by Gupta[12] is worth mentioning.

Application

Nemicandra applied formula [3] for finding the volume of a sphere. He says that a mustard seed shaped like a sphere, the diameter of which is unity, comes to be 9/16 cubic units. He further says that the volume of a mustard, shaped like a cube, a side of which is unity, becomes unity. If such mustard seeds are Nc in number, then their total volume will be Nc itself. What he has done, he has tried to recast mustered of cubic sizes (Nc), the diameter of which is unity into mustard seeds shaped like spheres. Let us suppose that mustards thus obtained will be Ns in number.

Nemicandra says :

चउरस्ससरिसवा ते णवसोडस भाजिदा वट्टं।

(TLS, Viśuddhamati, v 18 second half, p 22)

The above rule (or algorithm) rendered into Sanskrit[13] runs :

चतुरस्रसर्षपास्ते नवषोडश भाजिता वृत्तम्।

"Mustard seeds (*sarśapas*) shaped like a square (*caturasra*) (that is to say a cube) divided by 9/16 yield circular (*vṛtta*) (that is to say, spherical)".

That is :

$$Ns = Nc \div \frac{9}{16} \tag{4}$$

The rule (or algorithm) can not yield Ns with accuracy because it has been developed by including an incorrect formula (3).

However, we find here that he applied his knowledge and understanding of the formula [3] to a new problem and suggests an algorithm to solve it.

Mādhavacandra's Rationale

In his Sanskrit commentary on the TLS, Mādhavacandra Traividya gives the rationale (*vāsanā*) of the rule (v.18sh, p. 22) under the succeeding verse (*gāthā*) (v. 19, p. 25) in the following words[14] :

एकव्यासैकखातगोलकमर्धीकृत्यार्द्धमपहाय अवशिष्टार्द्धं पुनरपि खंडत्रयं कृत्वा तत्राप्येकखंडं गृहीत्वा तदप्यूर्ध्वादधश्छित्वा चतुरस्रं यथा तथा संस्थाप्य तत्र गोलकस्य बहुमध्येदेशे विवक्षितवेध सद्भावोस्ति। पार्श्वेषु क्रमहानिसद्भावात्समीकरणार्थं हीनस्थाने एतावत ऋणं निक्षिप्य समस्थले सति तदपि पुनस्तिर्यग्मध्यं छित्वा उपरि संस्थाप्य समच्छेदेन ऋणमपनीय "भुजकोटी" इत्यादिना खातफलमानीय एकखंडस्यैतावति षण्णां खंडानां किं फलामिति संपात्यापवर्त्य गुणिते गोलकस्य घनगुणितमेव नव षोडशभाजितेत्यस्य वासना जाता।

"Halving a sphere (*gola*) with diameter (*vyāsa*) equal to unity and with height (*khāta*) equal to unity, separating its half (*ardhamapahāya*), again trisect its remaining half (*avaśiṣṭārdha*); taking one section (*khaṇḍa*) from them, by cutting it too from up to downward and place any how so as to be a four sided figure (square) (*caturasra*). There is a mentionable depth (*vivakṣita vedha*) in the middlemost (*bahumadhyadeśa*) of the hemispherical pit (*golakasya*). Being gradually diminished on its sides (*pārśveṣu*) putting negative (or debt, *ṛṇa*) similarly on its diminution (*hīnasthāna*) for equivalence (*samīkarṇārtha*); cutting slantly (*tiryaka*) it too in the middle on becoming equivalent (*samasthala*), putting aloft, by bisecting, withdrawing negative (or debt, *ṛṇa*) ; the volume (*khātaphala*) of one section (*khaṇḍa*) is obtained by its length (or abscissa) (*bhuja*), breadth (or ordinate, *koṭi*) etc. Performing what it will be for six sections (*khaṇḍas*), multiply with multiple (*apavartya*). This is the volume of the whole sphere (*gola*) and is the rationale (*vāsanā*) of division by 9/16".

From this literal translation of his passage, it is seen that he regards a sphere as an aggregate of six cuboids.

Exposition of the Process

Āryikā Viśuddhamati[15] has exposed Mādhavacandra's rationale (*vā sanā*) in Hindi with ten accompanying diagrams.

His (Mādhvacandra's) rationale with the help of her exposition and with some improved diagrams is under four phases as follows in the words of the author of this paper.

Phase I :

Halve the sphere with diameter (*vyāsa*) equal to unity and with height (*khāta*) equal to unity.

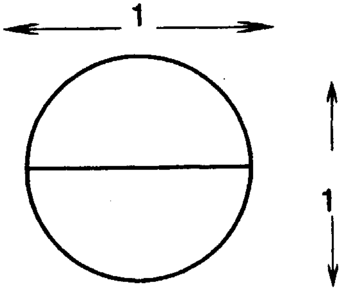

Fig. 1 : The sphere with diameter equal to unity.

After this, keep its one hemisphere and take the remaining other.

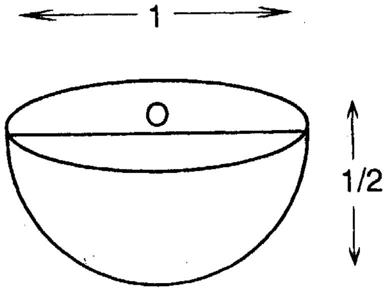

Fig. 2 : The hemispherical pit with diameter equal to unity and with height equal to half its diameter.

Phase II:

Trisect it (Fig. 2) as depicted below (cf. Fig. 3) :

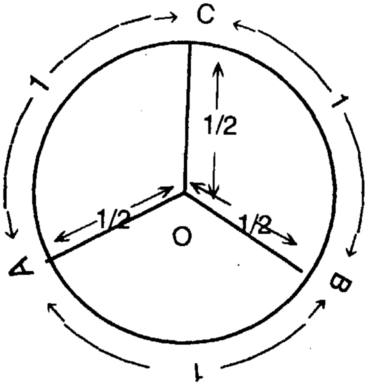

Fig. 3 : The base of the hemispherical pit with its radius (1/2) and circumference (3) and take a section AOB (cf. Fig. 4).

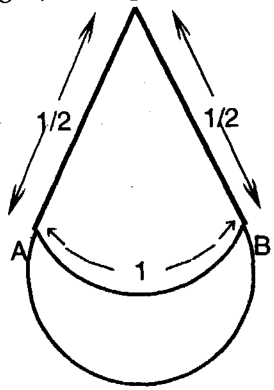

Fig. 4 : A section AOB obtained by trisecting the heispherical pit. Bisect the above (Fig. 4) too from top to bottom (cf. Fig. 5).

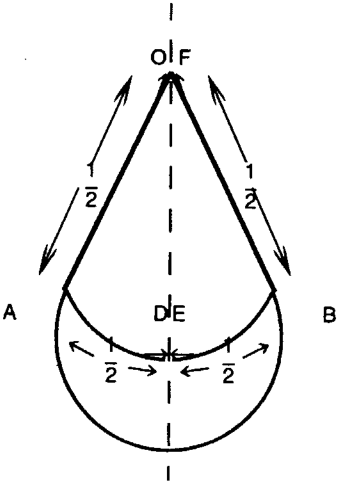

Fig. 5 : The section AOB bisected from top to bottom.

and place any how in such an order so as to be a square (*caturasra*) in upper face.

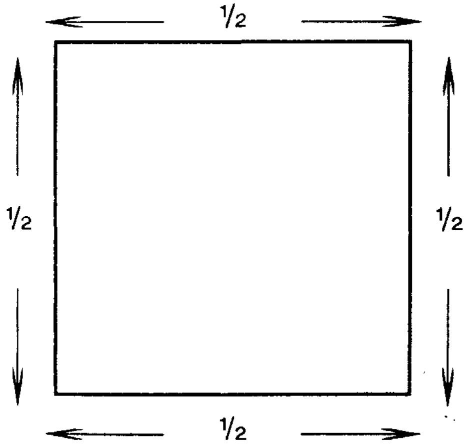

Fig. 6a

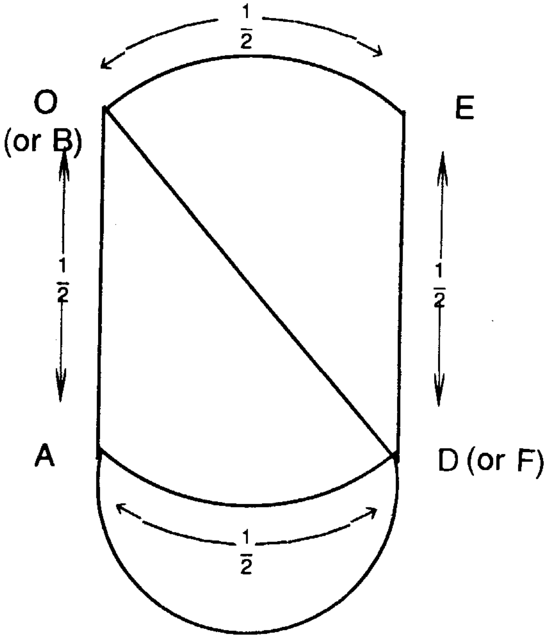

Fig. 6b

This body (Fig. 6b) is, now, considered as a cuboid of sides 1/2, 1/2 and h where h is its effective depth (or height, *vedha*) and is to be determined.

It is not possible to obtain the Fig. 6a that has been obtained by Viśuddhamati (p. 27) from the Fig. 5 through the method just mentioned above. After all, AD and BE (in Fig. 5) are non-linear. Therefore, we have designed the Fig. 6b.

Mādhavacandra[16] says in his rationale to adopt the following rule to calculate the circumference of the base of the hemispherical pit (Fig. 3) :

वासो तिगुणो परिही ....।

(TLS, Viśuddhamati, v. 17, p. 18)

The Sanskrit rendering of the rule is :

व्यासस्त्रिगुण: परिधि: ....।

"Diameter (*vyāsa*) multiplied by three becomes the circumference (*paridhi*) (of a circle)".

This is why the circumference 3 is shown (in Fig. 3).

Phase III:

तदपि पुनस्तिर्यग्मध्यं .... ऋणमपनीय

"Cutting slantly .... negative (or debt, *ṛṇa*)".

In the above part of his passage, what Mādhavacandra wants to say is technically not clear to the author of this paper.

This part needs explanation especially, because he himself did not give any accompanying diagram, which would have clarified the doubts.

Āryikā Viśuddhamati herself could not expose this part well although she has provided two accompanying diagrams in this connection.

However, with the help of her exposition, the third phase is as follows :

The hemisphercal pit has gradual diminution on its sides although there is 1/2 depth (or height, *vedha*) in its middlemost.

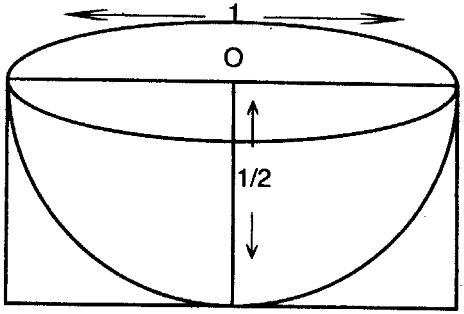

Fig. 7 : The hemispherical pit with its diminution on sides and with its middlemost depth (*vedha*).

For equivalence (*samīkaraṇārtha*), put negatively a quarter of its depth (*vedha*) (1/2) on diminutive place.

On cutting slantly this equivalent place (*samasthala*), putting aloft and withdrawing negative ; the effective depth (or height, *vedha*) remains

$$\frac{1}{2}-\left(\frac{1}{4}\cdot\frac{1}{2}\right)=\frac{3}{8}\ (=h).$$

Phase IV:

The length (*bhuja*), breadth (*koṭi*) and effective depth (or height) of a cuboid (see Fig. 6b) have been found to be 1/2, 1/2 and 3/8 respectively.

Hence,

the volume of cuboid (see Fig. 6b) $= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{8}$

$= \frac{3}{32}$ cubic units.

The volume (*khātaphala*) of the whole sphere (see Fig. 1) $= 6.\frac{3}{32}$

$= \frac{9}{16}$ cubic units.

Thus it gives the volume of the sphere and rationalizes the rule of division by 9/16.

Generalisation

Mădhavacandra regards a sphere as an aggregate of six cuboids and that the effective depth (or height, *vedha*) of a cuboid is equal to that of its hemispherical pit.

If there is a sphere with diameter (*vyāsa*) equal to d, then, according to him its volume :

$$V = 6.\frac{d}{2}.\frac{d}{2}.\left(\frac{d}{2} - \frac{1}{4}.\frac{d}{2}\right)$$

or

$$V = \frac{9}{2}\left(\frac{d}{2}\right)^3$$

This gives the rationale of the formula (3).

Why is the Formula (3) Incorrect ?

From the Fig. 5

OA = OD = FE = FB = radius each

and AD = EB = OA = OD = FE = FB = 1/2 each.

Hence the Fig. 6b by itself as a figure is correct in every possible way but OADE is not square at all. This result can be confirmed by the following wo instances in which we shall show that OADE does not obey the haracteristics of a square.

(a) In the Fig. 6b, OA and OD are shown as a side and diagonal of the square OADE respectively and therefore OD must be greater than OA. But the Fig. 5 shows OD = OA.

(b) In the Fig. 6b

$$\angle AOD = 45°$$

being angle between side and diagonal of a square. But it is 60° in the Fig. 5 because of the Fig. 3 of which

$$\angle AOB = 120°$$

Hence Mādhavacandra's assumtion that OADE is square is wrong and causes to make the formula (3) incorrect.

To determine the effective depth (*vedha*) of the hemispherical pit, Mādhavacandra subtracts one-fourth of its depth (or height, or semi-diameter, *vedha*) from its depth itself.

Gupta[17] thinks that the effective depth has been found by analogy to the effective altitude of a semi-circle for which

$$\frac{\pi}{2}\cdot\left(\frac{d}{2}\right)^2 = 2.\frac{d}{2}.h$$

which gives h equal to $\pi\frac{d}{8}$ (which will become $\frac{3}{8}$ d with $\pi = 3$).

If the hemispherical pit were actually recasted into an equivalent cylinder with height equal to h as in the last two diagrams[18] given by Viśuddhamati in her commentary intended to depict and as Gupta[19] suggests and imagines, the first rule-maker (of the formula [3]) would have had

$$\pi\left(\frac{d}{2}\right)^2 h = \frac{1}{2}\cdot\frac{4}{3}.\pi\left(\frac{d}{3}\right)^3$$

This gives :

$$h = \frac{d}{3}$$

or

$$h = \frac{d}{2} - \frac{1}{3}\cdot\frac{d}{2}$$

This expression implies that one third of the depth (or semi-diameter) of the hemispherical pit must have been subtracted from its depth (or semi-diameter) itself to determine h.

Nemicandra's Formula from Historical Point of view

(A) According to A.A. Sarasvathi Amma[20], we do not come across any authentic mention of sphere in India before the time of Āryabhaṭa I (476 A.D.).

Āryabhaṭa's rule for finding the volume of a sphere is contained in the *Āryabhaṭīya*[21] (499 AD), which is :

समपरिणाहस्यार्धं विष्कम्भार्धहतमेव वृत्तफलम्।
तन्निजमूलेन हतं घनगोलफलं निरवशेषम्॥

(Ā. 2.7, pp. 60-61)

"Half of the circumference *(pariṇāha)* (of a circle) multiplied by half the diameter *(viṣkambha)* is undoubtedly the area of the circle *(vṛttaphala)*; that (area) multiplied by its own (square) root is the volume of a sphere *(ghanagolaphala)* without a residue *(niravaśeṣa)*"[22].

That is,

$$V = \pi\left(\frac{d}{2}\right)^2 \sqrt{\pi\left(\frac{d}{2}\right)^2} \qquad (5)$$

Since $\pi = \frac{62832}{20000}$, according to him.[23]

Hence this formula reduces to :

$$V = 1.7752 \,..... \, \pi\left(\frac{d}{2}\right)^3$$

The rule (5) is also far removed from the exact formula (1).

While commentating on the above rule, Bhāskara I (c. 629 AD) quotes the following empirical *(vyāvahārika)* rule :

व्यासार्द्धघनं भित्वा नवगुणितमयोगुडस्य घनगणितम्।

(Ā, p. 61)

"The cube of half the diameter *(vyāsa)* (when) halved and multiplied by nine yields the volume *(ghanagaṇita)* of a sphere *(gola)*".

This rule finds the formula (3).

Yativṛṣabha (some date between 473 AD and 609 AD) and Brahmagupta (c. 628 AD) do not deal with the sphere.

Śrīdhara (c. 799 AD) sets down a rule in his *Triśatikā*[24] as follows :

गोलव्यासघनार्धं स्वाष्टादशभागसंयुतं गणितम्।

(*Triśatikā* v. 56 ff, p. 39)

"Half the cube of the diameter *(vyāsa)* of a sphere *(gola)*, then added with its eighteenth part, will give the volume *(gaṇita)*".

That is,

$$V = \frac{d^3}{2} + \frac{1}{18}\ \frac{d^3}{2} \qquad (6)$$

or

$$V = \frac{4}{3}\cdot\frac{19}{6}\left(\frac{d}{2}\right)^3$$

Since $\frac{19}{6}$ is an approximation to $\sqrt{10}$ (the wellknown Jaina value for π). This shows that the above formula (6) becomes quite comparable to the exact one (1).

It (6) reappears in the *Mahāsiddhānta*[26] (v. 15.108, p. 179) of Āryabhaṭa II (c. 950 AD).

Vīrasena (c. 816 AD) does not deal with the sphere.

Mahāvīra (c. 850 AD) prescribes two formulae through the following rule in his *Gaṇita-sāra-saṃgraha*.[27]

व्यासार्धघनार्धगुणा नव गोलव्यावहारिकं गणितम्।
तद्दशमांशं नवगुणमशेषसूक्ष्मं फलं भवति।।

(GSS v.8.28.5, p. 259)

"The half of the cube of half the diameter *(vyāsa)* multiplied by nine yields the empirical *(vyāvahārika)* volume of a sphere *(gola)*. Nine-tenth of that will be the subtle *(sūkṣma)* volume".

The first part of this rule finds the formula (3) and the second part finds :

$$V = \frac{9}{10}\cdot\frac{9}{2}\left(\frac{d}{2}\right)^3 \qquad (7)$$

On the ground of the above historical presentation, it comes into sight that India was advancing to give the correct shape to the formula for finding the volume of a sphere, till we reach Bhāskara II (c. 1114-1185 AD).

Therefore it will be better to say the formula (3) as very crude rather than incorrect.

But it (3) reappears even after *Bhāskara II* in the *Gaṇitasārakaumudī*[28] (v. 25, p. 71) of *Ṭhakkara Pherū* (1265-1330 AD) in the following form :

$$V = \frac{3}{4} \cdot \frac{3}{4} d^3 \qquad (8)$$

(B) We have noticed that the formula (3) was quoted by Bhāskara I (c. 629 AD) and given by Mahāvīra (c. 850 AD) before Nemicandra (c. 891 AD). It does not mean that Nemicandra might have got the formula (3) either from Bhāskara I's commentary *(bhāṣya)* on *Āryabhaṭīya* or from the GSS of Mahāvīra, If it were so, he would have also got either the *(niravaseṣa)* formula (5) or the subtle *(sukṣma)* one (7).

(C) According to Gupta[29], it is very likely that Āryabhaṭa I (476 AD) knew the formula (3) and was led to designate his own rule (5) as *niravaśeṣa* (without a residue) to proudly distinguish it from the approximate (*saviśeṣa* or *sthula*) (3).

Whatever we have presented and discussed so far is enough to firmly state that Bhāskara I (c. 629 AD), Mahāvīra (c. 850 AD) and Nemicandra might have got the formula (3) from a Jain source earlier than the fifth century.

After all, Bhāskara I (c. 629 AD), a Hindu mathematician, also quoted the well known Jain rule for finding the circumference C of a circle with diameter equal to d :

$$C = \sqrt{10d^2}$$

while commentating similarly on *Āryabhaṭīya* (v. 2.16, p. 72) and he was from present Mahārāṣtra[30]. Mahāvīra (c. 850 AD) and Nemicandra (c. 981 AD) were the Jain mathematicians themselves and were from South India[31].

(D) The Chinese work, *Chiu Chang Suan Shu* (Nine chapters on the Mathematical Art) (c. 1st Century BC) contains a rule which finds :

$$V = \frac{9}{16} d^3 \qquad (9)$$

for the sphere[32].

The commentator Liu Hui (c.3rd century AD) interprets this in a manner which finds[33]

$$V = \left(\frac{\pi}{4}\right)^2 d^3 \qquad (10)$$

Here we find that the formula (8) is quite comparable to the above (10).

The use of the formula (9) continued in China by mathematicians such as Yang Hui (c. 13th century AD) and Chu Shi-Chih (c. 13th century AD) although Tsu Keng-Chih (c. 5th Century AD) gave a rationale of the correct formula.[34]

Thus we trace the formula (3) in the form of the one (9) in China in the first century before the commencement of the Christian era.

Therefore it is very likely that the formula (3) might have been transmitted from China to India where it might have been held in the Jaina school of mathematics.

(E) Śrīdhara (799 AD) and Mahāvīra (c. 850 AD) were both predecessors of Nemicandra (c. 981 AD). Śrīdhara's formula (6) is more proximate to the exact one (1). Similarly, Mahāvīra formula (7) is better than the one (3). However Nemicandra did not get either of them. It implies that he was not in contact with their works.

It appears that they belonged to the exclusive class within the Jain school of mathematics whereas Nemicandra belonged to the canonical[35].

(F) Yativṛṣabha and Vīrasena were not only predecessors of Nemicandra in India but also were of his class within the Jain school of mathematics. Nemicandra was also in contact with their works, namely, the *Tiloyapannatti* and the *Dhavalā* respectively. But it is to note that these works did not deal with the sphere.

Therefore Nemicandra whose work contains a rule for finding the volume of a sphere appears as the first known mathematician in his class within the Jain school of mathematics.

References and Notes

1. His title is *Siddhāntacakravartī.*
2. As far as known, B.B. Datta is the first historian who introduced Nemicandra (c. 981 AD) as a mathematician to the world of Mathematics through his paper : Mathematics of Nemicandra, *The Jaina Antiquary,* 1.2 (1935), 25-44,
3. A celebrated Commander-in-chief and wise minister of the Gaṃga dynasty kings (c. 953-985 AD).
4. *Trilokasāra* (abbreviated as TLS), Ed. with Mādhavacandra's Sanskrit commentary and with Āryikā Viśuddhamati's Hindi commentary by R.C. Jain Mukhtar and C.P. Patni, Shree Mahavirji (Raj.) VNY 2501 (– 1975 AD).
5. TLS, Āryikā Viśuddhamati's p. 768.
6. The word 'sphere' comes to us from the Greek through the Latin. Ancient Indian word for it is *gola.*
7. Smith, D.E., *History of Mathematics,* Vol. 2, Dover Publications, Inc., New York, 1958, p. 294.
8. *Līlāvati* of Bhāskara II, Ed. with Gaṇeśa-I's and Mahīdhara's commentary by V.G. Apte, *Ānanadāśrama Sanskrit Granthāvalī*, Pune, 1937, verse *(śloka) No. 201.*
9. *TLS,* Viśuddhamati, p. 25.
10. Ibid, p. 26.
11. Refd. 2, p. 37.
12. Gupta, R.C., "On the Volume of a Sphere in Ancient India", *Historia Scientiarum,* 42 (1991) 33-44, See also p. 38 and pp. 40-41.

13. TLS, Viśuddhamati, p. 22.

14. Ibid, pp. 25-26.

15. Ibid. Viśuddhamati, pp. 26-28.

16. Ibid, p. 26.

17. Ref. 12, p. 41.

18. TLS, Viśuddhamati, pp. 27-28.

19. Ref. 12, p. 41.

20. Sarasvathi, T.A., *Geometry in Ancient and Medieval India*, MLBD, New Delhi, 1979, p. 208.

21. *Āryabhaṭīya* (abbreviated as Ā), Ed. with Bhāskara I's and Someśvara's commentary by K.S. Shukla, INSA, New Delhi, 1976.

22. Jha, Parmeshwar, *Āryabhaṭa I and His Contributions to Mathematics*, Bihar Research Society, Patna, 1988, p. 224.

23. Ref. 21, v. 2.10, p. 71.

24. *Triśatikā* of Śrīdhara, Ed. by Sudhakara Dvivedin, Benaras, 1899.

25. Datta, B.B. and A.N. Singh, *Hindu Geometry*, Revised by K.S. Shukla, *IJHS*, 15.2.(1980), 121-188, See also, p. 180.

26. *Mahā-siddhānta*, Ed. with his own commentary by Sudhakara Dvivedin, Chaukhamba Sanskrita Pratishthan, Delhi-Varanasi, 1995.

27. *Gaṇitasāra-saṃgraha* (abbreviated as GSS), Ed. by and Translated into Hindi by L.C. Jain, JSSS, Sholapur, 1963.

28. *Thakkara Pherū Viracita Ratnaparīkṣādi Sapta Grantha Samgraha*, Ed. by Agarchand Nahta and Bhanwarlal Nahta, *Prācya Vidyā Mandira*, Jodhpur, 1961. See also ref. 12, p. 38.

29. See ref. 12.

30. Mule, Gunakara, *Saṃsāra ke Mahāna Gaṇitajña*, Rajkamala Prakashan, New Delhi, 1992, pp. 86-87 and p. 95.

31. Mahāvīra (c. 850 AD) enjoyed the patronage of the *Rāṣṭrakuṭa king* Amoghavarṣa Nṛpatuṃga (814-877 AD) in South India, much of what is known as Karnataka today.

Mahārāṣtra is a neighbour state to Karnataka.

Nemicandra (c. 981 A D) belonged to the *deśiyagaṇa.* This sect flourished mainly in the Karnataka province of India.

32. As quoted by Gupta in ref. 12, p. 33 from :

Mikami, Yashio, *The Development of Mathematics in China and Japan,* Leipzig, 1913, Reprinted, Chelsea, New York, 1961, p. 44 and *Historia Mathematica,* 2 (1984), p. 44 for date.

33. As quoted by Gupta in ref. 12, p. 33 from :

Wagner, D.B., Liu Hui and Tsu Keng-Chih on the Volume of a sphere, *Chinese Science,* (1978), p. 60.

34. Ibid, p. 73; and from Lam Yang's papers in *Archieve Hist. Exact Sciences,* 6 (1969) 85 ; and 21 (1979) 18.

35. The author of this paper divides the Jain School of Mathematics into two classes : the canonical and the exclusive.

For details, consult : Jadhav, Dipak, 'Theory of A.P. and G.P. in Nemicandra's Works' (Unpublished).