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INDUS NUMERALS ON METAL TOOLS

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Nine numerical inscriptions on metal tools from Chanhu-daro, Kalibangan and Mohenjo-daro are shown to constitute a text genre of their own. It is tested whether the expressions relate to the weights of the tools and/or to the contemporary system of weight units. Both tests are negative. The number system appears as octal on internal grounds. Structures of numerical systems are discussed extensively.

Key words: Equation, Harappa culture, Octal number system, 't'-hypothesis, Weight.

1. INTRODUCTION

Numerals in the Indus script has often been regarded as constituting a decimal system; and for numeral inscriptions on utensils it has been suggested that they relate to the weight or volume of the inscribed object (Mackay 1938: 454; Parpola 1986: 403; 1994: 107). A fragment of a ruler strongly supports the hypothesis of an ancient base 10 system for the measurement of length (Mackay 1938: 404; Mainkar 1984: 146). However, when it comes to volume and weight the state of preservation of objects and/or inscriptions precludes any direct conclusion.

The paper will rely on some arguments based on internal evidence of the numerical system (Pettersson 1996:801) and bring in two specimens from Kalibangan which do not seem to have been considered before. The arguments speak against a decimal interpretation of what is most clearly numerals in the Indus script and also against the supposition that inscriptions of metal-ware relate to weight (the relation between volume and numerals on pottery will not be considered because of the lack of reliable material).

Terminology

Bold face **u** and **i** (and **t**) denote possible numerals which look like horse shoes (opening downwards) and strokes (strokes in tilted rows are designated with a **t**). Multiple occurrences of a symbol **u, i** or **t** will be indicated with a digit; e.g. 6**i** means I I I I I I.

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Italicized *u*, *i*, and *t* are used to denote possible numerical values of numerals **u, i** and **t**. These hypothetical values are sometimes system internal i.e., they show how the numerals relate to each other, sometimes external in that they relate to weight.

While standardization of inscriptions ease the work of the decipherer, it is important not to obfuscate any difference in form which could potentially reveal difference in function. The use here of a special transliteration letter, **t** instead of **i**, for strokes arranged in rows running at an angle to other rows of strokes should be seen as an attempt to overcome the weakness of a sequential rendering of symbol forms (Koskenniemi and Parpola, 1982, where an inscription 1**u** 5**i** 4**t** is rendered as **U IIIII IIII**). Reference numbers from lists of standardized texts have rarely been used here (except in a table in sec. 6 for cross-reference); instead references are made to publications where the relevant texts have been depicted in some way.

On the other hand, in seal inscriptions (which are the most common Indus script remains) and in inscriptions on copper and clay tablets (which are quite frequent too) one finds both long and short strokes, but this difference is not captured by the notation adopted here. The reason is simply that this difference does not seem to occur in the kind of numerical expressions the present study is concerned with (for a comment of the variation in the use of long and short strokes, see Parpola 1994: 82).

A numeral system in which different numbers are expressed by grouping of symbols is called *additive*. Historically, this is a very common way of expressing numbers. Since the Indus script displays many graphic units consisting of a number of strokes, it seems reasonable to assume that these units express numbers. In some cases, other symbols than the simple strokes are repeated or strokes appear in different groupings. It is very common in additive systems to express different magnitudes with different symbols; cf. Roman I,V,X,L,C,D,M. This is called *sign-value notation* Thus, it might be the case that the **u, i,** and **t** on Indus metals were part of a sign-value notation.

In some sign-value systems, small numerals denote the number of the bigger units of the system (perhaps weight or volume units) and these systems are then called *multiplicative*. If the purely additive XXX II in the Roman notation instead was written as III X II (= 3 x 10 + 2), this would be an example of a multiplicative notation. (for more on the terminology for numerical notations, see Pettersson 1996.)

2. Characterization of the numerical expressions

The numerical expressions treated here are distinguished not only by being incised on metal tools but also because they are not included in longer inscriptions but forming, as it seems, a separate message from the other inscriptions occurring on these objects. To the author's knowledge, excavators have found only eight copper or bronze tools inscribed with numerals arranged in this way (' numerals'= what one would suppose to be numerals from repetitions characteristic of additive systems or, in the case of K-

122 discussed in section 3, from resemblance to forms occurring repetitively in other inscriptions). One of these contains a problematic numeral and will be treated in this section only, as will also a ninth object with an odd but interesting numerical expression which is used here to justify the identification of a special class of numerical expressions.

The numerical inscriptions of all nine objects are tabulated in section 6.

Two objects from Chanhujo-dara: C-40 and C-39

A broken axe from Chanhujo-daro (C-40 in *CISI* 1, Joshi and Parpola 1987; Mackay 1943: 178), inscribed on both sides which on its A side bears two numerical expressions within a single, horizontal line of text ('horizontal' as judged from the individual signs which include a standing man). One of these is just an ordinary text-6: two rows of three small strokes, the whole compound made to fit neatly within the line of the text. The other expression looks more like the free-standing expressions found on the other tools: (possibly)5**i** on top of a column of 3**u** i.e. a vertical arrangement. Only one or two of the **u**'s fit into the horizontal text line, while the bottom **u** and the top row of **i**'s fall outside this line.(Again, this is not represented in lists of texts; see Mahadevan 1977: 557, text 6306, and Koskenniemi and Parpola 1982: 95, text 5075).

The very fact that this numerical expression, which looks like the others that are considered here, runs vertical in an otherwise horizontal text, motivates the treatment of all these expressions as a special category. However, since this particular numerical expression is in fact included in a text, it might be risky to regard it as the same as the ones considered here. In any event, the axe is broken and Mackay (1943) does not give its weight.

An ingot also from Chanhujo-daro (C-39 in *CISI* 1; Mackay 1943:187) displays a column of XXUUU. If this U is the **u** treated here, then, since the openings of the Us are towards the Xs, the Xs should be numerals 'on the other side' of **u** as compared to **i** (i.e., $x>u>i$ or $x<u<i$). Mackay does not give the weight of this object. Because of this and because it contains possible numerals of a sort, viz. X (actually a double triangle; Parpola's sign no. 234; 1994: 75), not found on the other specimens, it will not be brought into the calculations made here although it is very tempting to take an ingot to be marked as a weight !

Implications

The very fact that a special 'text genre' of numeral expressions exists on metal tools and that an ingot was marked by a series of numerals is really suggestive; it speaks *pro* the hypothesis that numerals were used to indicate weight. However, no numerals have been found on the always very carefully made weight units (see esp. sec. 6 below). In the rest of this paper, numerals on metal tools will be compared with the weights of the tools themselves as well as with systems of weight units from the Indus Civilization; they will also be evaluated system- internally. The evidence collected in this way actually speaks against the hypothesis that numerals were used to indicate weight. The reason for why tool-numerals were inscribed in such a distinct way is, however, not revealed by this investigation.

It could be mentioned that it has been suggested that numerals on copper implements are in fact inventory numbers (Mackay 1943: 178; Parpola 1986: 403). This is of course very hard to check since no slab incised with a list of items has been found. Such a hypothesis could probably best be supported by comparative evidence from other cultures of the same age, while the relation of numerals to weight (or, for vessels, to volume) should be testable by purely culture- internal evidence.

3. Evidence from Kalibangan: K-121 and K-122

There are two copper tools from Kalibangan in *CISI* 1 (Joshi and Parpola 1987) which possibly bear numerical inscriptions: a chisel (K-121) and an axe (K-122). The 'recto' sides bear miscellaneous signs while the 'verso' sides bear almost only numerals. In this they parallel DK-7856 (chisel) and DK-7835 (axe) from Mohenjo-daro which will be discussed in a separate section below. Both K-121 and DK-7856 have their inscriptions on the tang which makes it unlikely that if these inscriptions referred to weight, the intention was to include the weight of the shaft as well (of course, the expensiveness of the metal as compared to the probably wooden shaft makes this unlikely in any case). The same assumption will be made concerning the axe blades.

The number and the art of the numerals on K-121B may be disputed and there are numerals also on the 'recto' side K-121A. Since this complicates the matter, the discussion of it will be postponed until later in the present section. Similarly, K-122B may have contained more signs than those easily discernible on the photographs in *CISI* 1 (p.326). The arguements needed to deal with this possibility are somewhat different from the ones employed for the comparison of K-121 and K-122, and they will be presented in the next section.

The relationship between i and u in K-121B and K-122B

Initially the following possible sign- forms and weights will be recognized for the two Kalibangan objects (the weights have kindly been supplied by B.M. Pande, New Delhi through Asko Parpola, Helsinki):

K-121 weighs 210g and K-121B reads 2**u** or 1**u** (+ some other sign);

K-122 weighs 476g and K-122B reads 5**i**.

Since **u** and **i** are not mixed in the two numerical expressions, it is a very simple task to relate these symbols to the weight of the two objects:

u = either 125g or 210g; and

i = 95.2g (this is of course 476g /5).

Now it will be seen that if i and u belong to the same system, the relationship between them will be rather awkward. One would expect a relationship expressible with a whole integer. We get u / i = 1.1 or = 2.2; some of this might be explained by corrosion and wearing, but a relation like u / i =1 begs the question why different numerals were used at all and u / i =2 begs the question why the larger unit, **u**, was not used on the more heavy object to express its weight as **2u 1i.**

Admittedly, there is one possible explanation to all this, namely that **u** and **i** were used in conjunction with different sets of weight units. On the Mohenjo-Daro objects however, **u** and **i** appear in the same inscriptions—a fact which seemingly precludes that they belonged to different systems of weight, but one should not rush too quickly here; Hemmy (1938) established two (or possible three) parallel decimal series of weight units (extending from a binary system comprising units of circa 30 grammes and less), and these series were related to each other by a factor 2. Thus, two series of weight units could be referred by the two different numeral shapes. But it should be added that there is nothing that shows that the two series of weight units were not used together-rather, it seems probable that half-weights were used to quickly establish the precise weight of an object (compare the use in our own culture of 5-grammes weight units, and even 2-grammes and 3-grammes!; compare also our monetary systems where coins and notes align to a decimal system but not only at every power of ten but we may have, as in Britain, 1,2,5, and 10,20,50, pence and pounds, respectively).

Thus, the absence of the larger unit **u** on the heavier object speak against a relation of the numerals to weight.

Interpreting one of the u of K- 121B as 2i

Because one of the signs on K-121B looks very bad when regarded as a **u**, it should be considered if it is another relevant sign. However, there is no idea to invent a third type of numeral here, since that would not solve the mystery of why the heavier object, K-122, lacks one of the number signs appearing on K-121 (viz., **u** or the third type). On the other hand, interpreting it as a number of strokes, perhaps 2**i**, on which some incisions have been made accidentally, will make it possible to capture the bulk of the weights by a symbol also appearing on K- 122, namely **i**. The calculations would run as (presuming 2**i**):

K-121 weighs 210g and K-121B reads 1**u** 2**i**;

K-122 weighs 476g and K-122B reads 5**i**.

With i = 95.2g this implies 210 = u + 190.4, which yields u = 19.6g.

Here we get an interesting tilt of the relationship between u and i: i now becomes the larger unit. The proportion i/u is 4.9 but the exact relationship is hard to say anything about since the axe blade seems in addition to corrosion to have suffered from some wearing which might suggest an original weight of about 500g and with $5i = 500$g, u will be only 10g and i / u will be 10.

(Below, when solving a similar set of equations for the Mohenjo-darp objects, the result is however $u>i$.)

Other number signs on K-121

The 'recto' side of the chisel from Kalibangan contains 2**i** and above these there might be an **u** (in the beginning or end of the inscription). Following the 2**i** there is a 'fish' sign with and 'index' 2**i** at one of its side incised with two short parallel strokes somewhat slanted as if to fit graphically next to the fish's fin (not so in the Finnish corpus, text 7080, but cp. *CISI* 1). It seems thus as if the normal-sized 2**i** constitute a specification to the fish sign. Furthermore, one should bear in mind that the numerals on the other Kalibangan object are found solely on one side. Nevertheless, this expression will be considered here.

There are some problems when interpreting the numerical symbols on K-121A.

First, it is uncertain if it reads **2i** or **1u 2i**.

Secondly, since the possible **u** is at one end in the inscription (and rather near the end of the object) it might be that the (possible) numerical expression runs over to the other side, i.e. to K- 121B, although there is space enough after the possible **u** for the whole of the B inscription. In case the inscriber intended a continuous text the whole expression is either $2i + 3u$, or $4i + 2u$ if the alternative reading of the B side as 1**u** 2**i** is used. Both of these expressions are unlikely: $2i + 3u$ imply that the ancient Kalibangans would have incised **u** in three different ways even when the **u**'s appeared next to each other, and $4i + 2u$ with alterning **i**'s and **u**'s instead of heaping each symbol (magnitude) is never found in sign-value systems (actually, one could say that the users of such systems anticipated what is done by necessity in positional systems). Furthermore, it should be observed that the numerals ought not to be a part of a multiplicative system since **u** appear in two places as well as **i.**

Moreover, if the A side contains only 2**i** and no **u,** it might be the case that the last symbol is a unit (of which we have two, as indicated by 2**i**), and, again, there is a possibility that the expression continues on the 'verso' side. On that side we may count with another unit (perhaps x times bigger / smaller than the first one), so that we have only one **u** there. (The units are then redundant since different numerals are used to express their respective number. For an abstract system of numerals, this would perhaps be unlikely, but we are now testing the hypothesis that numerals relate to weight and this, it could be argued, make redundant notation less unlikely.)

As a last possibility we get 2**i** + unit sign on A, and 2**u** without unit sign on B (this is not unlikely in principle because there could have been a standard unit associated with numeral **u**).

We then get the following five possible expressions for K-121 (disregarding the B-side numerals when they do not belong to an expression partly found on the A side), of which only the first two or three are (modestly) possible on graphical grounds:

K-121A (+B): $2i$, $2i$ + u or $2i + 2u$; but hardly $2i + 3u$ or $4i + 2u$;

K- 122B: 5i.

Reading only $2i$ yields i = 105g which contradicts the $5i$ on K-122B, But considering the wear and corrosion of K-122, an original weight 5 x 105g = 525g is perhaps not out of the question, but K- 121 too might have been somewhat heavier than its present 210g which implies a higher value of i than 105g.

Reading $2i + 1u$ yields the same set of equations as found in the last subsection, and the result is very much like a $2i$ reading (u = 10-20g).

Finally, for the third possibility, $2i + 2u$ yields $u = 9.8$g.

In all, the hypothesis that the A side of the chisel K-121 contains relevant numerals cannot be refuted by pure calculations. However, as indicated in the beginning of this subsection, there are graphical arguments against it.

In sum

The problem with an interpretation of the numerals on K-121B and K-122B as indicating weight is the fact that the two expressions on the two B-sides contain different numerals such that the bigger one, **u**, is not present on the heavier object (the axe K-122).

Adding **i**'s to the lighter object (the chisel K-121) makes **u** a small numeral as compared to **i** and it is possible to establish a relation between numerals and weight without any embarrassing over-use of a smaller sign to express a heavy weight. There are, however, problems of a graphical sort when one reads **i**'s into possible numerical expressions of K-121.

As for the solution into grammes, two things should be observed: (1). the values are uncertain because of the state of preservation of the objects; and (2). it should also be noted that solving a system of equations consisting of two equations and two unknown (here: **u** and **i**) will always yield a solution (provided, i.e., the equations are neither 'linear dependent' as mathematicians call it nor contradicting each other). This solvability does not prove the solution to reveal any facts about the ancient use of these numerals. At least a third object, and thus a third equation, is needed to 'verify' (support) the mathematical solution of two unknown, as explained in a short paper by the author (Pettersson 1994).

If there were more objects (preferably from each site and each excavation level), then so-called regression analyses could be made to fit numerals 'on an average' to weights of objects and statistical analyses could be performed for estimating the probability that numerals and weights relate to each other by intention rather than by pure chance. As it is, the scarcity of the inscribed implements and their state of preservation make it necessary to use other methods.

4. More signs on K-122B?

The surface of the axe K-122 is partly very corroded. Judging from the photographs in *CISI* 1, p. 326, the corrosion on the A side is deep enough to have possibly eliminated signs inscribed before the corrosion started. The B side, i.e. the one bearing the five strokes here denoted as 5**i**, is perhaps equally badly corroded; could there possibly have been other inscisions than the ones visible? One should not attempt to settle that question without an examination of the original objects, but the following observations introducing a different principle can anyhow be worthwhile to make.

Alignment to a system of weight units

Hemmy's (1938) two parallel, decimal series for weight units consist of units like 137g and 274g. If **u** and **i** related to weight, one would assume that they were direct reflections of the system of weight units. Admittedly, only a few units of this magnitude has been found, while tenths and smaller fractions (which constitute a binary system) are more frequent; there could have been quite another system of units for metal-ware. In any event, the system established by Hemmy is what we have, and it is instructive to try to align the values of u and i to it. (In Marshall, 1931, Hemmy presents further possible units but few actual specimens are assigned to these classes.)

The value $i = 95.2$g (or $\approx$ 100g) inferred from K-122 does not match the system well at all: the nearest weight units are 2 x 27.4g = 54.8g and 137g. Assuming that **i** corresponds to the weight unit of 137g (since K-122 has lost some weight since it was made) is tantamount to ascribe an original weight of 5x 137g = 685g to this axe blade and that it has lost 685 - 476= 209 grammes through wear and tear. This seems unlikely from the photos of an overall well preserved axe in *CISI* 1.

Assuming that K-122B is so corroded that the specimen has lost numerals once inscribed upon it, one should test whether additions of numerals to the expression 5**i** will fit the weight system, i.e., whether such additions will make the expression to sum up to$\approx$500g while the numerals take their values only from Hemmy's series.

Introducing more **i**'s makes it possible to approach units like 27.4g and 54.8g: e.g. 500 / 54.8 = 9.1 (476 / 54.8 = 8.7), that is to say that i = 54.8g would imply nine occurrences of **i** and an original weight at 490g.

The surface of the axe in the vicinity of the 5**i** does not seem to allow for more **i**'s, however (see esp. the photo of these strokes in 200% of actual size in *CISI* 1, p. 326). Looking at the Mohenjo-daro objects where both **i** and **u** show up, one will find that the **u**'s are not aligned to the row (s) of **i**'s. What is more, on DK- 7835, an axe blade, the **u**'s actually forms a *column* at some distance from the row of simple strokes. (on the other object, DK-7856, the tang of a chisel, there is only one **u**.)

It could thus be hypothesized - very tentatively- that there was once a column of **u**'s on K-122B. Letting $u < i$ will imply that the five to nine **i**'s carry most of the weight and

u only a portion which is impossible to determine nowadays because of the wear and tear of the object (at least if its original shape cannot be very accurately determined). Letting $u > i$, u should be fitted to the weight units closest to the weight of K-122 to minimize the number of reconstructed numerals; thus u = 137 or 274 grammes. Of course, if u corresponds to the 137-grammes unit, then there is reason to suppose that it occurred thrice on K-122B since 3x 137 = 411 < 500 (476). Calculating along these lines we have n**i** (where $n >= 5$ because there are 5**i** on K- 122B nowadays, so there were probably at least 5**i** on K-122B originally), to take care of the remaining weight:

500 (476) - 411 or 274 = 89(65) or 226 (202) grammes.

There is a weight unit of 13.7g and taken five times it would represent a weight of 68.5 grammes. This seems to fit in nicely, since this makes i a tenth of u = 137g. For the larger u value, i.e. u =274g, i =27.4g would imply eight **i**'s, since 8 x 27.4 = 219.2, while a twice as big i corresponding to a not infrequent weight unit of 54.8g would imply four **i**'s which does not conform with the five which are actually there.

The above calculations seem to suggest that **u** and **i** relate to weight and furthermore that u / i =10. However, it must be remembered that these calculations were made with several extra numerals tentatively added to K-122B and that the fit to Hemmy's series of weight units was facilitated by the lack of a precise value for K- 122's original weight. Nevertheless, this section has demonstrated how one can go about to delimit the possible range of enigmatic numerals when one aims at interpreting such numerals in 'real' terms (here in grammes). Another kind of delimitation would aim merely at the numerical relations between the numerals ; this would still give idea of the *structure* of the numeral system, as will be demonstrated in section 6 below.

5. Evidence From Mohenjo-daro:DK-7856 and DK-7835

Yet another set of one chisel and one axe will now be considered. DK-7856 and DK-7835 both come from Mohenjo -daro together with some other inscribed bronzes which will be introduced in the text section. DK-7856 is actually only the tang of a broken chisel. The axe blade is fairly well preserved according to the excavator but he also says about this axe and some other specimen that "they were found corroded together into a mass and could only be separated in a chemical bath, which, naturally has reduced their weight considerably"(Mackay 1938: 454). If Mackay had weighed the whole mass before the bath it would have been possible to estimate each specimen's contribution to the total weight from their reduced individual weights (perhaps including each object's surface area in the calculations). Unfortunately, Mackay does not seem to have taken this step before the mass was let into the chemical bath (p.c. Michael Jansen, June 25, 1997).

In section 3, tentative values of u and i were given by solving a system of equations with two unknown. The same procedure will now be used to establish the values of u

and i in relation to the weights of the Mohenjo-daro chisel and axe. Furthermore, one could try to match the numerals against the weight units established by Hemmy, as was done in section 4 for Kalibangan inscriptions where both existing and tentatively added numerals were tested. Here, we will not add any numerals.

Of course, since DK- 7856 is merely the tang of a broken chisel, the weight of the whole chisel can only be estimated rather roughly. But it is also important to make calculations based on the actual specimen, the tang that is, because when it was broken it could very well have been marked as to weigh for the purpose of remoulding.

The relation between i and u in DK-7856 (tang) and DK-7835

DK-7856 weights 165.343g and the numerals reads **u** 5**i** 4**t;**

DK- 7835 weights 1910.030g and the numerals read 7**u** 6**i**.

The following system of equations can be set up.

$u + 5i + 4t = 165\text{g}$

$7u + 6i = 1910\text{g}$

This gives negative value of i: when neglecting t (since t might be fairly small as a fraction of i), this system yields $i = -26\text{g}$, $u = 295\text{g}$.

The tilted row of four strokes, which has here been designated by 4**t**, should perhaps be interpreted simply as 4**i**; then the numerical expression DK-7856 reads **u** 9**i.** With $t = i$, one gets $i = -13.25\text{g}$, $u = 284\text{g}$.

A negative value of i simple means that one preferred to indicate first the nearest higher unit and then how much should be subtracted from it by the lesser units; cp. Roman IX for 9 (10-1). On the other hand, it seems strange to give, for the tang, its weight by taking its double weight and then subtracting by its weight, which is what the first equation says! If the original chisel's weight was u, this seems more reasonable, but could a similar argument be made for the axe?

Instead, one may attempt an alignment to Hemmy's system of weight units: the two inscriptions, **u** 5**i** 4**t** for the lighter object and 7**u** 6**i** for the heavier, suggest that the numeral **u** stands for a heavier weight unit than does **i** and **t** regardless of whether **i** and **t** should be regarded as equipollent. There seems then to be two weight units of which **u** could possibly be a mark for, viz. the units at circa 137g and 274g.

The tang weighs 165b and contains one **u**, so, with positive i, **u** may stand for the weight unit at 137g, which leaves 28g to be accounted for by 5**i** 4**t.** However, for the other object, 7**u** accounts for only half the actual weight since 7 x 137g = 959g, which leaves 6**i** to represent too much (not even by assuming $u = i$ much is gained, since 13 x 137g = 1781g which is less than what the actual specimen weighs nowadays).

Alternatively, one may aim at the next larger unit, i.e. 274g. This will imply that the smaller unit must be negative. This is by no means impossible: the men holding the balance simple had to put the **i**-units in the same pan as the tang, while the 274g-unit was in the other pan of the balance. For the two possibilities $i \neq t$ and $i = t$ one could set i = 27.4g and i = 13.7g, respectively. However, the predicted values will be less than the actual weights of the corroded and weared objects:

Tang 274g - 5 x 27.4g = 137g or (t = i) 274 - 9 x 13.7g = 150.7g

Axe 7 x 274g - 6 x 27.4g = 1754g or 7 x 274g - 6 x 13.7g = 1836g

In conclusion, then the numeral inscription of DK-7856 does not seem to be a numerical expression for the weight of the tang. Almost certainly it is not a numerical expression for the weight of the tang as measured by the standard Indus weight units as established by Hemmy. The rest of section 5 will deal with computations based on a reconstruction of the original, unbroken chisel.

Estimating the original proportion between chisel and tang of DK-7856

One has to establish the shape of the object before any estimation is possible of how much is missing. According to Mackay (1938: 473) there are five types of chisels which he denotes by letters, (a)-(e). From the photograph and the drawings of the tang in Mackay's report it seems likely that the tang belonged to class (b): "rectangular or square in section, with flattened tang". Mackay does indeed discuss it under this subheading (pp. 473-474). The present author further makes the observation that what one could call the 'bridge' between the flattened tang and the (now lost) square part is quite short as compared to many type-(b) chisels. Looking for 'short-bridged' (b)-chisels to find a proportion p between tang and the whole length, no 5 in Mackay's Pl. CXIV and some specimen on Marshall's Pl. CXXXV (1931, vol. III) indicate that $p <$ 2.6 (Mackay's own guess of the original length- unsupported by arguments- and his tabulation of actual length (p.489) yields a proportion of 2.87. One object, no. 6 found in Mackay's Pl.CXVII, has the proportion 2.83 but the length of its tang is less then half of DK-7856.)

The thickness throughout any of the chisels seems to be rather uniform, except that the tang may be somewhat flattened; at the same time, the tang is broader than the rest of the chisel. Therefore, the original weight of DK-7856 was probably somewhat less than 2.6 times its actual weight; it could have been less than twice its present weight.

The relation between i and u in DK-7856 (chisel) and DK-7835

The following system of equations (or, rather, system of one equation and one inequality) can now be set up.

$u + 5i + 4t < p \times 165\text{g}, p = 2.6$

$7u + 6i = 1910\text{g}$

Solving this, one will eventually get $37.69 > i + 4 / 29\ t$.

Thus, i<38g if t is not negative (in fact, even if it is, the upper bound for i would probably be somewhere in the vicinity of 38 since t is scaled down by 4 / 29).

$u >$ 240g (provided that t is non-negative).

(With $p = 2.8$ one gets $i < 45.66$g and $u > 233.68$g.)

With $t = i$ one gets $i < 33.13$g and $u > 244.46$g.

The result of these computations obviously conflict with the result obtained for the Kalibangan numerals.

One could also in this case ask for a match with Hemmy's system of weight units. u above 240g suggests $u = 274$g and then the 'nearest' unit for i will be the one at 27.4g. Substituting these two values for u and i in the above system one gets:

274g + 5 x 27.4g + 4t < 2.6 x 165g <=> 411g + 4t < 429g

7 x 274g + 6x 27.4g = 1910g <=> 2082.4g ≈ 1910g (!)

That is, the axe in its present state weighs some 170g too little. If this is explainable by the chemical bath and wear is hard to judge from the report.

Finally, looking at the extreme values for u and i as calculated above, we get some hints of possible relations between u and i: $u / i > 240 / 37.7 > 6.3$, which one could have guessed since the numerical expression on DK-7835 reads 7**u** 6**i** which makes it unlikely that **u** could replace 6**i**, 5**i**, or less. But it is far from clear that the relation u / i equalled 10, as is tacitly assumed when **u** and **i** are regarded as weight indicators and aligned to Hemmy's system of weight units. Notably though, one gets no contradiction from the assumption that $t = i$, because then $u / i > 7.37$ which is a sound value since there are then 9i in one and the same expression, which suggests that $u / i > 9$.

However, one may ask whether it is sound to interpret a tilted row of **i**'s as just another row of **i**'s. To answer this, it is necessary to take a look at text- internal evidences to find out the nature to the Indus system of numerals (or of what appears to be a system). So far, six numerical inscriptions have been scrutinized. Three more will be treated in the next section—they differ from the four last ones in not being inscribed on the other side although one of them has a few apparently non-numerical signs on the 'numerical' side.

6. More evidence from Mohenjo-daro

Of the nine objects considered in the present study, two comes from Chanhujo-daro, two from Kalibangan, and five from Mohenjo-daro; four of the latter ones were found together "at the low level-24.4 ft." (Mackay 1938: 454; vol. II, Pl. CXXVI # 2, 3, and 5, Pl. CXXVII # 1, Pl.. CXXXI) and one, a heavily corroded copper knife, was found "18.4 ft. below datum" (p. 454, n. 4, and p. 463; vol.II, Pl. CXXXIII # 1). Listing the possibly numerical inscriptions of these objects will reveal an interesting structuring principle for Indus numerals. The following list starts with the two Chanhujo-daro objects briefly discussed in section 2; then it continues with the copper knife mentioned last followed by the rest of the Mohenjo-daro objects, and finally the list ends with the two Kalibangan objects. (The third Column to Host Text Corpus No.s. Finnish Corpus)

Object	Expression	Text no	Comment
C-39	2**x** 3**u** (?)	5083	Ingot, only example of X as numeral (?)
C-40	3**u** 5(?) **i**	5075	Broken axe (expr. crosses other inscr.)
DK-7800	4**i** 4**t**		Knife; very bad cond., shape uncertain
DK-7857	7**i** 4**t**		Spearhead (?), bad cond., very thin
DK-7855	6**i** (ı I I I I I)		Axe, weight 262g, other signs as well
DK- 7856	1**u** 5**i** 4**t**	2796	Tang (165g) of broken chisel (<430g)
DK-7835	7**u** 6**i**	2798	Axe (1910g)
K- 121	1**u** (or ???)	7080	Chisel (210g)
K-122	5**i**	7081	Axe (476g)

The four last objects are inscribed on the other side too, the axe and the tang from Mohenjo-daro with the same sequence of signs, a sequence also found on a seal and in an impression of another seal (Parpola 1994: 108). The tang furthermore bears two signs on the 'numerical' side. The other axe from Mohenjo-daro, DK- 7855, has three signs on the 'numerical' side, one close to the six strokes (it is the common terminal sign, see Parpola 1994:65, 82ff) which makes it difficult to extract this numerical expression from the rest of the inscription; the leftmost stroke is furthermore slightly lower than the other five. In any event, an equation 6*i* = 262g conflicts with the (competing) solutions obtained for the last four items in the list.

For the internal evidences as regards the structure of the (possible) numerical system, it is noteworthy that the expression 7**i** 4**t** on DK-7857 excludes the possibility that the two equalities $t = i$ and $u / i = 10$ both hold true. Mackay says "Perhaps a duo - decimal system" when commenting on the "no less than eleven digits." (1938: 454), but to introduce a 12- system here is not necessary. It is better to observe that whenever there are tilted rows of strokes these tilted rows contain four 'digits'. There are two interpretations of this (except for the one where **t** is regarded as the same as **i**, which leaves unexplained the fact why four strokes should be written in a tilted row):

One being that 'four strokes' had become a mark for something which the other numerals (i.e. **i** and **u**) counted, simply because the numeral **i** often occurred in such specifications. The 4**t** is then simply an icon standing for "here comes numerals for ....." but it is not a numerical expression itself. This interpretation can explain why there are 'always' (at least in the only three instances treated here) four **t** and neither more nor less (even the case of no **t**'s at all is explained by the icon-interpretation: the sign 4**t** was after all quite redundant). One can note that in Parpola's sign list (see e.g. 1994), there is a sign with a man 'holding' (being connected to) four strokes, but otherwise the four-strokes group does not seem to have any special status as compared to other groups of strokes. There is thus not much support for or against a non- numerical interpretation 4**t** in the rest of the Indus text corpus.

The other interpretation is the one used throughout the present study, namely that **t** stands for a number and that it has a specific, numeric relation to **i** and **u**. Now it may seem unexplainable why the **t**'s should always occur four together. However, there is one system in which this would be quite natural, namely in an octal system. If $i/t = 8$(regardless of what u/i equals), then 4**t** means 'half an **i**'. If there is one magnitude of some privilege status, the halves of the magnitude should be rather privileged too. The most privileged magnitude would probably be indicated with the simplest sign, the stroke (cp. our numeral 1, Roman I, etc.; Pettersson 1996), because this would be the most common sign.

How does the assertion $i/t = 8$ fit the hypothesis that the numerals indicate weight? Surprisingly well, since as already mentioned above the double (or triple) decimal series of weight units is an extension of a binary system. The units 13.7g and 27.4g (and 54.8g) constitute the start of the decimal series but also the end of the binary series. These weight units were the ones fitting i in most of the cases accounted for in the above calculations. Sadly, it would be impossible to test this by weight equations of the kind used above, since t as a fraction of i would fall within the uncertainties plaguing these calculations due to the state of preservation of the objects bearing inscriptions.

Still, it must be remembered that the solutions of equations presented above did not give values of u and i close to the weight units of Hemmy's system and it is time to question whether the numeral system really is of the same structure as the one proposed by Hemmy for weight units. Looking at the nine numerical expressions again, a quite striking feature is that neither the number of **i**'s nor the number of **u**'s exceed seven. This suggests that the whole numeral system was octal (probably such that, with * denoting a possible higher magnitude than u, $* > u > i > t$, i.e. $* / u = u / i = i / t = 8$). This is indeed supported by other inscriptions from the Indus culture: the frequency list given by Mahadevan (1977: 718) shows that sign consisting solely of strokes commonly contain 1-7 strokes, very rarely 8-10 (although a special 12-sign, written as 3 x 4**i**, is quite frequent). The fact that groups of more than seven strokes, appear sometimes indicates merely that for some purposes, numbers greater than seven could be indicated by strokes. But the normal employment of stroke-numerals were within an octal system.

The same seems to hold for more curviformed numerals. Fairservis (1992) observes that semicircles, which he calls *crescents* because he lumps together at least three sign forms, viz.), 'U', and **u**, never exceed the number of 7, which suggests "that eight was again the critical number" (1992:69). He also suggests that different orders between crescents and strokes indicate different uses—additive and multiplicative. It seems, though, as if there are no clear inscriptions like **iiiciiii** (**c** for crescent; cp. text 3274, Finnish Corpus) and only a few IIIAIIII where A is another sign (e.g. text 1471). It could be worth mentioning that he furthermore claims to have identified some pictographic signs as standing for 8-12 and suggests that these were used to denote months, but it should be added that these are very tentative identifications.

The numerals and number words used by a people rarely fit well—a point made by Menninger (1969:53)—and consequently there are in the present connection reasons to avoid comparisons with languages possibly spoken by the ancient users of the Indus script. Fairservis (op. cit.) does try to decipher the entire Indus script by reference to sign forms and homonyms in Dravidain proto-languages. While his self- proclaimed ability to see what the stylized pictograms depict and his quickness to present homonyms from different languages may repel many a reader, it should be noted that his observation that eight was "a critical number" is based on a quite uncontroversial sign count.

The method of counting individual sign forms can be combined with a count of co-occurrences between different signs, as has been done by Kak (1988: 142, 1989: 116, 1990) who claims that **u** never appears together with more than four strokes which implies a quinary system. However, as is evident from the list presented above, 5**i** and 6**i** actually do pair with **u** in some inscriptions.

It might be foolish to mix different kinds of texts—especially since the possibly numerical inscriptions on metal tools stand apart from other inscriptions on the very same objects—and in the present study the numerical inscriptions on metal tools have constituted a little corpus of their own, and only after analyzing this have references been made to a larger corpus of Indus inscriptions. The latter evidence could primarily be used only supportive, because if for instance eight and nine strokes were common signs in ordinary Indus texts, that would not necessarily speak against an octal interpretation of tool numerals; two systems could have been in use simultaneously. In the present study, moreover, drawings and photographs of inscription have been inspected instead of mere standardized corpus lists. this have made it possible to launch the '**t**' - hypothesis, i.e., that strokes incised in different ways may be tokens of different types, perhaps of different numerals.

Octal weight measures?

Considering the fact that the numerals seem to form a consistent octal system, could the weight units be re-evaluated so that the decimal part of the system starts somewhat later than at 13.7 grammes? if the weight-unit value for *i* obtained from the Mohenjo -daro chisel (as reconstructed from the tang) and axe is used to guess the *u*-value under the assumption the $u/i = 8$, then $u = 8 \; 27.4g = 219.2g$. This does not match the existent weight units well and what is more, the inscription on the axe must now be taken to indicate only 1700g which is less than its present-day weight. This again suggests that numerals did not relate to weight, at least not to the weight units as interpreted by Hemmy.

There is one 'text-internal' thing that speaks for a relation to weight. If there is a higher magnitude than u, let's again denote it by *, and $*/u=8$, then he DK-7835 axe inscription could be interpreted as that* units of metal were used to make DK-7835, but in the process, $2i$ was lost, which was noted on the axe after a control weighing ($1* - 2i = 7u + 6i$)! Such a use of standard weights for axe-making is not impossible even if the standard does not conform to the ordinary series of weight units. Tentatively then, one would have $* \approx 2$Kg, and indeed K-122 weighs a fourth of that which could be taken as supporting evidence for a * standard. However, the third axe weighs more than an eighth, so this case seems closed even without a look on other axes than the more three used here.

Parenthetically in could be mentioned that binary relations between lengths are found in the Indus civilization. Bricks, while sometimes of different size at different sites and even within one site, all have internal proportions 4:2:1 between length : width: height in the mature Indus civilization while in the formative phase bricks show the proportions 3:2:1 and bricks from other cultures may have no simple, whole-number relations between their sides (see Thapar 1985,pp. 53-83 for pre and mature Indus civilization and p. 91 for the more or less cotemporaneous Ahar culture). This adherence to binary relations between lengths is a bit surprising in the light of the (possibly) decimal ruler mentioned in the Introduction but by no means incompatible with it.

Hemmy's system of weight units

Mainkar (1984)—drawing on finds from Lothal (Rao 1973; not read by the present author)—questions the existence of a binary subsystem of weight units and suggests instead two different (triple) decimal series. If the suggestion turns out to be the best description of Indus weight units, then it is even less likely than indicated above that the numerals relate to weight or at least to weight units. But it is noteworthy that Mainkar has to leave out one weight class at 6.829g identified by Hemmy. This might be seen as no big issue since some odd examples puzzle even Hemmy. However, the unit left out by Mainkar were represented by 45 specimens clustered around 6.829g. These are too many not to be included in the systems proposed by Mainkar.

Hemmy used statistical methods to test the overall system: he ascribed each specific specimen to a unit and then normalized all the units to one unit to be able to detect deviation from the proposed binary+double-decimal system. He got a nice Gaussian curve for the whole set (which included only perfectly or near perfectly preserved specimen). Actually, he managed to detect a little peak—a small Gaussian curve in itself—with its mean value near the overall mean value; this indicates that at some time, a slightly false unit was made in Mohenjo-daro or Harappa and then some other tokens at both sites were made with this one as the norm.(For an often considered magnitude in the present study, this 'false' standard is ≈ 13.95g, which makes the corresponding original or dominant standard to 13.6g instead of the 13.7g used above.) For Chanhujo-daro, Hemmy reports that: "There is a single maximum, so only one standard is involved" (in Mackay 1943: 236).

Interestingly, the (very few) exceptional and abberrant weight units Hemmy discusses (in Marshall 1931, and Hemmy 1938) are not in the vicinity of the Mohenjo-daro *u* over 240g. (From Chanhujo-daro there is one unfinished piece of circa 390 g; Mackay 1943.) When fifteen odd specimens from Harappa, Mohenjo-daro, and Chanhujo-daro were considered together, the binary system seemed paralleled by a series of 1/3-fractions. "The limited number of fractional weights leads one to suppose that they had only a special use, and that like our Troy weight, they were used in weighing gold, silver, and precious stones." (A.R. Hall in Mackay 1943: 244.) Also Mainkar finds his proposed two decimal series to be related to each other by a proportion of 1/3 (though he does not refer to Mackay's 1943- volume).

Hemmy's approach is very sound, and he comments on other authors who have compared single tokens or types with single units in other cultures, the Egyptian and the Mesopotamian (1938b; 674ff). There is always a unit which is 'quite near' another unit. This is not to deny that systems of weight units in one place might be designed to fit foreign units, but only to say that individual types and tokens cannot be used to prove such connections. Much the same problem plagues the attempts to align **u** and **i** to weight of objects or of weight units: individual inscriptions might be fitted to 'possible', but highly insecure, weight of a worn and corroded object. Likewise, individual numerals may resemble numerals in other cultures: Mackay refers to the U-shaped 10 of the Egyptian numeral system (1938:454). But just as weight units normally form systems, so may numerals do, and this fact is the basis on which the present study rests.

7. Summary

The organization of the present paper was primarily according to objects. A summary arranged according to type of argument is in place.

Series of simple strokes and 'horse shoes' on Indus metal ware suggest that an additive, sign-value notation for indicating numbers was used. The present paper established the numerical expressions on metal tools as a specific text genre and indented nine such inscriptions. The questions addressed concerned what numerical values the possible numerals were of and whether they related to the weight of the inscribed objects.

Straightforward mathematical solutions are impossible because inscriptions and objects are in a bad condition. Reference to consistency in form and arrangement of numerals were made a few times to exclude some readings. Still, uncertainty of which sign forms have been incised and also of which the original weights might have been has made it necessary to compute several solutions. These solutions seem to suggest that numerals did not relate to weight:

For two Kalibangan objects, solving a system of equations of the form "expr. = weight" led to an awkward solution where the larger unit was not present on the heavier object. Furthermore, the solutions obtained for these objects did not conform to solutions obtained for two objects from Mohenjo-daro.

Likewise, attempts to match numerals with Indus weight units were not very successful; again an indication that numerals did not relate to weight.

But most conclusive is the circumstance that the structure of the system of weight units does not appear to match the structure of the system of numerals. While the system of weight units apparently is binary for small weight, it consists of a doubling/halving decimal system for the rest, and this decimal base is incompatible with the octal system which the numerals apparently constitute. Hemmy (i.a. 1938) has discussed the weight units. The present study gathered the following arguments that the numerals form an octal system:

There are two basic numeric symbols on metal tools, 'horse shoes' and strokes, here denoted by **u** and **i** respectively.

The number of occurrences in a single numerical expression of the lesser numerical unit should, as a rule, give a lower limit for the higher unit. Up to eleven strokes have been found in one single inscription. On three objects (including the one with eleven strokes), some strokes form a tilted row. Such strokes have here been denoted by a **t.** Their odd appearance indicates that **t** # **i** and speaks against the duodecimal interpretation otherwise suggested by the inscription with eleven strokes.

In fact, there are no more than seven **i** in any inscription, which suggests a relation by eight between the two magnitudes denoted by **u** and **i,** and there are also never more than seven **u** which again supports the octal interpretation. Similar observations have been made by Fairservis (1992) with respect to the whole of the Indus texts corpus. Here, the corpus was purposefully delimited not to conflate texts that might have been written by different standards.

The number of **t**-strokes are four in all three cases where tilted rows appear. This fact was here taken to indicate that **i** represented eight **t**, because the simple stroke **i** probably represented a frequently employed magnitude, and in any system, half of such a magnitude could be a rather frequent magnitude too, whence the appearance of 4**t** in some inscriptions. but it was also noted that the 4**t** might be a non-numeral sign. In either case though, **i** and **t** should not be conflated.

By internal evidence, then number of times a numeral can occur in sequence appears to be bound by eight. That is, if the different numerals together formed a system, as supposed here, that system was octal. This is an interesting fact in itself, but unfortunately, it demolishes the hypothesis that the numerals represent weight units, at least the common units described by Hemmy (1938). The mathematical treatment of the numerical expressions when these were inserted into weight equations furthermore speaks against any relation to weight, although no conclusive results can be reached with only a few objects per site, since any marks of weight or any weight units for metal could have been off local and fortuitous design.

To definitely refute or revive the hypothesis that numerals relate o weight, more objects to test the hypothesis on are needed. Another way to solve the puzzle would be that weight units inscribed with numerals were found. As it is, it is instead tempting to take the absence of numerals on weight units as indicative: numerals were not used together with any system of weight units.

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