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SUPPLEMENT

THE CYLINDRICAL SUNDIAL IN INDIA

(Sanskrit, English translation and exposition in terms of modern Mathematics)

by

YUKIO ÔHASHI

INDIAN NATIONAL SCIENCE ACADEMY

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THE CYLINDRICAL SUNDIAL IN INDIA

YUKIO ÔHASHI*

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The first Sanskrit work which mentions the cylindrical sundial is the *Yantra-prakāśa* (AD 1428) of Rāmacandra, but its description is very brief. After that, Hema (later half of the 15th century) wrote the *Kaśā-yantra*, which is a detailed Sanskrit monograph on the cylindrical sundial. Then, Gaṇeśa Daivajña (16th century) wrote the *Pratoda-yantra*, which is a concise Sanskrit monograph on the cylindrical sundial. Gaṇeśa's work was summarized by Munīśvara in his *Siddhānta-sārvabhauma* (AD 1646). In this paper, the full text of Hema's *Kaśā-yantra* and that of Gaṇeśa's *Pratoda-yantra* are presented for the first time with my English translation. It is not clear whether Indian cylindrical sundial is connected with Islamic sundial or not.

Key words : *Cābuka-yantra*, Cylindrical sundial, Gaṇeśa Daivajña, Hema, *Kaśā-yantra*, Munīśvara, *Pratoda-yantra*, Rāmacandra, Whip instrument, *Yantra-prakāśa*.

INTRODUCTION

India has a long tradition of observatinal astronomy since the Vedic perod,[1] and several astronomical instruments were described in the Classical Siddhāntas.[2] During the Delhi Sultanate and Mughal periods, several monographs specialized in astronomical instruments were composed in Sanskrit. The first of this kind is the *Yantra-rāja* (AD 1370) of Mahendra Sūri, the first Sanskrit work on the astrolabe and perhaps the first Sanskrit work which is apparently influenced by Islamic astronomy. Slightly later, Padmanābha wrote the *Yantra-rāja-adhikāra* (chapter I of his *Yantra-kiraṇāvalī*) (AD 1423), the *Dhruva-bhramaṇa-yantra-adhikāra* (chapter II of his *Yantra-ratnāvalī*), the *Diksādhana-yantra*, etc. His *Yantra-rāja-adhikāra* is probably the second Sanskrit work on the astrolabe. It may be noted here that Padmanābha's son Dāmodara was also an astronomer, who wrote two Karaṇa works, namely the *Bhaṭa-tulya* and the *Sūrya-tulya*, both in AD 1417. In AD 1428, Rāmacandra wrote the *Yantra-prakāśa*, where several astronomical instruments were described, among which the most detailed description is that of the astrolabe. As regards the early history of the astrolabe in India, I have discussed it in my previous paper.[3]

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* 3-5-26, Hiroo, Shibuya-ku, Tokyo 150-0012, Japan

Besides the astrolabe, several interesting astronomical instruments were made in the Delhi Sultanate and Mughal periods, one of which is the cylindrical sundial. Let us discuss the history of the cylindrical sundial, which was called *kaśā-yantra* or *pratoda-yantra* or *cābuka-yantra* in Sanskrit.

It is well known that Gaṇeśa Daivajña (born AD 1507) wrote the *Pratoda-yantra*,[4] which is a work on the cylindrical sundial with a horizontal gnomon. The Sanskrit word *"pratoda"* means whip. Later, Munīśvara (born AD 1603) summarized the description of the *Pratoda-yantra* of Gaṇeśa, and included it in his *Siddhānta-sārvabhauma* (AD 1646). The extract of Munīśvara's version was sometimes copied independently as the *Pratoda-yantra* or *Cābuka-yantra*, and this Munīśvara's version was published by S.D. Sharma.[5] The word *"cābuka"* is a loan word from Persian, and means whip. Gaṇeśa's original version of the *Pratoda-yantra* has hitherto been unpublished, and its full text with my English translation will be presented in this paper for the first time.

As far as I know, the existence of the "whip instrument" before Gaṇeśa had practically been unnoticed by historians of astronoiny, before I reported the probable date of the *Kaśā-yantra* of Hema as the later half of the 15th century.[6] The Sanskrit word *"kaśā"* (= *kaṣā*) also means whip. Actually, Rāmacandra described the *kaśā-yantra* still earlier in his *Yantra-prakāśa* (AD 1428), but his description is too brief, and it is difficult to investigate its construction. Therefore, Hema's *Kaśā-yantra* can be said to be the first detailed monograph of the cylindrical sundial in Sanskrit. Hema's *Kaśā-yantra* has also hitherto been unpublished, and its full text with my English translation will be presented in this paper for the first time.

Rāmacandra's Kaśā-Yantra

In AD 1428, Rāmacandra, son of Sūryadāsa and Viśālakṣī, wrote the *Yantra-prakāśa* at Naimiṣāraṇya (near Sitapur in present Uttar Pradesh).[7] It has six chapters, and the description of the *kaśā-yantra* is included in its chapter VI. His description of the *kaśā-yantra* consists of three quarters of *śārdūlavikrīḍita* metre, and a short prose commentary. Let us see its metrical text. I have used the following two manuscripts.

AS Calcutta G-1363 (hereafter Ms. "A"),

BORI 975 of 1886/92 (hearafter Ms. "B").

इत्थं धातुमयीं कशामृजुभिदाभिन्नेषु रन्ध्रेष्वृजु (?) ।
प्रोताच्छङ्कुत आदधीत घटिकाङ्कानावलीभिः पृथक् ॥
शङ्कुं च भ्रमिभाजिमूलसुषिरे प्रोतं सदा स्थापयेत् ।

Apparatus:

(1st quarter:) A: *ṣṭṛchu* (?), and B: *ṣṭalu* for "*ṣvṛju*".
(2nd quarter:) A: *ghaṭīkā* for "*ghaṭikā*".
(3rd quater:) B: *śuṣire* for "*suṣire*".

"Thus metallic "*kaśā*" [should be made]. From [the root ot] the gnomon, which is rightly inserted in the holes on the straight splitted parts (columns), one should mark *ghaṭikās* by segments severally. And also, one should always keep the gnomon for insertion in the hollow at the base of the round one (main body of the instrument ?)".

This text is too brief to investigate its construction. Let us proceed to the examination of more detailed text written by Hema.

The Kaśā-Yantra of Hema

(i) *Introduction*

The Oriental Institute, M.S. University, Baroda, has a manuscript of the *Kaśā-yantra* of Hema.[8] Its description is as follows.

Accession no.: 3258,
Name of the manuscript: *Kaśā-yantram*,
Author: Hemaḥ,
Leaves: 7,
Copied in Vikrama Saṁ. 1784 (AD 1727).

In the last verse of the *Kaśā-yantra*, Hema writes that his father is Viśvanātha, and that his grand father is Gaṅgādhara of Vātsya-kula, a resident of Jambhūvara (Jambūsara ?), who wrote three commentaries on arithmetic (*pāṭī*).

There is no doubt that this Gaṅgādhara is identical with Gaṅgādhara (fl. AD 1420),[9] son of Govardhana and grandson of Divākara of Vatsa-kula, a resident of Jambūsaronagara (present Jambusar in Gujarat, about 22° N). Gaṅgādhara's commeṇtary on the *Līlāvatī* of Bhāskara II is extant.

From the date of Gaṅgādhara, the date of Hema can be placed somewhere in the later half of the 15th century AD.

Hema quoted from two works in his *Kaśā-yantra*, namely the *Yantra-rāja* [-*adhikāra*] (AD 1423) of Padmanābha and the *Sūrya-tulya* (AD 1417) of Dāmodara, son of Padmanmābha. This fact does not contradict the aforesaid date of Hema.

(ii) *The Text of the Kaśā-yantra of Hema*

The following is the complete text of the *Kaśā-yantra* of Hema based on a manuscript (Baroda 3258).

॥ श्रीगणेशाय नमः ॥
नत्वा श्रीगणनायकं दिनमणिं वाणीं गुरुं चादराद्
देवं तं जनकं च सन्मतिवशाच्छ्रीविश्वनाथाभिधम् ।
ज्योतिर्विज्जनहर्षदं द्युसमयज्ञानप्रदं कौतुका-
च्छ्रीमद्यन्त्रवरं कशाभिधमिदं हेमाभिधो [ऽ] हं ब्रुवे ॥ १ ॥

शङ्कुः प्रकल्पो (ल्प्यो) [ऽ] र्कमिताङ्गुलो [ऽ] त्र
तन्मानतो बुद्धिमता च वृद्धिः ।
यष्टिः प्रकल्प्या[ऽऽ]कृति २२ संगुणा वा
स्यात्कल्पना सद्गुरुसंप्रदायात् ॥ २ ॥

यष्टिस्वरूपं खलु यन्त्रमेतत्
पूर्वोक्तमानाद्यदि यष्टिकोना ।
तदा दिनादौ च खमध्यगे ऽर्के
स्थूलात्र सिद्धिः समयस्य वेद्या ॥ ३ ॥

हैमं तारमयं सुपैत्तलमथो शौल्बं सुयन्त्रं तथा
कुर्याच्छै[च्छैं]शिपमप्यथात्र रुचिराः कार्यास्तु सप्तास्रकाः ।
आयामे खलु लम्बवच्च पृथुलत्वे ते यथार्धा (र्धो) न्मिता
अंक्यास्तत्र तु नाडिकाः स्वदिनमित्या स्वोत्क्रमच्छायया ॥ ४ ॥

मध्यास्रके शङ्कुमिताग्रभागे
स्थित्यै नरस्यैव नरप्रमाणम् ।
गर्तं विदध्यान्मतिमान्विधाय
शङ्कु(ङ्कू)र्ध्वमूर्धं दिनमानकार्यम् ॥ ५ ॥

द्यद्गर्तमूले सदृढं समं च
शङ्कुं शलाकाप्रतिमं निवेश्य ।
शङ्क्व (ङ्कूर्ध्व) देशे खलु मध्यकोणे
त्रिंशन्मितं संविलिखेद् द्युमानम् ॥ ६ ॥

ततो विलेख्यानि सदैव दध्यात्
कोणत्रये चोत्तरदिग्विभागे ।
क्रमा[द्] द्युमानान्यथ दक्षिणे च
कोणत्रये रूपविहीनकानि ॥ ७ ॥

सप्ताश्वि २७ मानद्युमितेस्तु कोणे
सार्धत्रिचन्द्रा १३/३० घटिका विलेख्या[:] ।

एवं क्रमात् स्वीयदिनार्धमित्या
शेषास्त्रकेष्वङ्कविदा नृमूलात् ॥ ८ ॥

स्वाक्षभावशतस्तत्र दिनमानविलेखनम् ।
घटिकालेखनं चापि स्वबुद्ध्या चिन्तयेद्बुधः ॥ ९ ॥

नाडीकोष्ठेष्वब्धि ४ भागेन तिर्य-
ग्रेखा वेदा ४ स्वस्वजेन प्रकुर्यात् ।
तन्मध्ये स्युः पञ्चचन्द्रा १५ णि सूक्ष्म-
कालज्ञानायानुभागं पलानि ॥ १० ॥

तद्यन्त्रशीर्षे खलु पट्टसूत्रं
बद्ध्वेह धार्यं सुधिया च लम्बवत् ।
स्याच्छङ्कुभा मध्यमकोणमध्ये
यथा तथा यन्त्रमिदं करेण सत् ॥ ११ ॥*

* This verse is metrically irregular.

एवं चाबुकसंज्ञिके [ऽ] त्र विधृते शङ्कोः स्पृशत्यस्त्रजा-
ङ्के भाग्रं वद यन्मिते तु घटिकास्तत्तुल्यसंख्याभिधः ।
याता प्र(प्रा)ङ्नत एव खार्धत इमे गम्यास्तथा पश्चिमे
छाया चात्र च वर्धते दिनदले त्वाद्ये परे हीयते ॥ १२ ॥

एवं नाडीखाग्नि ३० तल्यद्युमान(नं)
कोणे विध्यन् यन्त्रसंस्था (स्थां)ब्रवीमि ।
इष्टच्छायावासरे त[द्]द्युमानं
ज्ञात्वा कार्यं वक्ष्यमानं (णं) तु सम्यक् ॥ १३ ॥

शङ्को[ः] प्रभाग्रं स्पृशतीह यत्र
तच्चिह्नतो [ऽ] स्रे समसूत्रकं स्वे ।
ज्ञेयं द्युमाने [ऽ] त्र तु यन्मिताङ्कः
स्वतुल्यसंख्यां घटिका (कां) वदेच्च ॥ १४ ॥

इष्टद्युमित्या स्युरधः पलानि
बहूनि चेदत्र तदा गृहीत्वा ।
अर्धाधिकं त्वेकमितं तु तेषां
ततो विचिन्त्यं दिनमानमिष्टम् ॥ १५ ॥

सर्वेषु कोणेषु पृथक् पृथक् वा
स्यादुक्तवच्छङ्कुरिहाप्यथैवम् ।

इष्टा (ष्ट) द्युमाने निजशङ्कुभात:
खाग्नि ३० द्युमित्येव भवन्ति नाड्य: ॥ १६ ॥

छायाविलोक्यसमये ऽत्र नरस्तु तिर्य
ग्रक्ष (क्ष्य) स्तदे (द) न्यसमये खलु तत्प्रमाणे ।
गर्ते निवेश्य इव कोशगतं (त:) द्युर (कर:) स्या-
देवं त्वसौ कनकमुख्यसुयष्टिकासु ॥ १७ ॥

नृपसदसि तवेच्छा चेत्प्रतिष्ठादिकेच्छा
तदिदमिति सुयन्त्रं चाबुकाख्यं पठ त्वम् ।
गणकगणितजाया: पद्धतेर्नास्त्यपेक्षा
शुभमिह दिनवेलासाधनं सौख्यत: स्यात् ॥ १८ ॥

पवनजवतुरङ्गाधिष्ठितो भूमिपाल:
कलयति कुतुकाद्वाप्यात्मना नाडिकाद्यम् ।
करविधृतकषाख्ये धातुजे दारुजे वा
चतुररचितयन्त्रे मालिनीवृत्तवेत्ता ॥ १९ ॥

धर्मायत्तं त्रिभुवनमखिलं सो [ऽ] पि कालप्रधान:
कालोप्युच्चैर्ग्रहगतिवशतो [ऽ] भिन्नधर्माप्रमेय: ।
तस्य ज्ञानं भवति हि वशदं यन्त्रतश्चैव तस्मा-
देतत्तस्य प्रकटनपटुकं कस्य न स्यादुपास्यम् ॥ २० ॥

अथोत्क्रमाभानयनं ब्रुवे [ऽ] हं
प्रविष्टनाडीष्टदिने प्रमाणे ।
श्रीपद्मनाभोक्तसुयन्त्रराजान्
नाड्याङ्कनार्थं सुखतो [ऽ] ब्धि ४ वृत्ते ॥ २१ ॥

ये सायनांशेष्टखगापमांशा:
पलैक्यविश्लेषलवा: प्रसाध्या: ।
समान्यदिक्तेर् (क्त्वे?) नवते ९० र्विशोध्या-
स्तत्सं (च्छिं) जिनी मध्यनरज्यका सा ॥ २२ ॥

अभि (भी)ष्टनाड्यो गुणिता: खनन्दै ९०
र्दिनार्धभक्ताप्तगुणेन निघ्नी ।
मध्योन्नतज्या त्रिभजीवयाप्ता
चापं तदिष्टोन्नतभागका: स्यु: ॥ २३ ॥

इष्टोन्नतज्यार्कगुणा विभक्ता
नतज्यया लब्धमिहोत्क्रमा भा ।
एषा कषादावुपयोगिनी स्यात्
तत्रा तु नाडी (ड्यो) गणकैः प्रसाध्याः ॥ २४ ॥

इष्टोन्नतांशा नवतेर्विशोध्या
अत्रैवमिष्टा न्न (न) तभागका [:] स्युः ।
तत्सं (च्छिं) जिनी चेष्टनतज्यका सा
विलोमभार्थं सुधिया विचिन्त्या ॥ २५ ॥

भक्तात् फलं तस्य धनुः पलांशाः ।
सदैव याम्या [:] खनव ९० च्युतास्ते
लम्बांशका याम्यदिगू (गु) न्नता [:] स्युः ॥ २६ ॥*

* This verse has three *padas* only.

अथोच्यते किंचिदिहोपयोगी
श्रीसूर्यतुल्यात्करणा [ज्] ज्यकाद्यम् ।
दामोदरोक्ताद्रस ६ संख्यवृत्तैः
सिद्धैः सुखार्थं किमहो नवीनैः ॥ २७ ॥

रसाश्विनो २६ जिना २४ नखा २०
नृपा १६ दिशो १० [नदी]* श्वरा [:] ४ ।
ज्यकादलानि षट् क्रमात्
शत (तं) त्रिभज्यका भवेत् ॥ २८ ॥

* Two syllables in 28 b cannot be read in the original manuscript. *"Nadī"* has been supplied according to a manuscript (AS Calcutta IM-5356) of the *Sūrya-tulya* of Dāmodara.

बाणेन्दुभिर्भाग १५ मितेर्यदाप्तं
तत्संख्यखण्डानि गतान्यथैष्यात् ।
शेषांशनिघ्नाच्छरचन्द्र १५ लब्धं
तद्यातखण्डैक्ययुतं ज्यका स्यात् ॥ २९ ॥

शेषांशकाद्गम्यगतान्तरघ्नात्
खराम ३० लब्धोनयुतं क्रमेण ।
यातैष्यखण्डैक्ययुतं दलं स्यात्
क्रमोत्क्रमज्याथ (र्थ) मिहैष्यखण्डम् ॥ ३० ॥

स्फुटेन भोग्येन यथोक्तवच्च
संसाधिता ज्यातिपरिस्फुटात्र ।
विलोमखण्डैश्च विलोमजीवा
साध्या स्फुटार्थं स्फुटखण्डकेन ॥ ३१ ॥

संशोध्य खण्डानि यदत्र शेषं
बाणेन्दुनिघ्नं तदशुद्धभाज्यम् ।
विशुद्धसंख्याशरचन्द्र १५ घात-
युक्तं तु लब्धं भवतीह चापम् ॥ ३२ ॥

ज्यां शोध्य यातैष्यदलान्तरघ्नात् (घ्नं)
शेषार्धमेष्याप्तफलोनयुक्तम् ।
यातैष्यखण्डैक्यदलं क्रमाख्ये
विलोमचापे स्फुटभोग्यखण्डम् ॥ ३३ ॥

उक्त (त्क्र) मच्छायया प्रोक्तं चमत्कारि कषाभिधम् ।
कार्यं क्रमभयाप्येतद्यन्त्र(त्रं) त [ज्] ज्ञैः सुबुद्धिभिः ॥ ३४ ॥

अथ रात्रिज्ञानम् ॥

त्र्याशा १०३ इन्द्रभु ११४ गोऽर्क १२९ मंकगुणभु(भू)स्थां १३९ एवाद्यकं (खार्थकं) [१५०] पट (षट्) तिथि [१५६]
त्र्यष्टैकं [१८३]सशिवं[१९४]त्रियुक् [१९७]तिथियुतं[२१२]द्वित्र्यश्वि [२३२] साङ्गं [२३८] लवाः ।
अङ्काब्ध्यश्वि [२४९] सविश्व [२६२] मध्य (ब्ध्य) गभुजं [२७४] साशं [२८४] द्विखाग्निं [३०२] त्रियुक् [३०५]
तत्त्वत्री [३२५]षुयुगाग्नि[३४५]साङ्ग[३५१]मथ खं [०]रुद्रा[११]जिना[२४]गेशराः[५९]॥ ३४´ ॥*

* Although the previous verse number is 34, the present verse has also been numbered 34 in the manuscript, and the following verses are numbered successively from this verse. Tentatively, I number the present verse 34'.

पञ्चाङ्गं [६५] सदशं [७५] रसाङ्क [९६] मितिके दस्त्रादिके नु क्रमात्
स्पष्टर्क्षं निशि नि (नी) लकण्ठविदुषा [ऽ] वादीय (ह?) हर्षप्रदा (दम्?) ।
रात्रौ षड्भचलांशयुक्तरवितो भोग्यां खमध्यस्थभे
युक्तं (क्तः) सायनलग्नतश्च समये मध्योदयाढ्यो गतः ॥ ३५ ॥

स्पष्टास्त्रं (सप्तास्त्रं?) करणीं (णी) यमेय (त?) दपि च ज्ञातु (तुं) निशानेसहं (?)
नागास्ते (स्त्रे) विलिखेत् खगेन्दुसमयस्यांक्तांश्च (कांश्च) लग्नांशकान् ।
विध्येदम्बरमध्यगोदु (डु) समति (समितं ?) यन्त्रेण तात्कालिका-
ल्लग्नात् साध्य इहार्थतश्च समयः श्रीनीलकण्ठोक्तवत् ॥ ३६ ॥

सप्तास्त्रेनाग्नि (णात्र ?) यन्त्रेण रात्रेर्वा तु खमध्यमात् ।
श्लोकोक्तैरेव लग्नांशै [:] कालः स्यात्पुनरुक्तवत् ॥ ३७ ॥

गर्तोर्ध (र्ध्व) देशे खलु सूक्ष्मरन्ध्रं
कुर्यात् खमध्यस्थितभव्यधार्थम् ।
तदास्य शीर्षेण च यन्त्रकस्य
विध्यो (ध्ये)त् खमध्योडु गू (गु)रूपदेशात् ॥ ३८ ॥

अथ नक्षत्रा[णा]मन्तरपलानि ॥

खाङ्कं [९०] बाणगुणेन्दु [१३५] पञ्चरसभू [१६५] तुल्याः खसूर्या [१२०] द्विधा [१२०]
पञ्चाब्ध्य [४५] भ्रनगाक्षि [२७०] भोगगुणभु (भू) [१३८] त्वङ्काब्धि [४९] बाह्वङ्कभू [१९२] ।
गोधृत्य [१८९] ग्निखगास्तु [९३] खार्क [१२०] मिषुदिक् [१०५] सर्पेन्द्र [१४८] मब्ध्यङ्गकं [६४]
खाष्टैकं [१८०] खरसा [६०] शराग्निभु [१३५] शराङ्गैकं [१६५] विनाड्यः क्रमम् [७१] ॥ ३९ ॥

खं [०] बाणाङ्कभुवश्च १९५ बाणरहितः [१९०] पूर्णाब्धिरामा [३४०] स्तथा
नागाक्षा [५८] गजदिक् [१०८] च पर्वतनखा [२०७] आभिश्च दस्रादिकम् ।
पौष्णाद्यात् क्रमशो भमेति नभसो मध्यं खमध्यस्थितात्
प(ष)ष्ट्या वी (ई) षुयुगाब्धि ४४५ शाकरहितः स्युश्चायनांशाः सदा ॥ ४० ॥

त्रिंशद्विभक्तास्तु लवाः स्फुटानि
राश्यादिकानीह विलग्नकानि ।
भवन्ति भानां तु खमध्यगानां
विनाडिका षष्टि ६० हतास्तु नाड्यः ॥ ४१ ॥

अथ घटीलक्षणम् ॥
ताम्री घटार्धाकृतिरम्बुमज्जने (नी)
वक्राल्प (?) रन्ध्रा घटिका द्युरात्रजैः ।
तन्मज्जनैरभ्रखषड्गुणा [३६००] हता
लब्धं प्रमाणं पलपूर्वके (कं) भवेत् ॥ ४२ ॥

अथेष्टसूर्यानयनं ब्रवीमि
क्रान्त्यंशसिद्ध्यै तु नभोगसिद्धिः ।
श्लोकैस्त्रिभिश्चेष्टदिनप्रमाणात्
कषाख्ययन्त्रे गणकप्रतुष्ट्यै ॥ ४३ ॥

दिननिशोर्विवरं शरभू [१५] गुणं
चरपलं भवति द्युमणिं विना ।
पृथु दिनं निशितो घटि (यदि?) तदृणं
पृथु न चेन्निशितश्च तदा धनम् ॥ ४४ ॥

चरात् खण्डानि संशोध्य
शेषं खाग्नि [३०] गुणं भवेत् ।
अशुद्धेन लवादि [:] स्यात्
शुद्धराशिभुजो मु(यु)त: ॥ ४५ ॥

चरे हानौ धनर्णाख्ये
चक्राद् भार्धाद्विशुद्धयेत् ।
वर्धमाने द्वयं व्यस्तं
सायनो ऽर्क: स्फुटो भवेत् ॥ ४६ ॥

त्रिंशन्मितेश्चेदधिकं द्युमानं
तदा भवेदुत्तरसंज्ञगोल: ।
न्यूनं तदा दक्षिणगोलको [ऽ] त्र
गोलक्रमादस्तधनं चरं स्यात् ॥ ४७ ॥

ज्योतिश्चिकित्सितमहागमशिल्पविद्या
विज्ञाय शास्त्रममलं निखिलं च सम्यक् ।
लोकोपकारकरणाय सुबोधमल्प-
माश्चर्यकाणि (रि?) * न कृतं यदि किं कृतं तै: ॥ ४८ ॥

* A manuscript (Baroda 3160) of the *Dik-sādhana-yantra* of Padmanābha has the same verse. The *Dik-sādhana-yantra* reads *kāri* in place of *kāṇi* in the *Kaśā-yantra*.

यो यन्त्रविद्यारहितश्च तस्य
दैवज्ञसंज्ञोदरपूरणाय ।
तस्माच्चमत्कारकषादियन्त्रं
ज्ञानं सुधीभि: समयार्थमु (मू) ह्यम् ॥ ४९ ॥

अस्राष्टकं चेत्क्रियते [ऽ] त्र यन्त्रं
तदा खमध्यस्थभलग्नभागा: ।
शङ्कुस्थितिर्यत्र भवेच्च कोणे
तद्बुध्नकोणे सुधिया विलेख्या ॥ ५० ॥

मूर्खादिकंच (चु) कखलातिकृतघ्नपापी-
हिंस्रातिरोगिगुरुतल्पकनिन्दि (न्द) केभ्य: ।

एभ्यो न देयमिदमेव रहस्यमस्य
नाशो धियश्च ददतः सुकृतायुषः स्यात् ॥ ५१ ॥

आसीद् वात्स्यकुले निजस्थितिलसज् जम्बूव (स?) रस्थो द्विजः
श्रीगङ्गाधरसंज्ञिको [ऽ] तिविदितो व्याख्यातपाटीत्रयः ।
तत्सूनुः सविताप्रसादसुमतिः श्रीविश्वनाथाभिधा (ध)-
स्तज्ज्ञो यन्त्रमिदं कषाख्यमकरोद्धेमाधिकः (भिधः?) कौतुकात् ॥ ५२ ॥

इति श्रीज्योति [र्वि] द्विश्वनाथसुतहेमाख्येन कृतं चाबुकयन्त्रम् ॥

संवत् १७८४ वर्षे शके १६४९ प्र० आषाढकृष्ण १ प्रतिपदाभृगौ ज्योतिर्विद्याहेश्वरसुतामरेश्वरेण लिखितम् ॥

After the above main text, there are some appendices in the manuscript of the *Kaśā-yantra* as follows.

[Folio 6 a:]

अथ लग्नांशाः कोष्टकाः लिख्यन्ते, अश्विन्यादौ खमध्यस्थिते स्पष्टलग्नांशाः ॥

अ	भ	कृ	रो	मृ	आ	पु	पू	अ	म	पू	उ	ह	चि
१०३	११४	१२२	१३९	१५०	१५६	१८३	१९४	१९७	२१२	२२९	२३८	२४९	२६२

स्वा	वि	अ	ज्ये	मू	पू	उ	अ	श्र	ध	श	पू	उ	रे
२७४	२८४	३०२	३१५	३२५	३४५	३५१	०	११	२४	५९	६५	७५	९६

अथ ऽश्विन्यादौ खमध्यस्थितानामन्तरपलानि नक्षत्राणामुदयान्तराः ॥ नक्षत्रखमध्ये पलान्तराः ॥

अ	भ	कृ	रो	मृ	आ	पु	पू	अ	म	पू	उ	ह	चि
९०	१३५	१६५	१२०	१२०	४५	२७०	१३८	४९	१९२	१८२	९३	१२०	१०५

स्वा	वि	अ	ज्ये	मू	पू	उ	अ	श्र	ध	श	पू	उ	रे
१४८	६४	१८०	६०	१३५	१६५	६१	०	१९५	१०५	३४०	५८	१०८	२०७

वलययन्त्रे ॥
वृत्तार्धं तिथिभागं १५ कुर्यादेकैकषड्गुणितं ॥
तस्यांशके चतुर्थे छिद्रं सूच्यग्रभागेन ॥
तत्सूर्यबिम्बाच (भि?) मुखं च धार्यं तन्मण्डलं यत्र पतद्विभागे ॥
विभाजिते तत्र घटीप्रमाणं ज्ञेयं गतं मध्यदिनावशेषम् ॥ १ ॥

[Folio 6 b:]

अथ यत्र सार्धचस्र (स्रो) [ऽ] क्षभा ४/३० तत्र सप्तविंशत्या दिनमानेषू (षु) ई (इ) ष्टघटी प्रतिसिद्धा उत्क्रमच्छाया लिख्यते चाबू (बु) कयन्त्राङ्कनार्थं परमोन्नतांशा: ९३ अक्षकर्ण: १२/४९ अक्षांशा: २०/ ३३/४ मध्योन्नतज्या ९३ इष्टोन्नतज्या ८३/४२ ई (इ) ष्टनतेन (नत) ज्या १०२/१५

१	२	३	४	५	६	७	८	९	१०	११	१२	१३	१३॥	१४	१४॥	१५	१५॥	१६	१६॥	
१ ०	१ ५९	२ ५५	४ १	५ ५	६ ९	७ १३	८ १३	९ ३४	१० २८	११ २७	११ ४८	१२ ५	१२ ४३	० ०	० ०	० ०	० ०	० ०	० ०	मकर
१ ६	२ १८	३ १७	४ २५	५ ३८	६ ५३	८ १८	९ ३८	११ १०	१२ ३६	१३ ५८	१५ ६	१६ ०	० ०	१६ ५९	० ०	० ०	० ०	० ०	० ०	धन.कुंभ
१ १०	२ १७	३ ३०	४ ४४	६ ७	७ ३०	९ ४	१० ५१	१२ ४३	१४ १४	१७ ५	१९ १	२० ४०	० ०	२१ ४०	३३ २१	० ०	० ०	० ०	० ०	वृ.मि.
१ ११	२ २४	३ ३७	४ ५५	६ ३०	७ ५२	९ ३५	११ १८	१३ ९	१६ ५२	१९ ३७	२३ २७	२७ २	० ०	२९ ३९	० ०	३२ १०	० ०	० ०	० ०	मे.तु.
१ १२	२ २४	३ ३९	४ ५८	६ ३७	८ ७	९ ४८	१२ ०	१४ २४	१७ ३८	२१ २१	२७ २१	३२ १५	० ०	४५ १९	० ०	४९ ४	५२ १०	० ०	० ०	वृ.क.

[Folio 7 a:]

चाबुकयन्त्रनीपोथी

*	*	*	४	५	६	७	८	९	१०	११	१२	१३	१३॥	१४	१४॥	१५	१५॥	१६	१६॥ **
* *	* २३	* ३७	४ *	६ ३१	८ ०	९ ४०	११ ४५	१४ १५	१७ १९	२१ ३६	२७ ३०	३५ ३९	० ०	५१ ३०	० ०	६७ १५	० ०	१४ ११	३२ सिं
* *	* *	३ ३१	४ ४७	६ ११	७ ४२	९ ३५	११ २६	१४ २०	१६ ५२	२० ५८	० ०	४८ ४४	० ०	४८ ४४	० ०	८२ ३८	० ०	२५ ०	३३ कर्क

* The left side of this folio is defective, and some figures cannot be read.

** The figure 16 1/2 might have been cancelled in the manuscript.

(iii) *An English Translation of the Kaśā-yantra of Hema*

(a) The main text

[I. Invocation]

1. Having saluted respectfully to Śrī Gaṇanāyaka (= Gaṇeśa), to the sun, to Vāṇī (= Sarasvatī), to my preceptor, to the deity called Śrī Viśvanātha, and to my father also called Śrī Viśvanātha on account of his noble mind, I, named Hema, eagerly describe this precious instrument called *kaśā* ("whip") which gives knowledge of time of day and delights astronomers.

[II. Construction of the kaśā-yantra]

2. An intelligent man should construct a gnomon (*śaṅku*), whose length is twelve *aṅgulas* or increased length from this measure. A rod (*yaṣṭi*) whose length is twenty two times should be made. Or, it may be designed according to [the instruction of] the proper preceptor, or tradition.

Note : The length of 12 *aṅgulas* is the common length of the gnomon in Hindu astronomy. If the length of the rod is really twenty two times of the gnomon, it becomes 264 *aṅgulas*, which look too long for this instrument. As we shall see later, this instrument should be handy so that the time can be read even when riding on a horse. If it means 22 *aṅgulas*, it will then be too short.

3. The rod is the main body of this instrument. If the rod is shorter than the aforesaid length, [only] a rough value of time will be known at the beginning of the day or at the time when the sun is at midheaven.

4. This fine instrument should be made of gold, silver, good brass, copper, or *śiṁśapa*-timber. And it should be made septangular with fine edges. The vertical [edges] should be [straight] like a plumb line. The columns should be measured by halves (i.e. each column has a central vertical line). *Nāḍikās* should be marked according to [the length of] reverse shadows corresponding to a particular day.

Note : The method of the calculation of the length of the gnomon-shadow will be explained in vs.24. The *śiṁśapa*-tree is the *Dalbergia sissoo*. It is used for furniture etc. as it is strong, elastic, and durable.[10] The "reverse shadow" means the shadow of the horizontal gnomon in contrast with the shadow of the vertical gnomon.

5. In the middle of [each] side (vertical column), at the upper portion which is as wide as [the diameter of] the gnomon, a wise man should make a hole, whose size (diameter) is the same as that of the gnomon, in order to erect the gnomon. The [graduation of] length of the day, having its top at the height of the gnomon [on the column], is divided.

6. Then at the bottom of the hole, the needle-like gnomon should be fixed firmly and evenly. On the central column, at the place above the gnomon, the length of the day, which is measured by 30 *[nāḍikās]*, should be written.

Note : The "central column" is used for the equinoctial solar month.

7. Then the marks should be given to three columns in the north (i.e. columns for months when the sun is in the northern hemisphere). And also, the measurement of days, which is decreased by the same rate, to three columns in the south.

Note : Since the sun's declination is symmetrical before and after the solstice, one column can be used in two corresponding solar months except for the solstitial month. So, three columns are used for five solar months.

8. When the length of daytime is 27 *ghaṭikās*, 13 1/2 *ghaṭikās* should be marked on the column. Similarly, on the remaining days, half the length of daytime should be regularly marked by an arithmetician from the root of the gnomon.

Note : Since the altitude of the sun is symmetrical during forenoon and afternoon, a half of the day-length is marked on the column.

9. According to the equinoctial midday shadow at [the observer's] own latitude, a wise man should consider marking the length of the day and marking the *ghaṭikās* also by his own intelligence.

10. In order to divide each interval of a *nāḍī* into four parts, four horizontal lines should be drawn. This interval is 15 *palas* in order to know minute divisions of time.

Note : A *nāḍī* is 60 *palas*. Therefore, one fourth of a *nāḍī* is 15 *palas* which are equal to 6 minutes.

11. An intelligent man should tie a silk thread to the top of this instrument, and hold it with his hand [suspending] like a plumb line in such a way that the gnomon-shadow falls in the middle of the column.

12. By holding the *cābuka* instrument like this, the tip of the shadow of the gnomon touches the mark on the column. Then, tell the *ghaṭikā* which is equal to the number shown on the column. When [the sun] is in the east, these (*ghaṭikās*) are elapsed. In the west, these are to elapse. The shadow increases in the first half of the day, and decreases in the second.

13. Thus the length of daytime, which is 30 *nāḍīs*, is marked on the column. I shall tell the rule of the instrument. On a day with a given length of the shadow, after knowing the length of daytime, the following should be done properly.

14. From the mark, where the tip of the gnomon-shadow touches, one should know the line (graduation of *ghaṭikā*) on the column corresponding to the given length of the day. One should tell *ghaṭikā* which is equal to the number thus observed.

15. In case there are additional *palas* for certain day-length, count them as one [*ghaṭikā*] if they exceed a half [of a *ghaṭikā*]. The desired day's length should be considered like this.

16. On each column, a gnomon is to be placed respectively as was told. When the length of the day is certain amount, *nāḍīs* which are 30 in number are read from the corresponding gnomon's shadow.

17. At the time of observing the shadow, the gnomon [is fixed] horizontally here. At other times, it (gnomon) should be protected by inserting it in a cavity of the same size [made in the body of the instrument] just like revenue is deposited in the treasury. Thus it [should be kept] in the rods (main body of the instruments) which are like the best of gold.

Note : I have reconstructed a probable figure of this instrument in Fig. 1

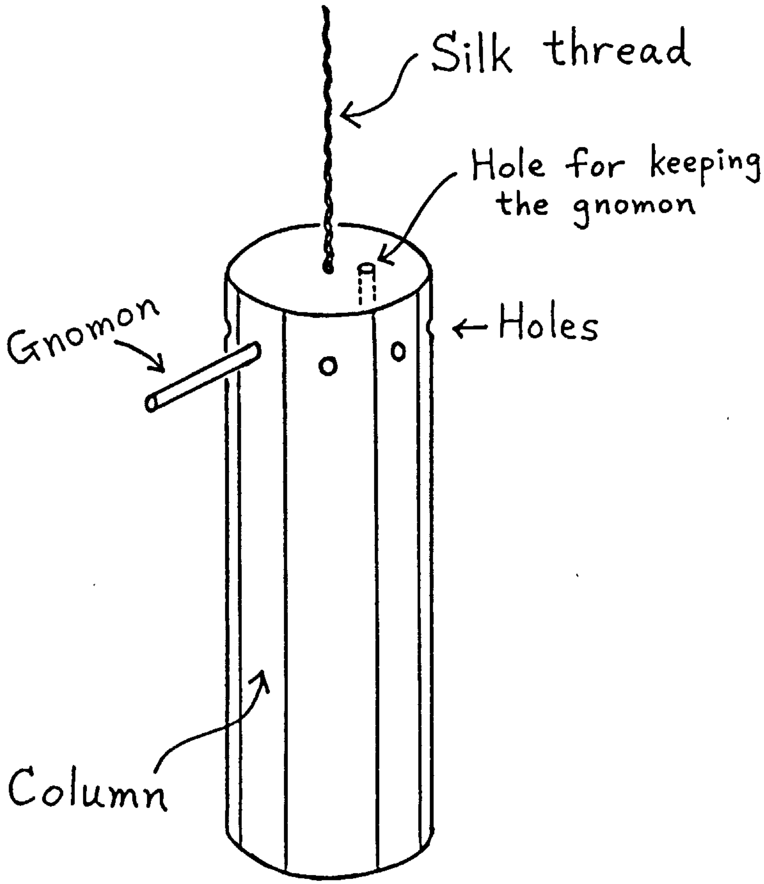


Fig.1 The Kaśā-yantra

[III. Praise of the kaśā-yantra]

18. If you want to be at the royal palace, or if you want to be at high position, study this excellent instrument named *cābuka*. There is no need of the process of calculation by mathenraticians. The determination of time of the day can be done very easily here.

19. The king, who knows the *mālinī* metre, sitting on a horse running like a wind, counts *nāḍikā* etc. by himself eagerly with the cleverly constructed instrument *kaśā*, which is made of metal or wood, in his hand.

Note : The vss. 18 and 19 have been written in *mālinī* metre.

20. All the three worlds are based on law (*dharma*). For it (law), then, time is the most important. Time, uniform yet immeasurable, highly depends on the motion of the planets. Its knowledge indeed can be obtained through the instrument. Therefore, who will not honour this [instrument] which is capable of revealing that (time)?

[IV. Calculation of the shadow]

21. Now I shall tell the computation of reverse shadow according to the length of the desired day expressed in *nāḍīs*. Quoting from the excellent *Yantra-rāja* written by Śrī Padmanābha, marking of *nāḍīs* is easily explained in four verses.

Note : The *Yantra-rāja* [*-adhikāra*] (AD 1423) is chapter I of the *Yantra-kiraṇāvalī* of Padmanābha. Although Hema wrote to quote 4 verses, only the following 2 verses (vss. 22-23) are found in the extant manuscripts of the *Yantra-rāja-adhikāra* of Padmanābha.

22. The declination of the sun corresponding to its tropical (*sāyana*) longitude is to be added to or subtracted from the terrestrial latitude (*pala*) according as they are in the same or opposite directions. The resulting degrees should be subtracted from 90. Its R-sine is the R-sine of the midday altitude.

Note : Equinoctial midday zenith distance of the sun is the same as the terrestrial latitude (φ) of the observer. When the declination (δ) of the sun is south, midday zenith distance of the sun is ($\varphi+\delta$). If north, it is ($\varphi-\delta$). Hence the above rule.

This verse is a quotation from the *Yantra-rāja-adhikāra* of Padmanābha (vs. 98 of Lucknow 45888, and vs. 90 of Lucknow 45892). For the readings of Lucknow manuscripts, see my previous paper.[11]

23. The given *nāḍīs* are to be multiplied by 90, and divided by half a day time. The R-sine (*guṇa*) of the result is to be multiplied by R-sine of the midday altitude, and divided by the Radius. The corresponding arc is the desired altitude [corresponding to the given *nāḍīs*].

Note : This formula is correct on equinoctial days. On an equinoctial day, R-sine of the latitude of the sun is proportional to the R-sine of time elapsed since sunrise in the forenoon and time to elapse until sunset in the afternoon. Therefore,

$$R.\sin a = \frac{R.\sin m \times R.\sin \lambda}{R}$$

where *a* is the altitude of the sun, *m* is the midday altitude of the sun, and,

$$\lambda = 90 \times \frac{t}{T/2}$$

where *t* is the *nāḍīs* elapsed or to elapse, and T is the length of daytime in terms of *nāḍīs*. This verse is a quotation from the *Yantra-rāja-adhikāra* of Padmanābha (vs. 100 of Lucknow 45888, and vs. 92 of Lucknow 45892).[12]

24. The R-sine of the given altitude is to be multiplied by 12, and divided by the R-sine of the zenith distance. The result is the corresponding reverse shadow. This is

useful in the *kaśā* [*-yantra*]. From it (shadow), the *nāḍī* should be determined by mathematicians.

Note : The length of the horizontal gnomon is 12 *aṅgulas*. Then the length of its vertical shadow is obtained as follows.

$$12 : s = R . \sin z : R . \sin a,$$

where *s* is the length of the shadow, *z* the zenith distance of the sun, and *a* the altitude of the sun. (See Fig. 2.) Therefore,

$$s = \frac{12 \times R . \sin a}{R . \sin z}$$

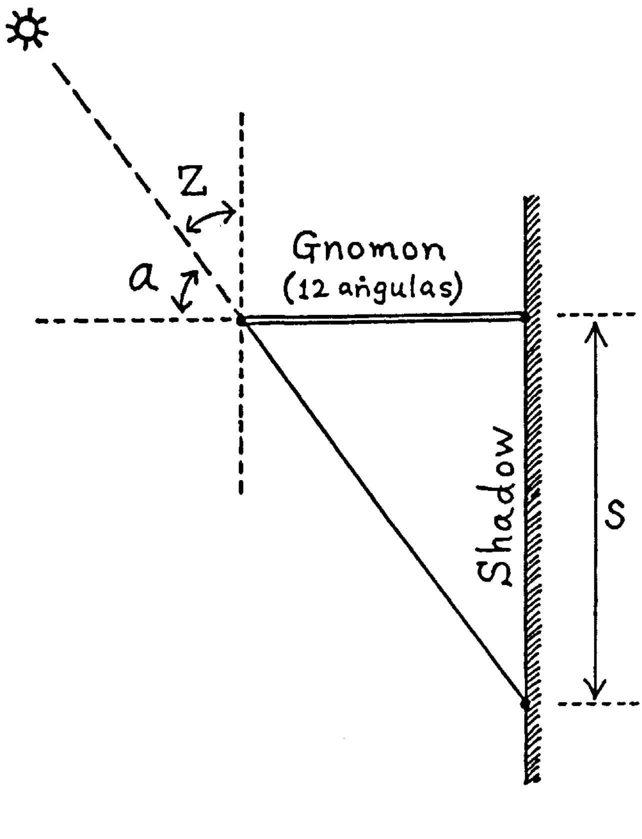


Fig. 2 The horizontal gnomon

25. The given altitude should be subtracted from 90. It is the corresponding zenith distance, and its R-sine is the R-sine of the corresponding zenith distance. Intelligent man should calculate this for the sake of reverse shadow.

26. (... One *pada* might have been missing at the beginning .....) After dividing, the result. Its corresponding arc, that is the latitude (*pala*, the zenith distance of the equinoctial midday sun), is always in the south direction. When it is subtracted from 90, it is the co-latitude (*lamba*) which is the altitude [of the equinoctial midday sun] in the south direction.

Note : This verse consists of 3 *padas* only. Since the first word "after dividing" (*bhaktāt*) is not connected with the previous verse, one *pada* might have been missing at the beginning.

[V. Calculation of the R-sine]

27. Now some useful things like the R-sine etc. are explained through six verses, which are easily understandable, from the *karaṇa* work *Sūrya-tulya* written by Dāmodara. What is the need of new verses !

Note : The following six verses (vss. 28-33) are quotations from the *Sūrya-tulya* (AD 1417) of Dāmodara, son of Padmanābha. I have seen a manuscript of the *Sūrya-tulya* (The Asiatic Society, Calcutta, IM-5356), and the quotations have been identified with vss. 4-9 of its second chapter (*Sūryendu-spaṣṭīkaraṇa*).

The verses explain the calculation of sine and arcsine.[13]

28. The numbers 26, 24, 20, 16, 10, and 4. These are the six differences of successive half chords (i.e. R . sines) for the Radius one hundred.

Note : The above verse can be tabulated as below. (R = 100)

Degree	R . sine	Differences
90	100	
75	96	4
60	86	10
45	70	16
30	50	20
15	26	24

29. The degrees are to be divided by 15. The quotient is the number of segments (i.e. the serial number of the differences which have been given in vs. 28) already passed. The next (i.e. the value of the next segment in the series of the differences) is to be multiplied by the remainder (i.e. the degrees minus 15 times of the "quotient"), and divided by 15. The result is to be added to the sum of the values of already passed segments. This is the R . sine.

Note : This is the first order interpolation. Let the values of the argument be $\alpha_1, \alpha_2, ...$, the values of their function be $f_1, f_2, ...$, the common difference of the values of the argument be *h* $(= \alpha_2 - \alpha_1 = \alpha_3 - \alpha_2 = ...)$, and the differences of the values of the function be :

$$\Delta_1(= f_2 - f_1), \Delta_2(= f_3 - f_2), ...$$

We have to find out the value of the function $f(\alpha)$, when $\alpha_2 < \alpha < \alpha_3$ say. The first order interpolation is expressed as follows.

$$f(\alpha) = F_2 + \frac{r}{h} \Delta_2 , ...(1)$$

where $r = \alpha - \alpha_2$. In this case, *h* is 15.

30. The degrees of the remainder (*r*) are to be multiplied by the difference between the segment to be passed(Δ_2, say)and the segment already passed (Δ_1), and divided by 30. It is to be subtracted from(if $\Delta_1 > \Delta_2$)or added to (if $\Delta_1 < \Delta_2$) the half of the sum of the already passed segment (Δ_1) and the next segment (Δ_2). This is the segment to be passed [which has been corrected for the second order interpolation] for the purpose of the R-sine and R-versed-sine.

Note : This is the method of the second order interpolation, where Δ_2 of the first order interpolatin is substituted by Δ_2' . The value of Δ_2' is expressed as follows.

$$\Delta_2' = \frac{\Delta_1 + \Delta_2}{2} \mp \frac{r}{15} \left(\frac{\Delta_1 \sim \Delta_2}{2} \right) ....(2)$$

Therefore,

$$f(\alpha) = f_2 + \frac{r}{15} \left\{ \frac{\Delta_1 + \Delta_2}{2} \mp \frac{r}{15} \left(\frac{\Delta_1 \sim \Delta_2}{2} \right) \right\} ....(3)$$

The negative or positive sign is taken according as

$\Delta_1 >$ or $< \Delta_2$.

This is the same as Brahmagupta's method.[14] It is equivalent to Stirling's formula.

The manuscript (IM-5356) of the *Sūrya-tulya* reads *dalaṃ sphuṭaṃ* in place of *yutaṃ dalaṃ*.

31. The R-sine obtained according to the instruction from the correct segments is very correct. By the reverse series of the segments, the R-versed-sine should be obtained correctly from the correct segments.

Note : The R-versed-sine of θ is equal to $(R - R . \cos\theta)$. Therefore, the reverse series of the segments given in vs.28 can be used for the R-versed-sine as follows.

Degree	R-versed-sine	Differences
90	100	
		26
75	74	
		24
60	50	
		20
45	30	
		16
30	14	
		10
15	4	
		4
0	0	

32. The values of the segments are to be subtracted [from the given R-sine, as far as possible]. The remainder is to be multiplied by 15, divided by the uncompleted value (i.e. the value of the next segment), and added to the product of 15 multiplied by number of already completed (already have been subtracted) segments. The result is the arc.

Note : This is the method to obtain the arcsine. From the relation stated in vs. 29 (the equation (1)), we can express the value of the arc (α) as follows.

$$\alpha = \alpha_2 + \frac{15}{\Delta_2} \left\{ f(\alpha) - f_2 \right\} \qquad \ldots\ldots\ldots\ldots(4)$$

This is what vs. 32 says. The value of α_2 is of course "the product of 15 multiplied by number of already completed segments".

The manuscript (IM - 5356) of the *Sūrya-tulya* reads as follows in place of the above verse of Hema.

खंडानि शोध्य शेषं यत् घस्र १५ घ्नं तदशुद्धहृत् ।
लब्धं तच्छुद्धसंख्येषु चंद्र १५ घाते युतं धनुः ॥

33. After subtracting [the values of segments as before], the difference between the already passed segment (Δ_1) and the next segment (Δ_2) is to be multiplied by a half of the remainder (f (α) - f_2). It is to be divided by the next segment (Δ_2), and the result is to be subtracted from or added to a half of the sum of the already passed segment (Δ_1) and the next segment (Δ_2). This is the correct segment to be passed for the calculation of the arc corresponding to the R-sine or R-versed-sine.

Note : This is the method of the second order interpolation in order to obtain the arcsine.[15] The above rule can be expressed as follows.

$$\Delta_2^* = \frac{\Delta_1 + \Delta_2}{2} \mp \frac{f(\alpha) - f_2}{2\,\Delta_2} \left(\Delta_1 - \Delta_2 \right) \qquad \ldots\ldots\ldots\ldots(5)$$

and this Δ_2^* is to be substituted in place of Δ_2 in the equation (4). The negative or positive sign is taken according as

$$\Delta_1 > \text{ or } < \Delta_2.$$

34. This astonishing instrument, named *kaśā*, has been prescribed by using reverse shadow. Those who have knowledge of the instrument and are endowed with excellent intellect may also construct the instrument using direct shadow.

Note : The "direct shadow" means the shadow of an usual vertical gnomon in contrast with the horizontal gnomon of the *kaśā-yantra* on which the shadow is called "reverse shadow".

[VI. Longitude of the lagna corresponding to the transiting nakṣatra]

Now, the knowledge of the night.

34'-35. [The following figures are the tropical longitudes of the *lagna* (the rising point of the ecliptic) when the various *nakṣatras* are on the meridian.]

(Aśvinī:) 103, (Bharaṇī:) 114, (Kṛttikā:) 129, (Rohiṇī:) 139, (Mṛgaśiras:) 150, (Ārdrā:) 156, (Punarvasu:) 183, (Puṣya:) 194, (Āśleṣā:) 197, (Maghā:) 212, (Pūrva Phalgunī:) 232, (Uttara Phalgunī:) 238, (Hasta:) 249, (Citrā:) 262, (Svātī:) 274, (Viśākhā:) 284, (Anurādhā:) 302, (Jyeṣṭhā:) 305, (Mūla:) 325, (Pūrva Āṣāḍhā:) 345, (Uttara Āṣāḍhā:) 351, (Abhijit:) 0, (Śravaṇa:) 11, (Dhaniṣṭhā:) 24, (Śatabhiṣaj:) 59, (Pūrva Bhādrapadā:) 65, (Uttara Bhādrapadā:) 75, and (Revatī:) 96 degrees.

These values of the true positions of the stars, beginning from Aśvinī (*dasra*), have been told successively by the learned Nīlakaṇṭha. For finding time when the star is on the meridian at night, the period to be passed (i.e. degrees to be passed by the sun) from the six-sign-added sun's position (longitude) [upto the end of the same sign] is to be added to the sum of [oblique] ascensions of already passed signs from the tropical (*sāyana*) *lagna* (i.e. the total oblique ascension of the signs between the horizon and the opposite point of the sun).

Note : There is a table of the longitudes of the *lagnas* corresponding to the transit *nakṣatras* in the manuscript (folio 6a). There are three differences of the value as follows.

(1) The table gives 122 for Kṛttikā.

(2) The table gives 229 for Pūrva Phalgunī. However, 232 must be correct, because the text for the next *nakṣatra* (Uttara Phalgunī) reads "*sa-aṅgam*" (plus six), and its number is 238, which implies that the previous number was 232.

(3) The table gives 315 for Jyeṣṭhā. The text for Jyeṣṭhā reads *triyuk* (plus three), and three plus previous 302 for Anurādha is 305.

The last statement of the text means that the ascensional difference between the *lagna* and the opposite point of the sun should be obtained. This angle gives time elapsed since sunset.

The verse number 34 occurs twice in the manuscript, and I have tentatively called the latter verse vs. 34'.

36. Therefore, seven columns are to be made in order to know [time] at night (?). The degrees of the *lagnas* should be written on seven columns along with 15 divisions of time. The star which is passing the meridian should be observed by the instrument always (?), and the time should be determined from the *lagna* of that moment as was told by Śrī Nīlakaṇṭha.

Note : This text seems to be defective, and its meaning is not quite clear. The astronomer Nilakaṇṭha mentioned in vss. 35-36 has not been identified.

37. With this instrument of seven columns, the time can easily be known at night from [the star] on the meridian and from the degrees of *lagna* that have been told in the verses (vss. 34-35).

38. A small slit should be made above the cavity, through which the object on the meridian is observed. Then with the top of this instrument, the star on the meridian should be observed following the instruction of the preceptor.

[VII. Right ascensional differences of nakṣatras]

Now the [right ascensional] differences of *nakṣatras* in terms of *palas*.

39-40. (Aśvinī:) 90, (Bharaṇī:) 135, (Kṛttikā:) 165, (Rohiṇī:) 120, (Mṛgaśiras:) 120, (Ārdrā:) 45, (Punarvasu:) 270, (Puṣya:) 138, (Āśleṣā:) 49, (Maghā:) 192, (Pūrva Phalgunī:) 189, (Uttara Phalgunī:) 93, (Hasta:) 120, (Citrā:) 105, (Svātī:) 148, (Viśakhā:) 64, (Anurādhā:) 180, (Jyeṣṭhā) 60, (Mūla:) 135, (Pūrva Aṣāḍhā:) 165, (Uttara Āṣāḍhā:) 71, (Abhijit:) 0, (Śravaṇa:) 195, (Dhaniṣṭhā:) 190, (Śatabhiṣaj:) 340, (Pūrva Bhādrapadā:) 58, (Uttara Bhādrapadā:) 108, and (Revatī:) 207 *vināḍīs*.

These are begun from Aśvinī (*dasra*). Beginning from Revatī (*pauṣṇa*) situated on the meridian, successive bright stars pass the meridian. The Śaka year minus 445 divided by 60 is always the *ayanāṁśa* (the amount of precession since Śaka 445).

Note : A *vināḍī* (= *pala*) is one 60th of a *nāḍī*. There is a table of the right ascensional differences of the *nakṣatras* in the manuscript (folio 6a). There are three differences of the value as follows.

(1) The table gives 182 for Pūrva Phalgunī.

(2) The table gives 61 for Uttara Āṣāḍhā. If the text can be read as *kvaṅgam*, it gives 61, but it becomes metrically defective. The reading *kvagam*, which means 71, is metrically correct.

(3) The table gives 105 for Dhaniṣṭhā. The text for Dhaniṣṭhā reads *"bāṇa-rahitaḥ"* (minus five). If it is connected to the previous number 195, the result is 190.

The total of the given numbers in the text is 3752, and that of the table is 3650. There must be some mistakes, because the total should be 3600 *vināḍīs*.

The last statement of the text means that the rate of the precession is one minute per year, and the *ayanāṁśa* was zero in Śaka 445 (AD 523).

41. When the number of degrees is divided by 30, it is the *vilagna* in terms of signs (*rāśis*). When the *vināḍikās* of the stars on the meridian are divided by 60, they are *nāḍīs*.

Note : One zodiacal sign is 30° (= 5 *nāḍīs*), and one *nāḍī* is 60 *vināḍikās*. Hence the above rule. The *vilagna* is the same as *lagna* (the rising point of the ecliptic).

[VIII. The clepsydra]

Now the description of the clepsydra.

42. A copper vessel resembling a half pitcher (*ghaṭārdha*), which has a fine hole at its bottom (?), sinks into the water. By the number of its sinkings during a whole day

and night, 3600 is divided. The result is the measure [of the clepsydra] (i.e. time required to sink into the water) in terms of *palas*.

Note : This is an usual floating type clepsydra.[16] Since a whole day and night is 3600 *palas*, it is divided by the number of the clepsydra's sinking in a whole day and night, and the result is time required to sink once.

[IX. Calculation of the sun]

43. Now I shall tell the calculation of the desired [day's] sun, and the determination of the heavenly body (sun) for the determination of its declination by means of the length of daytime of the day measured by the instrument named *kaśā*, in three verses for the satisfaction of mathematicians.

Note : The calculation of the declination (*krānty-aṁśa*) is not directly explained in the following verses. What follows is the determination of the sun's longitude.

44. The difference between the length of daytime and nighttime (in terms of *nāḍīs*) multiplied by 15 is the ascensional difference (*cara*) in terms of *palas*, [which is obtained] without [the observation of] the sun. If the daytime is longer than the nighttime, [the value of the ascensional difference is] negative. If the daytime is not longer than the nighttime, it is positive.

Note : The ascensional difference is equivalent to the arc of the celestial equator lying between its intersection with eastern horizon and its intersection with the hour circle which passes through the sun at sunrise. (The former intersection is the east cardinal point, and the latter is a point on the celestial equator which indicates the sun's right ascension). Therefore, the ascensional difference is equal to the arc corresponding to a quarter of the difference between the length of the daytime and nighttime. Since one *nāḍī* is 60 *palas*, a quarter of 60, that is 15, is multiplied by the difference between the length of the daytime and nighttime in order to obtain the ascensional difference in terms of *palas*.

45. From the ascensional difference, segments (*khaṇḍas*, finite differences of the series of the ascensional difference of zodiacal signs) are to be subtracted as far as possible. The remainder is to be multiplied by 30. (And divided by the next segment). By the remainder (*aśuddha*), degrees etc. are obtained. This is added to the passage of completed (*śuddha*) signs.

Note : In the actual calculation, only the finite differences of the ascensional difference of the first three signs are used. Let the ascensional difference of the first three signs be ω_1, ω_2 and ω_3 respectively, and the finite differences be

$$\Delta_1 = \omega_1 - 0 = \omega_1$$

$$\Delta_2 = \omega_2 - \omega_1,$$

$$\Delta_3 = \omega_3 - \omega_2 .$$

Now, these differences (*khaṇḍa*) should be subtracted from the observed ascensional difference (ω) serially as far as possible. The number (n) of the subtracted differences, which is either 0, 1, or 2, is the number of the zodiacal signs which has been passed (or to be passed) by the sun. Then the remainder ($\omega - \sum_{0}^{n} \Delta_n$; let Δ_0 be 0) is multiplied by 30, and divided by the next difference (Δ_{n+1}). The result is the degrees which have been passed by the sun in the next sign, because one 30th of the difference corresponds to the mean difference of the ascensional difference of each degrees in the sign. These degrees should be added to the total arc of the completed signs (= 30n°). The result is the sun's *bhuja*. If the sun is in the 1st or the 3rd quadrant, the *bhuja* is the angle which has been passed by the sun from 0° or 180°. If the sun is in the 2nd or the 4th quadrant, it is the angle to be passed upto 180° or 360°.

46. When [the absolute value of] the ascensional difference is decreasing, be it positive or negative, one should subtract the value [which was obtained by the previous verse] from 360° (*cakra*) of 180° (*bhārdha*). When the ascensional difference is increasing, for both (the 1st and the 3rd quadrants), the opposite operation should be done. [The result] is the true tropical (*sāyana*) [longitude] of the sun.

Note : The ascensional difference decreases when the sun's longitude is in between 90° and 180°, or between 270° and 360°. In the case of the former, the *bhuja* should be subtracted from 180°, and in the case of the latter, from 360°. The ascensional difference increases when the sun's longitude is in between 0° and 90°, or between 180° and 270°. In these cases, the *bhuja* should be added to 0° or to 180° respectively.

47. When the length of daytime is more than 30 [*nāḍīs*], [the sun] is in the northern hemisphere. When the length is less, [the sun] is in the southern hemisphere. According to [the position of the sun on] the hemispheres, the ascensional difference is negative or positive.

[X. Conclusion]

48. Even after knowing correctly the sciences of astronomy, medicine, tradition, and craft, and also the pure comprehensive learning, if they do not produce a lucid, small, and wonderful work for the benefit of people, what is considered to be done by them?

Note : A manuscript (Baroda 3160) of the *Diksādhana-yantra*, composed by Padmanābha, has the same verse. Since Hema is later than Padmanābha,Hema might have borrowed it from the *Diksādhana-yantra*. It is quite possible, because Hema already had quoted from another work of Padmanābha, the *Yantra-rāja-adhikāra*, as vss. 22-23 of this *Kaśā-yantra*.

49. The title of astronomer/astrologer (*daivajña*) for those who lack the knowledge of instruments is just for filling his belly. Therefore, intelligent men should understand the knowledge of wonderful instrument like *kaśā* etc. for determining time.

50. If an instrument with 8 columns is constructed, the degrees of the meridian *lagna* (intersection of the ecliptic and the meridian) can be represented by the gnomon on the column. On the column under it (gnomon), an intelligent man should mark [the longitude of the meridian *lagna*].

Note : If the longitude of the meridian *lagna* is marked on the 8th column according to its transit altitude, the longitude of the sun can be known by the observation of the sun at noon, because the position of the sun at noon is the meridian *lagna*.

51. This secret knowledge should not be given to foolish, libidinous (?), mischievous, too ungrateful, sinful, cruel, too sick, and to the defilers of the preceptor's marital bed. If one gives, his intellect, merit, and longevity will be lost.

52. There was a twice-born called Śrī Gaṅgādhara of Vātsya-kula who is famous for his three commentaries on *pāṭī* (arithmetic), resident of Jambūvara (Jambūsara ?) which is shining by its location. His son is Śrī Viśvanātha who is good-minded by the grace of the sun. And his son, Hema, constructed this instrument called *kaśā* out of [intellectual] curiosity.

This is the *Cābuka-yantra* composed by Hema, son of Viśvanātha the astronomer.

In the year [Vikrama] Saṃvat 1784, or Śaka 1649, in the month of the 1st Āṣāḍha, waning fortnight (*kṛṣṇa-pakṣa*), on the 1st day, Friday, Amareśvara, son of Māheśvara the astronomer, copied.

(b) Appendices

[Folio 6 a:]

Now the table of the longitudes of the *lagna* is written, that is the true longitudes of the *lagna* when [the *nakṣatra*] Aśvinī etc. are on the meridian.

Note : In the manuscript, the table follows. It will not be necessary to translate the table into English here, because we already have seen the values of the longitudes of the *lagna* in vss. 34-35.

Now the [right] ascensional differences in terms of *palas* when the *nakṣatras* are on the meridian, beginning with Aśvinī. The [right ascensional] differences in terms of *palas* for the *nakṣatras* on the meridian.

Note : In the manuscript, the table follows. It will not be necessary to translate it, because we have seen the values in vss. 39-40.

In the Valaya-yantra :

One half of a circle should be divided into 15, and each division should be subdivided into 6. At the 4th point (i.e. the middle of unmarked half, as the marked half includes three cardinal points (?)) of this marked instrument, a very small hole is to be made with the tip of a needle. It should be so held that it faces the sun. From the subdivision on which the sunlight falls, the *ghaṭīs* already passed since sunrise or to pass in the afternoon should be known.

Note : This description is too brief to investigate the construction of the *valaya-yantra*. It is not clear whether this is from some work of Hema or from some work of a different author. This point is discussed in the remarks below.

[Folio 6 b:]

Now at the place where the *akṣa-bhā* (equinoctial midday shadow of the 12-*aṅgula* gnomon) is $4\frac{30}{60}$, at the time when the length of daytime is 27 [nāḍikās], the reverse shadow determined for desired *ghaṭīs* for the graduation of the *cābuka-yantra* [is as follows].

The maximum altitude [of the sun] : 93°.

The equinoctial midday hypotenuse : $12\frac{49}{60}$.

The [observer's] latitude : 20°33'4".

The R-sine of the midday altitude : 93.

The R-sine of desired altitude : $83\frac{42}{60}$.

The R-sine of desired zenith distance : $102\frac{15}{60}$.

Note : It appears that this is an example for the latitude 20°33'4" at the winter solstice. The equinoctial midday hypotenuse is the hypotenuse of the right angled triangle made by the 12-*aṅgula* gnomon and its equinoctial midday shadow, that is $\sqrt{12^2 + (4.5)^2}$ in this case. Since the maximum altitude of the sun (at the summer solstice) has been gives as 93°, it appears by adding 3° to the observer's latitude that the obliquity of the ecliptic has been taken as 23°33'4". As theR-sine of the midday altitude (at the winter solstice) has been given as 93, it appears that the Radius of the trigonometric function is 130, because 130 x sin {90° - (20°33'4") - (23°33'4")} ≒ 93.35. After calculating the R-sine of desired altitude and the R-sine of desired zenith distance, the reverse shadow can be obtained by the method shown in vs. 24 of the *Kaśā-yantra*.

Probably, this example is not originated with Hema, because Hema used R = 100 in vs. 28 of his *Kaśā-yantra*. We should also note that the above latitude 20°33°4" is different from the latitude of Jambusar (about 22°N) where the grandfather of Hema resided.

In the manuscript, the table of the reverse shadow follows. In the table, the values for 16 and $16\frac{1}{2}$ *ghaṭīs* seem to have been measured by a larger unit than *aṅgula*, whose dimension has not been mentioned in the text.

It will not be necessary to translate the table here.

(iv) Remarks

As we have seen, the *Kaśā-yantra* of Hema (later half of the 15th century AD) is a very detailed work on the cylindrical sundial with full theoretical exposition. It should be noted that he did not compose his own verses for some theoretical topics, but quoted from the works of Padmanābha and Dāmodara (son of Padmanābha), both of whom belong to the first half of the 15th century. As I have described in my previous paper,[17] Padmanābha composed some works on astronomical instruments, one of which is on the newly introduced astrolabe. This work was quoted by Hema. Hema's quotation shows that some astronomical works in this period were widely circulated among contemporary astronomers, and that astronomers obtained new knowledge quickly.

As regards the author of the *Valaya-yantra* in folio 6 a of the manuscript of the *Kaśā-yantra*, there is a possibility that he is also Hema. The Asiatic Society of Bombay has a manuscript of two anonymous works, both of which are entitled *Valaya-yantra*. (They are included in AS Bombay 245 (BD 298/21) which contains three other works also). These two works are different from the *Valaya-yantra* in the manuscript of the *Kaśā-yantra*. The first *Valaya-yantra* in the Bombay manuscript consists of three verses, and the second *Valaya-yantra* consists of eight verses. They were copied in Saṃvat 1716 (AD 1660). The first verse of the first *Valaya-yantra* in the Bombay manuscript is as follows.

हेमाद्यैः सुकृतं सुरम्यवलयं सद्व्यङ्गुलप्रसृतं (प्राकृतं ?)
तत्खण्डे प्रथमे खखेचरमिता ९० भागाः सदङ्कयस्तथा ।
कुर्याद्भागमितं सुरन्ध्रमपरार्धार्ध च संकारयेत्
द्र(र)न्ध्राद्यंशदले सुरन्ध्रमिति तद्रज्जूर्ध्वतार्थ(र्थं) स्फुटम् ॥ १ ॥

"The very beautiful *"valaya"* (ring) with the standard measure of *vyaṅgula*, excellently made by *"hema"* (literally means "gold") etc., [should be made]. On its first part (half), 90 degrees should be marked correctly. One should make a hole [whose position is] measured with degrees. And also, one should make the middle point of the second half. At the middle of the part (half) which begins with the hole, [another] hole [is made]. It is for hanging with a cord rightly".

I have reconstructed a probable figure of this instrument in Fig. 3. As the light through the hole falls on the lower half of the ring, the lower half is graduated with 90 degrees in order to indicate the sun's altitude.

At the beginning of the above quoted verse of the *Valaya-yantra* in the Bombay manuscript, the word *"hema"* appears. It literally means gold, and the word *"hema"* in the above verse may simply mean gold. However, there is also a possibility that this *"hema"* is the astronomer Hema. If so, the author of the *Valaya-yantra* in the manuscript of the *Kaśā-yantra* may also be Hema. This point should be investigated further. There are some other Indian sources and specimens of the ring instrument,[18] and its history is also an interesting topic.

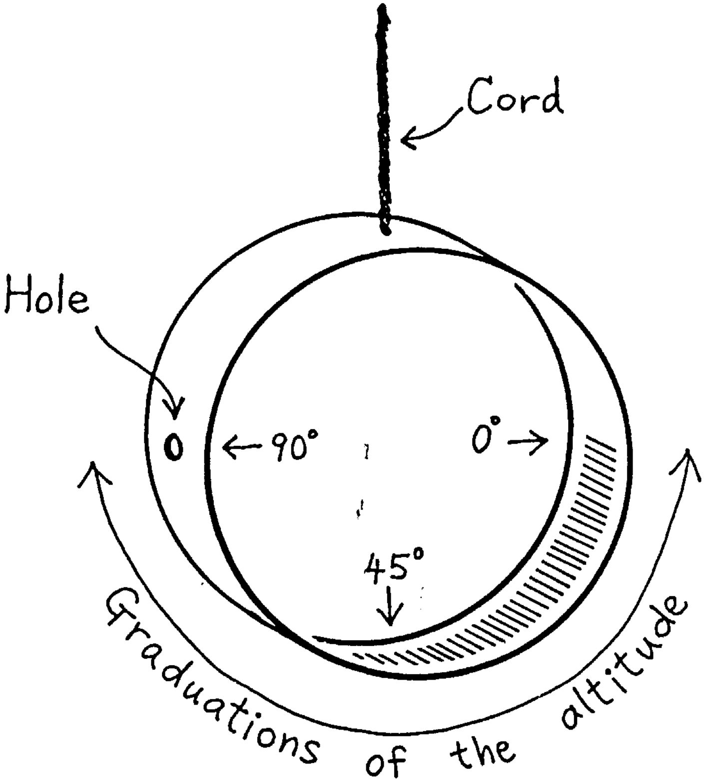


Fig. 3 The valaya-yantra

THE PRATODA-YANTRA OF GAṆEŚA

(i) *Introduction*

Gaṇeśa Daivajña[19] (born AD 1507), son of Keśava and pupil of Vaijanātha, is well known by his *karaṇa* work *Graha-lāghava* (AD 1520). Gaṇeśa's father Keśava was also an astronomer, and composed a *karaṇa* work *Graha-kautuka* (AD 1496). Gaṇeśa resided at Nandigrāma. S.B. Dikshit identified it with Nāndgāon in Janjīrā state, about 40 miles to the south of Bombay.[20] On the contrary, D. Pingree identified it with Nandod in Gujarat.[21] I think that the identification of Dikshit is right, because Gaṇeśa himself used 18°N as the latitude of the observer in his *Pratoda-yantra* (vs. 6) as we shall see below. This latitude agrees with Janjīrā in Maharashtra, but disagrees with Nanded (about 22°N) in Gujarat.

Gaṇeśa composed two works on astronomical instruments, namely the *Pratoda-yantra* and the *Sudhīrañjana-yantra*. The *pratoda-yantra* is a cylindrical sundial, and the word *"pratoda"* mean whip. So, its name as well as its construction is similar to Hema's *kaśā-yantra*, but it is not certain whether Gaṇeśa was directly influenced by Hema or not.

The *Pratoda-yantra* of Gaṇeśa has been mentioned by S.B. Dikshit as follows.

Pratoda Yantra

> Gaṇeśa Daivajña, the author of the *Grahalāghava*, compiled this work on instruments. It contains 13 verses.[22] The author claims that even while riding a horse one can find time by observation with the help of this Pratoda yantra, and also the shadow cast by the gnomon at that time. Its construction is not describhed here for want of space. Sakhārāma and Gopīnātha have written commentaries on this work". (Translated by R.V. Vaidya)[23]

I have seen a manuscript of Sakhārāma's commentary at the library of Ānandāśrama, Pune, but I have not come across any manuscript of Gopīnātha's commentary.

David Pingree listed three works of Gaṇeśa on the "whip instrument" as nos. 8-10 of Gaṇeśa's works listed in his *Census* (1971) as follows.[24]

"8. *Cābukayantra*. There is a commentary by Munīśvara (*b*. 1603)".

"9. *Pratodayantra*. There is a ṭīkā by Gaṇeśa himself".

"10. The ṭīkā on the *Pratodayantra*".

Pingree listed the data of the manuscripts of each work under respective titles. In this *Census*, Pingree quoted the 13th verse of the *Pratoda-yantra* (work no. 9). (It agrees with Gaṇeśa's original *Pratoda-yantra*, which I shall present below). Pingree wrote in his *Jyotiḥśāstra* (1981) that he had consulted a manuscript of the *Pratoda-yantra* (ff. 15-16 of India Office 1989), but he had not seen any copy of the *Cābuka-yantra*.[25]

In 1982, Shakti Dhara Sharma published a Sanskrit text as the "Pratoda Yantra (Chabuka Instrument) by Sh. Ganesha Daivajna (A Gnomonic Whip-Shaped device to know time)".[26] This text consists of 8 verses with a commentary, and has been edited from two manuscripts, one of which is entitled *Cābuka-yantra* and the other is entitled *Pratoda-yantra*. (Their contents are basically identical). As regards the original source of this text, Sharma wrote as follows.

> "Pratoda Yantra has different holes for fixing gnomon for every month of the year. It was developed by Ganesh Daivajna who wrote Grahalaghava in Shaka

> 1444 (1522 A.D.) (sic).[27] The same was included in Yantradhyaya of Siddhanta-Sarvabhauma by Acharya Munishwara (as evidenced from last colophon of the manuscript no. 1)"[28]
>
> "This form of the book is reproduced from two manuscripts but the details as given in Siddhanta Sarva-bhauma could not be made use of as the Yantradhyaya of Siddhanta Sarva-bhuma is not yet available in properly edited form. The original work by Ganesh Daivajna is not available these days and we have to depend only on the informations available through the work of Acharya Munishwara". (S.D. Sharma)[29]

Now I shall show that Gaṇeśa's own work on the cylindrical sundial is only the *Pratoda-yantra* consisting of 13 verses without extant auto-commentary as far as I know, and that the *Pratoda-yantra* or *Cābuka-yantra* consisting of 8 verses with a commentary is actually an extract from the *Siddhānta-sārvabhauma* of Munīśvara with Munīśvara's own commentary.[30]

I have consulted 8 manuscripts[31] of the *Pratoda-yantra* which consists of 13 verses. This work is evidently Gaṇeśa's own work, because he clearly wrote in its last verse that Gaṇeśa himself had composed it. I have never come across its auto-commentary. I shall present its full text with my English translation.

Munīśvara wrote the *Siddhānta-sārvabhauma* in AD 1646. Althugh its first half has been published, its second half which includes the *Yantrādhyāya* has not been published. I have consulted 4 manuscripts[32] of its *Yantrādhyāya*, and I confirmed that Munīśvara described the *pratoda-yantra* in vss. 63-70 of the *Yantrādhyāya* of his *Siddhānta-sārvabhauma*. It was Munīśvara's own commentary As we shall see later in the section of Munīśvara, his version is not a quotation from Gaṇeśa's work, but Munīśvara's own composition, because Munīśvara himself wrote "I describe this instrument "*pratoda*", which was told by Gaṇeśa, ...".

The vss. 63-70 of the *Yantrādhyāya* of the *Siddhānta-sārvabhauma* were copied independently, sometimes as the *Pratoda-yantra* and sometimes as the *Cābuka-yantra*. Munīśvara called this instrument *pratoda-yantra* in the text, but *cābuka-yantra* in the commentary. I have seen 9 manuscripts[33] of the Munīśvara's version entitled *Pratoda-yantra*, and 2 manuscripts[34] of the Munīśvara's version entitled *Cābuka-yantra*. In the case of the former, the name of the instrument in the commentary has been changed into "*pratoda-yantra*", but it is identical with the latter otherwise. This Munīśvara's version is the text published by Shakti Dhara Sharma. David Pingree listed both the *Pratoda-yantra* and the *Cābuka-yantra* as Gaṇeśa's works, but it is due to his confusion of Gaṇeśa's original work with Munīśvara's version, and it will be necessary to check each manuscript listed in Pingree's *Census* in order to identify whether it is Gaṇeśa's or Munīśvara's. Pingree also listed the "ṭīkā on the *Pratoda-yantra*" as Gaṇeśa's work, and listed its 10 manuscripts. I have seen 5 manuscripts[35] among them, but all of them were Muniśvara's version. Therefore, the existence of Gaṇeśa's own commentary on his *Pratoda-yantra* is quite doubtful.

From the above discussions, it is now clear that Gaṇeśa's own work on the cylindrical sundial is only the *Pratoda-yantra* consisting of 13 verses. Let us see its text.

I have used the following eight manuscripts for this edition. If the same manuscript has two numbers, I give the number mentioned in Pingree's *Census* first, and give the other number within bracket.

Ms. A: Bombay U 375 (Account no. 833), 15ff (with the *Yantra-cintāmaṇi* of Cakradhara). The *Pratoda-yantra* is in 13b - 14b. Beginning: "Atha Pratoda-yantraṁ likhyate". End: "iti Pratoda-yantra-nirmitiḥ. daivajñavarya-Ramākānta-ātmaja-Vidyādhara-daivajñena likhitā samāptam. Śrīḥ". This is in the Library of the University of Bombay, Bombay.

Ms. B: Benares 35298, 1 f (with a fragment of the *Yantra-cintāmaṇi* of Cakradhara). Beginning: "Śrī-Gaṇeśāya namaḥ". This is in Sarasvati Bhavan, Sampurnanand Sanskrit University, Varanasi.

Ms. C : Benares 35702, 4 ff (with Munīśvara's version of the *Pratoda-yantra*). Gaṇeśa's *Pratoda-yantra* is in 1-2a. Beginning: "Śrī-Gaṇeśaṁ bhaje". The end of the whole manuscript: "iti Pratoda-yantraṁ śubham". This is in Sarasvati Bhavan, Sampurnanand Sanskrit University, Varanasi.

Ms. D : AS Bombay 245 (BD 298/21), 10 ff (with *Dhruvabhrama-*, *Valaya-*, and *Sudhīrañjana-yantras*). The *Pratoda-yantra* is in 9a - 10a. Beginning: "Śrī-Bhavānī satya". End: "iti Śrīmad-Gaṇeśa-daivajña-viracitaṁ tat Kaśākhyaṁ yantraṁ sampūrṇam. Ādya-māsa (-e) pare pakṣe trayodaśyāṁ śani-vāsare. Śukācārya-padāmbhoja-Kaśā-yantraṁ śubhaṁ param. Śubhaṁ bhavatu. Kalyāṇam astu. Cha. Cha. Śrī. Cha". According to the colophon of the *Valaya-yantra* in this manuscript, it was copied in Saṁvat 1716 (AD 1660), Māgyha month, white fortnight, 14th *tithi*, Saturday. This is in the Asiatic Society of Bombay, Bombay.

Ms. E : BORI 43 of 1898/99, 13 ff (with the *Yantra-cintāmaṇi* of Cakradhara). The *Pratoda-yantra* is in 12a - 13a. Beginning: "Atha Cābuka-yantram". End: "iti Cābuka-yantram. Śubhaṁ bhavatu. Cha. Śrī. Cha. Saṁvat 1812, Śake 1677 pravarttamāne dvitīya-jyeṣṭha-māse śukla-pakṣe 9 budhe 'yaṁ grantho likhitaḥ". The year Saṁvat 1812 is AD 1755. This is in Bhandarkar Oriental Research Institute, Pune.

Ms. F : SOI 9416, 4 ff (with the *Yantra-cintāmaṇi* of Cakradhara). The *Pratoda-yantra* is in 3b - 4b. Beginning: "Atha Cābuka-yantram". End: "iti Cābuka-yantraṁ samāptam". This is in Scindia Oriental Institute, Vikram University, Ujjain.

Ms. G : LDI 7041 (Accession no. 4038). 1 f. Beginning: "Śrī-Gaṇeśāya namaḥ". End: "iti śrī-Gaṇeśa-daivajña-kṛtaṁ Pratoda-yantram. Saṁvat 1753 varṣe phāgaṇa (phālguna) - vadi 7 dine Nalinagare. Bha. Trīkamajī tā (tal) likhitam". The year Saṁvat 1753 is AD 1697. This is in LD Institute, Ahmedabad.

Ms. H : Ānandāśrama 6673, 18 pages, with Sakhārāma's commentary. Beginning: "Śrī-Gaṇeśāya namaḥ". End: "iti śrīmat-sakala-gaṇaka-cakracūḍāmaṇi-śrī-Gaṇeśa-daivajña-viracitaṁ Pratoda-yantraṁ savivaraṇaṁ samāptam. Anabhyasta-vilekhena śrī-Gaṇādhīśa-sevinā. Sakhārāmeṇa racitaṁ Tarjanī-yantra-ṭippaṇam. Āṣṭegrāma-sthasya [space] Svasti śrī-nṛpa-śālivāhana-śake 1756 Jaya-nāma-saṁvatsare āśvina-kṛṣṇa 8 śanau Marīcī-kṣetre Aṣṭagrāmasya jyotirvid-Gopāla-bhaṭṭa-sūnu-Nārāyaṇa-daivavidā likhitāt pustakān Māhādeva- [ā] tmaja-Bhikū-dīkṣitena likhitam idam. Lekhana-samāptis tu Dvādaśī-kṣetre Śake 1807 śrāvaṇa-śukla 5 indau (25 Julaī 1887) abhavat". This is in Ānandāśrama, Pune.

(ii) The text of the Pratoda-yantra of Gaṇeśa

विष्ण्वर्कमुख्यत्रिदिवेशमूर्तेः
स्मृत्वा गुरोः सच्चरणारविन्दम् ।
प्रतोदयन्त्रं गणकाग्रतुष्ट्यै
वक्ष्ये चमत्कारकरं नृपाणाम् ॥ १ ॥

Apparatus:

(1a:) G: *natvā* for "*viṣṇva*".
G: *tridiśeśa* for "*tridiveśa*".
C: *mūrttiḥ*, D: *pūrvaṁ*, G: *mūrttīḥ* for "*mūrteḥ*".
(1c:) C: *gaṇakāya*, H: *gaṇakāgrya* for "*gaṇakāgra*".

हयाद्यारूढेनापि च विधृतमात्रे ऽत्र स नरे
घटीशङ्कुच्छायादिकसकलबोधः प्रभवति ।
अतो युक्त्या युक्तं नृपसहचरैः प्रौढगणकैः
सदा धार्यं भूपैरपि सुमतिभिस्तर्जनवशात् ॥ २ ॥

Apparatus:

(2a:) C: *'dhyārū*, D, G: *dyāru* for "*dyārū*".
D omits *vi* before "*dhṛta*".
G: *ṇa ca kare* for "*'tra sa nare*".
C : *naro* for "*nare*".
(2b:) B: *vedhaḥ*, C: *yodhaḥ* for "*bodhaḥ*".
(2c:) D: *muktāmuktaṁ* for "*yuktyā yuktaṁ*".
(2d:) D: *tarjuna* for "*tarjana*".
D: *miṣān*, E, F, G, H: *miṣāt* for "*vaśāt*".

यन्त्रं यष्ट्यनुकारि दन्तिरदज वा शिंशवृक्षादिजं
नेम्यां षोडशपट्टिकाः कुरु यथेष्टा वा यथेष्टं समम् ।
दैर्घ्ये ऽथोन्नतभागशङ्कुटचरणच्छायास्त्रिपट्टीष्वतः
शेषासु क्रमशो द्युमानवशतो नाडीः सुधीरङ्कयेत् ॥ ३ ॥

Apparatus:

(3a:) F omits anusvāra after "*yantra*".
C: *datti* for "*danti*".
A, B: *śiṃsa*, D: *sisu*, E, F, H: *śiṃśi* for "*śiṃśa*".
G: *dārudhātvādijaṃ* for "*śiṁśavṛkṣādijaṃ*".
A, B: *dikaṃ* for "*dijaṃ*".

(3b:) A: *nemyāḥ*, B: *nemyā* for "*nemyāṃ*".
C: *śoḍaśa* for "*ṣoḍaśa*".
B, F, G. omit visarga after "*paṭṭikā*".
A, D, E: *kurū* for "*kuru*".
A: *yatheṣṭaṃ* for "*yatheṣṭā*".
H: *dṛḍhaṁ* for "*samam*".

(3c:) H: *dairghyaṁ* for "*dairghye*".
H: *connata* for "'*thonnata*".
D omits *ta* after "'*thonna*".
G gives "*śaṅku*" twice.
A omits *yā* after "*caraṇacchā*".
D omits *s* after "*chāyā*".
C, G: *paṭṭi*, D, F: *paḍī* for "*paṭṭī*".
D: *ṣṭya* (?) for "*ṣva*".

(3d:) D: *kramato* for "*kramaśo*".
D, G omit visarga after "*nāḍī*".

कुर्यान्मूर्ध्नि सुरन्ध्रमत्र रचयेदाधारसच्छृङ्खलां
तस्याधो नृनिवेशनाय परितश्छिद्राण्यणून्युत्किरेत् ।
गर्भे तानि यथा परस्परमसंस्पृष्टानि कुर्यात्तथा
गर्भान्तर्विदधीत मूर्ध्नि सुषिरं शङ्कोः सदा लुप्तये ॥ ४ ॥

Apparatus:

(4a:) G: *yaṣṭer murdhni* for "*kuryān mūrdhni*".
G omits anusvāra after "*chriṅkhalā*".

(4b:) G: *niveṣaṇāya* for "*niveśanāya*".
F: *pariteḥ* for "*paritaś*".
B, F: *urūṇy*, C: *uruṇy*, D, E: *ajūṇy*, G: *ṛjūṇy* for "*aṇūny*".
C: *utkaret*, F: *uktiret* for "*utkiret*".

(4c:) C: *garbho* for "*garbhe*".
D: *parasphara* for "*paraspara*".
D: *saṁsṛṣṭyāni*, F: *saṁsṛṣṭani* for "*saṁspṛṣṭāni*".
F: *kuyāt* for "*kuryāt*".

(4d:) D: *garbhāṁtaṃ*, G: *garbhāṃtā* for "*garbhāntar*".
G: *viniveśanāya ca tathā* for "*vidadhīta mūrdhni suṣiraṃ*".
C: *muddhni* for "*mūrdhni*".
C omits visarga after "*śaṅko*".
H: *guptaye* for "*luptaye*".

क्षिप्तो रन्ध्रे यन्त्रबाह्ये यथा स्या-
द्यन्त्राङ्गां ६ शासन्नदैर्घ्यस्त्वभीष्टः ।
शङ्कुः कार्यो यन्त्रबाह्यस्थशङ्को-
रर्का १२ शोमानं भवेदङ्गुलस्य ॥ ५ ॥

Apparatus:

(5b:) G: *yaṁtrādyāṁ* for "*yantrāṅgāṁ*".
C, D: *dīrgha* for "*dairghya*".
A, D: *abhiṣtaḥ* for "*abhīṣṭaḥ*".

(5c:) F: *bāhyasya*, G: *bāhye stha* for "*bāhyastha*".

अर्कापमाक्षांशकसंस्क्रियोनाः
खाङ्का ९० द्युमध्योन्नतभागकाः स्युः ।
द्युमानदन्ता ३२ न्तरमङ्क [९] निघ्नं
स्थूला भवेयुः खनव ९० च्युतं वा ॥ ६ ॥

Apparatus:

(6a:) A: *samskrīyo*, C: *saṃskṛ(?) to*, D: *saṃstriyo*, G: *saṃskṛi(sic)yo* for "*saṃskriyo*"
B, H: *nā*, E, F: *na* for "*nāḥ*".

(6b:) a: *svāṃko* for "*khāṇkā*".
B, D, F omit visarga after "*bhāgakā*".

(6c:) E, F: *arka* for "*aṅka*".

(6d:) C: *khana*, D: *khanakha* for "*khanava*".
G: *cyutā* for "*cyutaṃ*".

पूर्णनन्द ९० गुणितेष्टघटीभ्यो
वासरार्धविहृताप्तगुणेन ।
संगुणद्युदलजांशकमौर्व्या-
स्त्रिज्ययाप्तधनुरिष्टलवाः स्युः ॥ ७ ॥

Apparatus:

(7a:) A: *ghaṭi* for "*ghaṭī*".

(7b:) F: *guṇene* for "*guṇena*"

(7c:) C: *saṃguṇā*, G: *saṃguṇajya* for "*saṃguṇa*".
F: *maurvīṃ* (but has "*rvyā*" on its margin), G: *maurvī* for "*maurvyās*".
C, E omit *s* after "*maurvyā*".

(7d:) A: *trijjya* for "*trijya*".
C, G omit visarga after "*lavā*".

तदूनखाङ्का ९० नतभागकाः स्युः
कार्योन्नतांशोद्भवसिञ्जिनी सा ।
सूर्या १२ हताप्ता नतभागमौर्व्या
स्युरत्र यन्त्रे घटिकाङ्गुलानि ॥ ८ ॥

Apparatus:

(8a:) A: *tadūnna* for "*tadūna*".
F: *khākā* for "*khāṅkā*".
C, E omit visarga after "*bhāgakā*".
(8b:) D: *kāyā* for "*kāryo*".
F: *nnatāṃśau* for "*nnatāṃśo*".
G: *saṃjanī* for "*śiñjinī*".
(8c:) G: *jīvayā* for "*maurvyā*"
(8d:) D: *sphur* for "*syur*".

एकादिकेष्टोन्नतभागकेभ्य-
स्तदङ्गुलान्यत्र भवेयुरेवम् ।
एकादिहृद्वेदगजा ८४ भवेयु-
र्यन्त्रे ऽत्र पादद्युतिजाङ्गुलानि ॥ ९ ॥

Apparatus:

(9a:) C, D, F, G: *koṣṭo* for "*keṣṭo*".
B omits *ga* after "*nnatabhā*".
(9c:) F omits *di* after "*ekā*".
F: *vegajā* for "*vedagajā*".
G: *gaje* for "*gajā*".
D omits *r* after "*bhaveyu*".

एकादिभिर्भाजितवेदशक्राः १४४
स्युरत्र यन्त्रे नरभाङ्गुलानि ।
एकादिनाडीषु च पट्टिकासु
रन्ध्रान्निजोत्थाङ्गुलकैः सताङ्क्यम् ॥ १० ॥

Apparatus:

(10a:) B omits visarga after "*śakrā*".
(10a-b:) D wholly omits vs. 10a-b.
(10c:) A: *nāḍayāsu*, B: *nāḍīsu*, D: *nāḍyādi*, E, F: *nāḍyāsu*, H: *nāḍyopi* for "*nāḍīṣu*".
G: *nāḍyādika* for "*nāḍīṣu ca*".
C: *paṭṭī* for "*paṭṭi*".
(10d:) E, F: *raṃdro nijo* for "*randhrān nijo*".
B: *chrā* for "*tthā*".
C: *kai*, E, F: *kau* for "*kaiḥ*".
D: *dhiyā* for "*satā*".
H: *ṃkyāḥ* for "*ṃkyam*".

क्षिप्त्वाथो निजपट्टिकोत्थसुषिरे शङ्कुं प्रधार्यं यथा
यन्त्रं शृङ्खलिकादिना नरभवच्छाया स्वट्टयां पतेत् ।
छायाग्रावधि रन्ध्रतश्च गणयेन्नाडीस्तथेष्टांशकान्
यन्त्राग्रान्नरभावधीह गणयेत्पादप्रभां शङ्कुभाम् ॥ ११ ॥

Apparatus:

(11a:) A: *kṣiptvāyaṁ*, B, C, E, F: *kṣiptāyā* for "*kṣiptvātho*".
F: *padvikā* for "*paṭṭiko*".
B: *kosya*, D: *kotha* for "*kottha*".
F: *ricare* for "*suṣire*".
C, D, E, F omit anusvāra after "*śaṅku*".
G: *dhāryāṁ* for "*dhāryaṁ*"
H: *tathā* for "*yathā*".
(11b:) A, H: *śṛṁkhala* for "*śṛṅkhali*".
G: *layā dṛḍhaṁ (?)* for "*likādinā*".
A: *narabhā* for "*narabha*".
A: *svapaṭyāṁ*, C: *tvapaṭyāṁ*, D: *syapaṭyāṁ*, G: *dyumāne* for "*svapa ṭṭyāṁ*".
(11c:) D: *nāḍis* for "*nāḍīs*".
G: *ṣṭīṁśa* for "*ṣṭāṁśa*".
(11d:) C: *atrā*, D: *yatrā* for "*yantrā*".
B: *grā* for "*grān*".
E, F: *valī* for "*vadhī*".
B omits anusvāra after "*prabhā*".

रन्ध्रादभीष्टघटिकावधि या शलाका
संस्थाप्य तामपि नृभादिकपट्टिकासु ।
प्रागवत्तदग्रत इहेष्टघटीसमुत्था
शङ्कुप्रभेष्टलसवपुंश्चरणप्रभा वा ॥ १२ ॥

Apparatus:

(12a:) E: *yaṃtrād*, F: *radhrād* for "*randhrād*".
D: *bhiṣṭa* for "*bhīṣṭa*".
F makes gap for "*ṭikāva*".
F: *ṇa* for "*yā*".
(12b:) E, F, G: *bhāvadhi* for "*bhādika*".
(12c:) F: *tagrata* for "*tadagrata*".
A, D: *tthāḥ*, H: *ththā* for "*tthā*".
(12d:) C: *śaku* for "*śaṅku*".
B: *pūṁś*, F: *yuś* for "*puṁś*".
C: *nā*, D: *vāḥ* for "*vā*".

नन्दिग्रामनिवास्यभूद् द्विजवरो दैवज्ञचूडामणि-
र्नानाशास्त्रकलाकलापचतुर: श्रीकेशवस्तत्सुत: ।
तत्पादाम्बुजसेवनाप्तनिगमज्ञानो गणेश: कृती
चक्रे यन्त्रमिदं चमत्कृतिकरं भूपादिकानां स्फुटम् ॥ १३ ॥

Apparatus:

(13a:) G: *naṃdī* for "*nandi*".
C: *nivasvi*, E, F, G: *nivāsa* for "*nivāsya*".

A, B, C, D, E, F omit *d* after "*abhū*".
G: *bhūr* for "*bhūd*".
D omits *r* after "*maṇi*".
(13b:) G: *kuśalaḥ* for "*caturaḥ*".
D omits visarga after "*catura*".
(13c:) C: *dāvuja* for "*dāmbuja*".
A, B, H: *sevayā* for "*sevanā*".
B omits visarga after "*gaṇeśa*".
C: *kṛtaṁ(?)*, D: *kṛti* for "*kṛtī*".
(13d:) F makes gap for "*cakre*".
G: *sphaṭaṁ* for "*sphuṭam*".

(iii) An English translation of the Pratoda-yantra of Gaṇeśa

1. Remembering the lotus-like feet of my preceptor who is an incarnation of Viṣṇu, the best of heavenly gods or the lord of the heaven, I shall explain the *pratoda-yantra*, which makes kings get surprised, for the satisfaction of the best of mathematicians.

2. Even if a man is riding on a horse, he gets the complete knowledge of time, gnomon-shadow etc., when he is holding [this instrument]. Hence [this instrument] should always be possessed skilfully and rightly by the associates of the king, experienced mathematicians, and also by wise kings just like [a whip] to control [a horse].

3. The stick-like [body of the] instrument [should be] made of ivory or *śiṁśa*-timber. On its surface, make 16 [vertical] columns, or of any desired number according to necessity, equally. Along its length, a wise man should mark the degree of the altitude, the [vertical] gnomon's shadow (*śaṅkucchāyā*), and the man's shadow in terms of feet (*caraṇacchāyā*), on three columns , and then *nāḍīs*, according to the length of daytime, on other columns regularly.

Note: The *caraṇacchāyā* is a man's shadow measured by his own feet. It was used in order to know rough time in ancient India. According to the above verse, the *nāḍīs* are marked on 13 columns. If they are marked for every half solar month from the winter solstice to the summer solstice, 13 columns are required.

4. A fine hole should be made at the top, and a good chain for supporting should be attached there. Below it, fine holes should be made all round (i.e. on all columns) in order to put the gnomon in such a way that they do not meet each other inside. At the top, a hole should be made in order to hide the gnomon inside always.

5. One should make a gnomon, which is to be attached to the hole, in such a way that the length of its exterior part is about one sixth of the instrument or of desired length. One twelfth of the length of the exterior part of the gnomon is the unit of *aṅgula*.

6. The formation (sum or difference) of the declination of the sun and the [observer's] latitude, subtracted from 90°, is the midday altitude of the sun. Otherwise,

the difference between the length of daytime and 32 is multiplied by 9, and subtracted from 90. This is gross.

Note: If the declination is south, the sum of the declination of the sun and the observer's latitude is the zenith distance of the midday sun. If the declination is north, the difference between the declination of the sun and the observer's latitude is the zenith distance of the midday sun. The zenith distance subtracted from 90° is the altitude. Hence the above first rule.

The second rule is the method to obtain gross midday altitude of the sun. This method can be expressed as follows. Let *a* be the sun's altitude, and *d* the length of daytime in terms of *nāḍīs*. Then,

$$a = 90 - \{|32 - d| \times 9\}.$$

It appears that Gaṇeśa assumed that the midday altitude of the sun is 90° when the length of daytime is 32 *nāḍīs*, and that the observer's latitude is 18°, because the zenith distance of the midday sun at the equinoctial day becomes (32 - 30) x 9 = 18° by this rule. This latitude agrees with the latitude of Nandigrāma where Gaṇeśa resided, which is about 40 miles to the south of Bombay according to S.B. Dikshit.

7. The *iṣṭa-ghaṭīs* (*ghaṭīs* elapsed since sunrise in the forenoon, or to elapse until sunset in the afternoon) is multiplied by 90 and divided by a half of the length of daytime. The R-sine of the result is multiplied by the R-sine of the degrees of the midday [altitude of the sun], and divided by the Radius. Its corresponding arc is the *iṣṭa-lavas* (degrees of the sun's altitude corresponding to the *iṣṭa-ghaṭīs*).

Note : The above method can be expressed as follows. Let *a* be the sun's altitude, *m* the midday altitude of the sun, *t* the *iṣṭa-ghaṭīs*, and *d* the length of daytime in terms of *ghaṭīs*. Then,

$$R \,.\, \sin a = \frac{R.\sin\left(\frac{90\,t}{d/2}\right) \times R.\sin m}{R}.$$

This method is the same as the method given by Hema in his *Kaśā-yantra* (vs. 23).

8. Ninety minus this (altitude) is the zenith distance. The R-sine of the altitude is multiplied by 12, and divided by the R-sine of the zenith distance. Then the result is the [length of the shadow in terms of] *aṅgulas* on the instrument, corresponding to the *ghaṭikās*.

Note: This is the method to calculate the length of the vertical shadow cast by the horizontal gnomon of 12 *aṅgulas*. This method can be expressed as follows. Let 's' be the length of the shadow in terms of *aṅgulas*, and *a* the altitude of the sun. Then,

$$s = \frac{12 \times R.\sin a}{R.\sin(90 - a)}.$$

This method is the same as the method given by Hema in his *Kaśā-yantra* (vs. 24).

9. Thus, by the *iṣṭa-unnata-bhāga* (the sun's altitude corresponding to the *iṣṭa-ghaṭīs*), numbered one etc., the [length of the shadow in terms of] *aṅgulas* will be

obtained. Eighty four divided by one etc. are *aṅgulas* on this instrument corresponding to the man's shadow in terms of feet (*pāda-dyuti*).

Note: The second half of this verse is the method to obtain the man's shadow in terms of his own feet (*pādas*). The rationale of this method is as follows. Let s be the length of the shadow of 12-*aṅgula* horizontal gnomon, h the height of the man in terms of feet, f the length of the man's shadow in terms of feet, and a the sun's altitude. Then,

$$s = \frac{12 \times R.\sin a}{R.\sin(90 - a)}.$$

$$f = \frac{h \times R.\sin(90 - a)}{R.\sin a}$$

Therefore,

$$s = 12h/f.$$

According to the above verse, it appears that Gaṇeśa has taken the value of h as 7, because (12x7) = 84. Thus,

$$s = 84/f.$$

Substituting 1, 2, 3, etc. into f, the corresponding value of s can be obtained.

10. One hundred forty four divided by one etc. are *aṅgulas* on this instrument corresponding to the [vertical] gnomon's shadow (*nara-bhā*). A wise man should graduate by the obtained *aṅgulas* for each *nāḍī*, numbered one etc., on each column from the hole (i.e. the gnomon's root) [downwards].

Note: The first half of this verse is the method to calculate the length of the shadow of the horizontal gnomon corresponding to the shadow of the vertical gnomon. The rationale of this method is as follows. Let s be the length of the shadow of the horizontal gnomon whose length is g, v the length of the shadow of the vertical gnomon whose height is g, and a the sun's altitude. Then,

$$s = \frac{g \times R.\sin a}{R.\sin(90 - a)}.$$

$$v = \frac{g \times R.\sin(90 - a)}{R.\sin a}.$$

Therefore,

$$s = g^2 / v.$$

If g is 12 as usual,

$$s = 144/v.$$

Substituting 1, 2, 3, etc. into v, the length of the shadow of the horizontal gnomon corresponding to the shadow of the vertical gnomon can be obtained.

11. Then, inserting the gnomon into the hole of the proper column, the instrument should be held by the chain etc. in such a way that the gnomon-shadow falls on the column. One should count *nāḍīs* or the *iṣṭāṁśaka* (degrees of the sun's altitude corresponding to the *iṣṭa-ghaṭīs*) from the hole upto the tip of the shadow. One should measure [the length of] the man's shadow in terms of feet (*pāda-prabhā*) and the [vertical] gnomon's shadow (*śaṅku-bhā*) from the top of the instrument (the root of the gnomon) upto the tip of the gnomon-shadow.

12. Placing also a stick, which [is of the length] from the hole upto [the mark of] the *iṣṭa-ghaṭīs*, on the column for the [vertical] gnomon's shadow etc. also, [one should measure], from its tip as before, the [vertical] gnomon's shadow (*śaṅku-prabhā*) corresponding to the *iṣṭa-ghaṭīs* or the man's shadow in terms of feet (*puṁś-caraṇa-prabhā*) corresponding to the *iṣṭa-lavas* (the sun's altitude).

Note: The vertical gnomon's shadow in terms of *aṅgulas* and the man's shadow in terms of feet are graduated on respective columns. So, their values can be read by placing a stick, whose length is equal to the shadow of the horizontal gnomon, along the graduation from the hole. Of course, the value can directly be read by observing the shadow itself, if the horizontal gnomon is fixed to the hole of respective columns.

13. There was the best twice-born astronomer Śrī Keśava, dweller of Nandigrāma, who was well versed in the techniques of various sciences and arts. His son, clever Gaṇeśa, who acquired the knowledge of the science by the service of the lotus-like feet of his father, constructed this accurate instrument which astonishes the king etc.

(iv) *Remarks*

It appears from the text that the *pratoda-yantra* of Gaṇeśa is quite similar to the *kaśā-yantra* of Hema. There are, however, some differences also. Hema wrote to make 7 columns, but Gaṇeśa wrote to make 16 columns. Hema recommended gold, silver, brass, copper, or *śiṁśa*-timber as its material, but Gaṇeśa recommended ivory or *śiṁśa*-timber. Hema wrote to hang it by silk thread, but Gaṇeśa wrote to hang it by chain etc. Hema wrote to mark the *lagnas* on the column besides the time, but Gaṇeśa wrote to mark the gnomon-shadow and the man's shadow besides the time.

As there are many differences, it may be that Gaṇeśa was not directly influenced by Hema. However, there must have been at least indirect relationship between them, because there are many similarities, such as the name of the instrument which means "whip instrument", the hole for hiding the gnomon at the top of the instrument when it is not in use, etc.

The Cylindrical Sundial Described by Nityānanda and Munīśvara

(i) *Introduction*

Some important Siddhāntas were composed in the 17th century, namely, the *Siddhānta-sindhu* (AD 1628) and the *Siddhānta-rāja* (AD 1639) of Nityānanda, the *Siddhānta-sārvabhauma* (AD 1646) of Munīśvara, and the *Siddhānta-tattvaviveka*[36] (AD 1658) of Kamalākara.

Nityānanda resided in Indrapurī (= Delhi) under the reign of Shāh Jahān. He composed the Sanskrit version of the Shah Jahan Table (*Zīca-nityānandī Śāhajahānī*)[37] also. The Persian version of this table is known as the *Zīj-i shāhjahānī* of Farīd ud-Dīn, which is based on the *Zīj-i Ulugh Beg*.[38] Munīśvara also seems to have obtained the patronage of Shāh Jahān.[39] Kamalākara[40] wrote his work at Varanasi, and was a contemporary rival of Munīśvara.

Among the above mentioned Siddhāntas, the *Siddhānta-rāja* and the *Siddhānta-sārvabhauma* have the chapter of astronomical instruments. (I regret that I have not seen any manuscript of the *Siddhānta-sindhu*). Both Nityānanda and Munīśvara described the cylindrical sundial in their works.

(ii) *The cābuka-yantra in the Siddhānta-rāja of Nityānanda*

Nityānanda,[41] son of Devadatta, wrote the *Siddhānta-rāja* in AD 1639. It has not been published, but its manuscripts are extant.[42] It consists of two parts, i.e. the *Gaṇita-adhyāya* and the *Gola-adhyāya*. The *Gola-adhyāya* includes the chapter of astronomical instruments, i.e. the *Yantra-adhyāya*. The *Yantra-adhyāya* is subdivided as follows.

(1) *Gaṇita-adhyāya*,
(2) *Ghaṭana-adhyāya*,
(3) *Racana-adhyāya*,
(4) *Śodhana-adhyāya*,
(5) *Yantra-nirīkṣaṇa-adhyāya*.

It appears that Nityānanda mainly followed the *Yantra-rāja* of Mahendra Sūri regarding the naming of the subdivision. Nityānanda explained the principle, construction, and application of the astrolabe (*yantra-rāja*) in this *Yantra-adhyāya*. He also mentioned other instruments, viz. the *turīya-yantra*, the *yantra-cūḍāmaṇi*, and the *cābuka-yantra*, in the last section.

The *turīya-yantra* is the quadrant, and the time is determined from the observation of the altitude. The *yantra-cūḍāmaṇi* is a circular instrument, and also used for determination of the time. The *cābuka-yantra* is a cylindrical sundial, which has been described by Hema and Gaṇeśa Daivajña also

Nityānanda explained the *cābuka-yantra* in the *Yantra-adhyāya, Yantra-nirīkṣaṇa-adhyāya* (vss. 74-75 of AS Bombay 264 (BD 311), and vss. 146-147 of BORI 206 of

A. 1883/84) as follows.[43] I follow the readings of AS Bombay manuscript, because BORI manuscript has some defects here.

अथेष्टनाडीप्रभवोन्नतांश-
जातां प्रभां वा विगणय्य धीमान् ।
यावद्दिनार्धं घटिका[:] प्रकोष्ट (-ष्टे)
लिख्या: पृथक् चाबुकयन्त्रदण्डे ॥ ७४ ॥
चाबुकस्य किल वेधविधानं
सर्वलोकविदितं बहुधात्र ।
तेन विस्तरमपास्य यदुक्तं
तत्स्वबुद्धिवशत: प्रथयन्तु ॥ ७५ ॥

Apparatus:

(74b:) BORI Ms. : *jyatāṃ* for "*jātāṃ*".
(74b-c:) BORI Ms. wholly omits "*vā*"......*prakoṣṭe*".
(74d:) BORI Ms. : *cāvaka* for "*cābuka*".
(75a:) BORI Ms.: *cāvaka* for "*cābuka*".

74. "Now, the wise man, calculating the gnomon-shadow which is produced by the altitude [of the sun] corresponding to the desired *nāḍīs* should separately graduate a half-day's *nāḍīs* on the rod of the *cābuka-yantra*".

75. "Since the method of the observation of the *cābuka* is known to the whole people in many ways, its detail is omitted here. The aforesaid matter should be supplemented by one's own intelligence".

From the above statement, it appears that the *cābuka-yantra* (whip instrument) was quite well known at his time.

(iii) *The pratoda-yantra in the Siddhānta-sārvabhauma of Munīśvara*

Munīśvara[44] (b. AD 1603), son of Raṅganātha, wrote the *Siddhānta-sārva-bhauma* in AD 1646. It consists of two parts, i.e. the *Pūrvārdha* and the *Uttarārdha*. The *Pūrvārdha* has been published.[45] The *Uttarārdha*, which includes the *Yantrādhyāya*, has not been published, but its manuscripts are extant.[46] The following instruments have been described in the *Yantrādhyāya*. The *cakra, phalaka, yaṣṭi, pratoda, cakranibha, dhanus* and *buddhi*.

The description of the *pratoda-yantra* by Munīśvara is a summary of the description of the same instrument by Gaṇeśa Daivajña as we already have discussed under the section of Gaṇeśa. Munīśvara has mentioned the name of Gaṇeśa in the text (vs. 63). The extract of the description of the *pratoda-yantra* by Munīśvara was sometimes separately copied, and was published by Shakti Dhara Sharma.[47] (For the front cover of Sharma's book, see Fig. 4.)[48] I shall quote its text here, and append my English rendering and comments about its relationship with earlier works. (For the full text of its Sanskrit commentary and detailed English exposition, see Sharma's book).

(Sanskrit text):

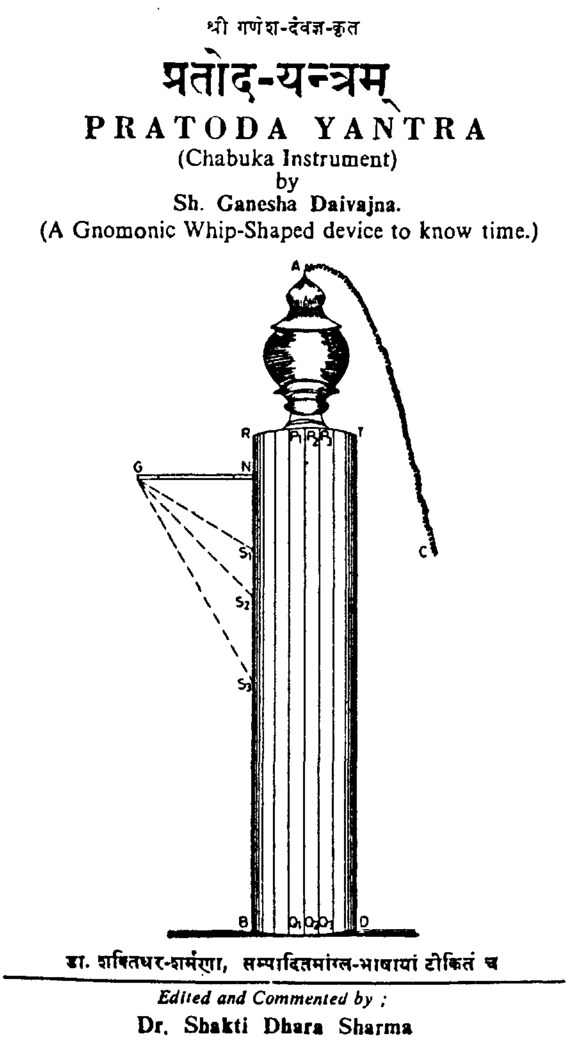


Fig. 4 The front cover of S.D. Sharma's edition of the Pratoda-yantra

गणेशोदितं यन्त्रमेतत्प्रतोद-
मनायासकालावबोधं प्रवच्मि ।
निजेच्छावशादिष्टदैर्घ्यः सुदण्डो
ऽनतिस्थूलकः शिंशिवृक्षादिजातः ॥ १ ॥

परात्पाहमानापरद्युप्रमाणं
यया संख्यया तत्प्रमित्यस्त्रकैः सः ।
समस्थानदैर्घ्यो विधेयो ऽस्य शीर्षे
सदा शृंखला धारणार्थं नियोज्या ॥ २ ॥

आधारतो ऽधः परितः समस्य
दैर्घ्यादिभागे नरवेशनार्थम् ।
छिद्राणि कार्याणि तथा यथा ऽत्र
स्पृष्टानि गर्भे न भवन्ति तानि ॥ ३ ॥

शीर्षे छिद्रं गर्भमध्ये विधेय-
माधारान्ते शङ्कुलोपार्थमस्मिन् ।
पूर्वच्छिद्राद् बाह्यतः शङ्कुमानं
यन्त्राङ्गांशासन्नमेव नरः स्यात् ॥ ४ ॥

बाह्यस्थशंक्वर्कलवः प्रतोद-
यन्त्रे भवेदङ्गुलमानमस्मिन् ।
पूर्वोक्तरीत्येष्टघटीनतांशो-
न्नतांशजीवे सुधिया प्रसाध्ये ॥ ५ ॥

सूयघ्नयुन्नतलवमौर्विका नतज्या-
संभक्ताभिमतघटीभवाङ्गुलानि ।
एकादिष्वभिमतनाडिकोन्नतेषु
रन्ध्रात् स्वाङ्गुलकमितिः समस्थले ऽङ्क्या ॥ ६ ॥

स्वाहर्मानसमस्थलस्य सुषिरे शङ्कुर्निधेयस्तथा
यन्त्रे शृंखलिकाधृते पतति तद्भा तत्समस्थानके ।
छायाग्रावधि रन्ध्रतश्च गणयेन्नाडीर्दिनाद्यार्धके ।
याताः शेषमिताः परत्र यदि भा मध्ये ऽनुपातो भवेत् ॥ ७

दण्डो भूमिस्तत्र सूर्योदये स्याद्
भा ऽभावो भा ऽनन्तरूपा खमध्ये ।
तस्माद् दृग्ज्यारूपशङ्कोर्नृरूप-
दृग्ज्यातो भा या ऽङ्गुलान्युक्तरीत्या ॥ ८ ॥

Apparatus: The verse 8 c-d in S.D. Sharma's edition is as follows.

तस्याः दृग्ज्या-शंकोर्नृरूप-दृग्ज्यातो भायामंगुलान्युक्तरीत्या ॥

This reading is unmetrical and must be wrong. My reading is attested by the photograph of manuscripts appended to S.D. Sharma's edition.

(English translation):

1. I describe this *pratoda-yantra*, which was told by Gaṇeśa, with which time is known easily. [One should make] an excellent rod made of *śiṁśi*-timber etc., which is not too thick [and not too thin], whose length is to one's wish according to one's desire.

Note: For the material of the rod, see Hema's *Kaśā-yantra* (vs. 4) and Gaṇeśa's *Pratoda-yantra* (vs. 3). For the size of the rod, see Hema (vss. 2-3) and also *Gaṇeśa* (vs. 5)

2. [One should make] uniform columns [which are divided] by edges whose number is the same as the difference between the longest daytime and shortest daytime [in terms of *nāḍīs*]. At its top, a chain should always be attached in order to hang it.

Note: Hema (vs . 4) writes to make the rod septangular, and Gaṇeśa (vs. 3) wrote to make 16 columns or of any desired number. Gaṇeśa writes to use 3 columns as additional columns and use other columns for the graduation of *nāḍīs*. Hema (vs. 50) also suggests to add one additional column and make 8 columns as a whole. Hema (vs. 11) writes to hang it by silk thread, and Gaṇeśa (vs. 4) by chain.

3. Under the supporter [to which the chain is attached], towards all the directions(i.e. on all the columns), holes should be made at the upper portions of the columns at the same height, in such a way that they do not touch each other in the middle.

Note: For the holes, see Hema (vs. 5) and Gaṇeśa (vs. 4).

4. A hole should be made towards the middle at the top [of the rod], near the supporter, in order to store the gnomon. The gnomon should be made in such a way that the length of its portion appearing outside the previous hole (i.e. the hole mentioned in vs. 3) is about one sixth of the instrument (i.e. rod).

Note: For the hole to store the gnomon, see Hema (vs. 17) and Gaṇeśa (vs. 4). For the size of the gnomon in contrast with the rod, see Hema (vs. 2) and Gaṇeśa (vs. 5). Gaṇeśa writes that the appearing portion of the gnomon should be about one sixth of the rod or of desired length.

5. One twelfth of the outer portion of the gnomon is the length of *aṅgula* in this *pratoda-yantra*. A wise man should determine the R-sine of the [sun's] zenith distance and altitude corresponding to desired *ghaṭīs* by the method explained previously (i.e. in the *Tripraśnādhyāya* of the *Siddhānta-sārvabhauma*).

Note: For the measurement of *aṅgula*, see Hema (vs. 2) and Gaṇeśa (vs. 5). For the method to obtain the sun's altitude (or its R-sine), see Hema (vss. 22-23) and Gaṇeśa (vss. 6-7).

6. The R-sine of the [sun's] altitude multiplied by twelve and divided by the R-sine of the [sun's] zenith distance is the *aṅgulas* [of the vertical shadow of the horizontal gnomon] according to desired *ghaṭīs*. On the column, the position of the *aṅgulas*, which corresponds to the altitude of desired *nāḍikās* beginning with one, should be marked below the hole [for inserting gnomon].

Note: The above method is the same as Hema (vs. 24) and Gaṇeśa (vs. 8).

7. The gnomon should be inserted to the hole on the column corresponding to the length of daytime [of the day], and the instrument should be hung by the chain in such a way that its (gnonon's) shadow falls on the column. From the hole to the tip of the shadow, one should count *nāḍīs* elapsed in the forenoon or remaining in the afternoon. If the [tip of] shadow is at the middle [of two marks], [apply] the proportion (rule of three)

Note: For the method of observation, see Hema (vss. 11-16) and Gaṇeśa (vss. 11-12).

8. Here, the rod is the base. At sunrise, there is no shadow, and when [the sun is] at the zenith, the shadow is infinite. Therefore, [in contrast with the vertical gnomon], consider the R . sine of the altitude as the R . sine of the zenith distance, and vice versa, [and calculate] the *aṅgulas* of the shadow by the method which was already explained.

Note: For the relationship between the vertical gnomon's shadow and the horizontal gnomon's shadow, see Gaṇeśa (vs. 10).

The Origin of the Indian Cylindrical Sundial

We have seen some Sanskrit texts on the cylindrical sundial. Let us discuss about its origin. The theory of gnomon was quite advanced in the Classical Siddhānta period in India. So, it is possible that it was invented in India. On the other hand, the cylindrical sundial is also found in Arabic texts etc. Therefore, it is also possible that the cylindrical sundial was introduced from the Islamic world.

Let me briefly review the portable sundials described in some Western works first. Famous Roman architect Vitruvius (dead after AD 27) wrote in his *De Architectura* (IX. 8.1) as follows.

> " ...... Many have also left us written directions for marking dials of these kinds for travellers, which can be hung up. ....." (Translated by Morris Hicky Morgan).[49]

The construction of this traveller's dial is not clear from this text. In connection with this text, Hermann Diels refers to a specimen of traveller's dial which was found in Forbach (in Lorraine, near the border of France and Germany, France). (For its figure shown in Diels's book, see Fig. 5).[50] And also Diels shows a figure of a Roman hanging dial, whose shape is just like a piece of ham, which was found in Portici (near Napoli, Italy) in 1755. (For its figure shown in Diels's book, see Fig. 6).[51] It is made of bronze, and the length of the shadow of horizontal gnomon on the vertical plane has been marked for different seasons. The plane is directed towards the sun, and the time is read from the graduation. This is quite interesting in our context, because its principle is the same as that of the cylindrical sundial. According to Hugh Godfrey and Robert Theodore Gunther,[52] this Roman Ham dial was excavated at Herculaneum under the Vesuvian muds of the eruption of AD 79, and is marked with the months of July and August and must therefore be more recent than 27 BC.

There is a specimen of a Saxon sun-dial, which is a vertical dial with a horizontal gnomon, which was made in the 9th or 10th century. It was found at Canterbury Cathedral in 1939. It has six columns for 12 months, each of which has marks of "tides" at noon, 9 am, and 3 pm. (For its figure of one side given in Derek J. Price's article, see

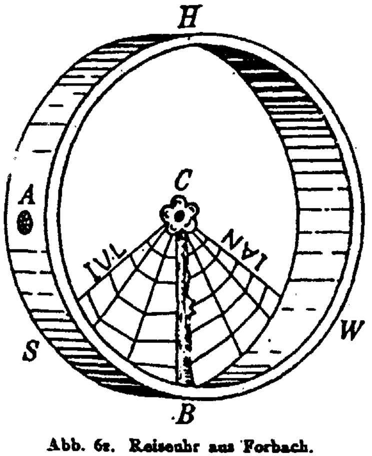


Fig. 5 The traveller's dial found in Forbach (from Diels : Antike Technik)

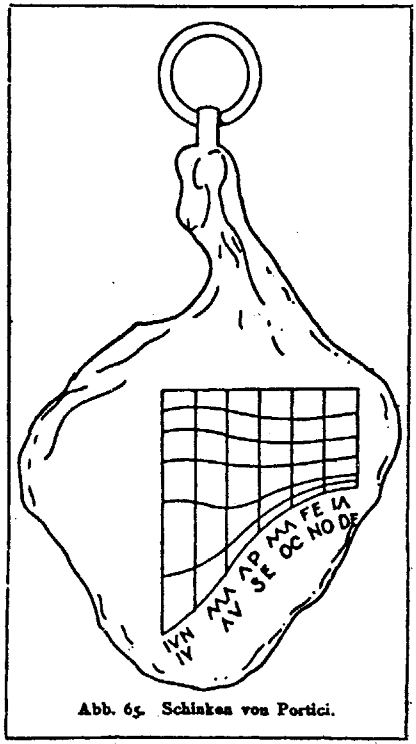
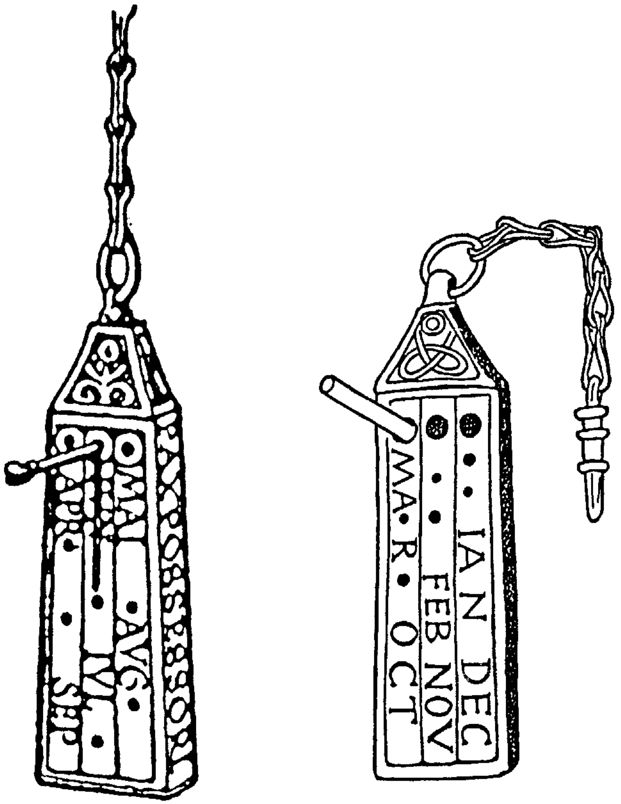


Fig. 6 The Roman hanging dial found in Portici (from Diels : Antike Technik)

Fig. 7(a). And for rough sketch based on the photograph of the other side in S.A. Goudsmit and R.Claiborne's book, see Fig. 7(b)).[53]



(a) (b)

Fig.7 The Saxon sundial found at Canterbury Cathedral

(a) from A History of Technology;

(b) rough sketch based on the photograph in Goudsmit and Claiborne's book

There is another similar Syrian sundial made in 554 AH (= 1159/1160 AD) for Sultan Nūr al-Dīn. It has 12 columns. (For its figure given in Paul Casanova's article, see Fig. 8).[54]

The above mentioned two sundials may be a kind of forerunner of the cylindrical sundial. According to H. Godfrey and R.T. Gunther,[55] the earliest description of the cylindrical sundial is by Hermannus Contractus (1013-1054 AD).

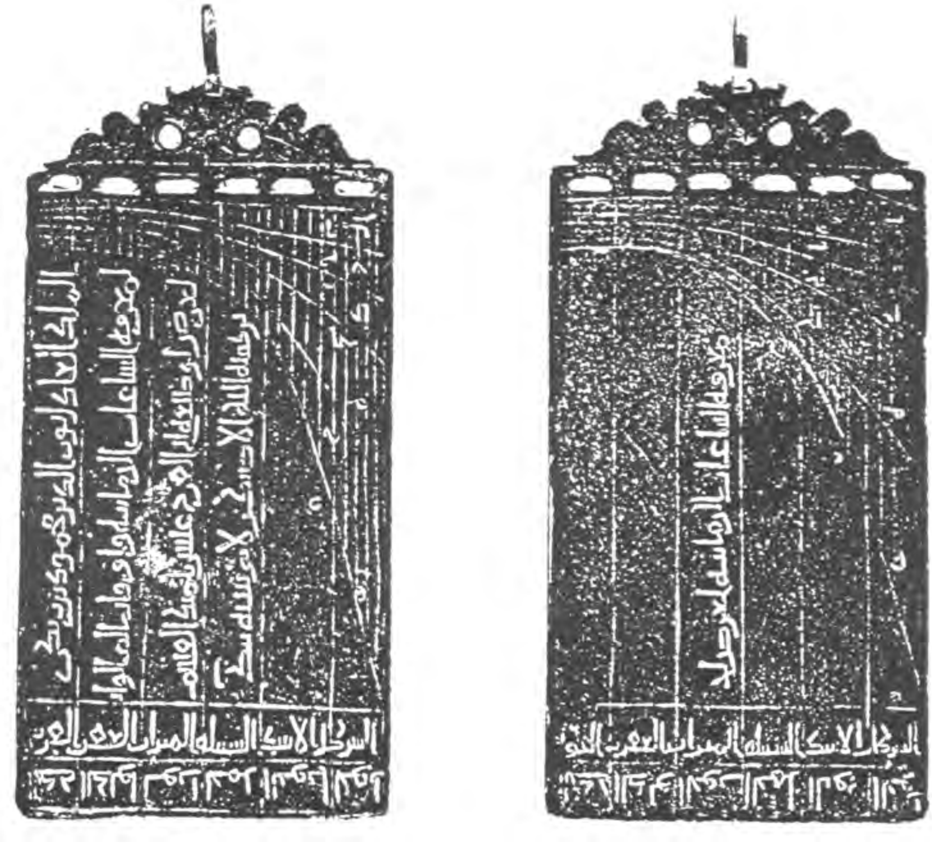


Fig. 8 The Syrian sundial made for Nūr al-Dīn (from Casanova's paper)

The science of the sundial was quite developed by Arab astronomers. In the 13th century AD, an Arab astronomer Abul-Ḥasan wrote the *Jāmi 'al-mabādi' wa-l-ghāyāt fī 'ilm al-mīqāt'* which is well known by its French translation by J.J. Sédillot and L.A. Sédillot under the title of *Traité des instruments astronomique des Arabes*.[56] In this book, Abul-Ḥasan described some types of sundials where the time is obtained by the length of the shadow, such as a horizontal board with a vertical gnomon, a cylinder with a horizontal gnomon, a vertical board with a horizontal gnomon, a cone with a horizontal gnomon etc. These types of the sundial are movable, and are directed towards the sun so that the shadow of the gnomon falls on the proper graduation. Among them, the cylindrical type is interesting in our context, because it is similar to the *kaśā-yantra* of India. According to Abul-Ḥasan, the cylinder is divided into vertical columns for each zodiacal sign where the sun stays, and the gnomon is put at the top of the column horizontally. Under the root of the gnomon, the length of its shadow is marked for each hour. (For its development given in Sédillot's book, see Fig. 9. The development is already given in Arabic manuscript of the original work).[57] After describing these types of the sundial, Abul-Ḥasan also described several types of the sundial which are mounted horizontally, vertically, or on the plane of the equator, where the dial is a plane, a convex cylinder or cone, or a concave hemisphere.

The cylindrical sundial was popular in Europe also. It was called "chilindre . "shepherd's dial" etc. For example, famous Geoffrey Chaucer (ca. 1343/44 .... 1400) wrote in *The Canterbury Tales, The Shipman's Tale* as follows.

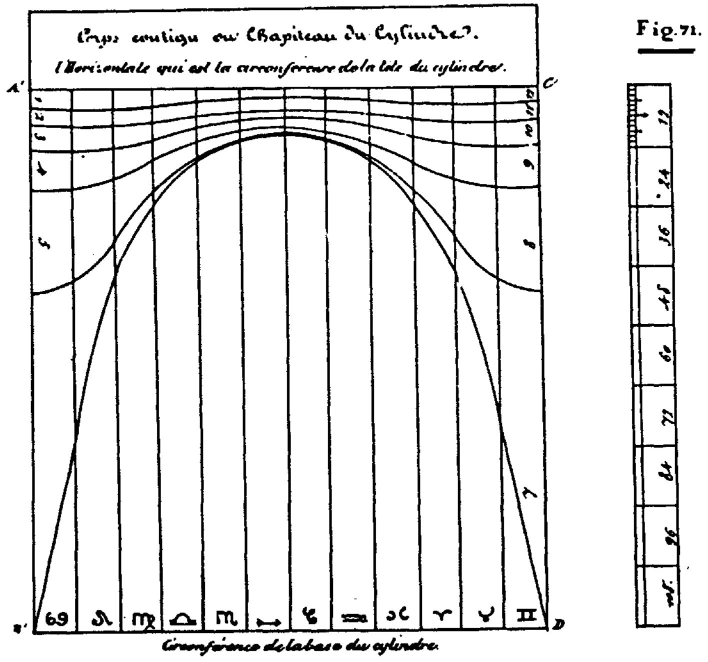
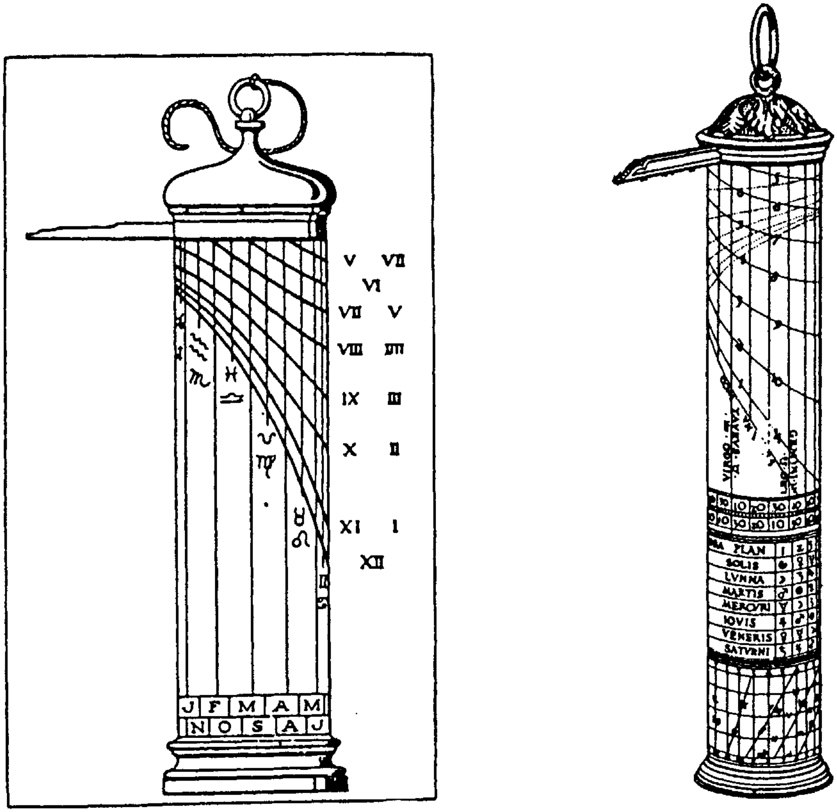


Fig. 9 Abul-Ḥasan's development of cylindrical sundial (from Sédillot's translation)

"'Gooth now youre wey', 'quod he, 'al stille and softe,
And lat us dyne as soone as that ye may;
For by my chilyndre it is pryme of day.
Gooth now, and beeth as trewe as I shal be'".[58]

According to Derek J. Price,[59] the earliest extant example of the cylindrical sundial is dated 1455, which is in the National Museum at Munich.

For the figure of "cylinder" or "shepherd's dial" (AD 1531) given in H. Godfrey and R.T. Gunther's article in *The Encyclopaedia Britannica* (14th edition), see Fig. 10(a).[60] And for the figure of the "chilindre" in the Whipple Museum of the History of Science, Cambridge, given in D.J. Price's article in *A History of Technology* (Volume III), see Fig. 10(b).[61] According to Paul Casanova, the cylindrical sundial was still used



(a) (b)

Fig. 10 European cylindrical sundials
(a) from the Encyclopaedia Britannica, 14th ed. (b) from A History of Technology.

in the Pyrénées until recent times. For its figure given in Casanova's article, see Fig. 11.[62]

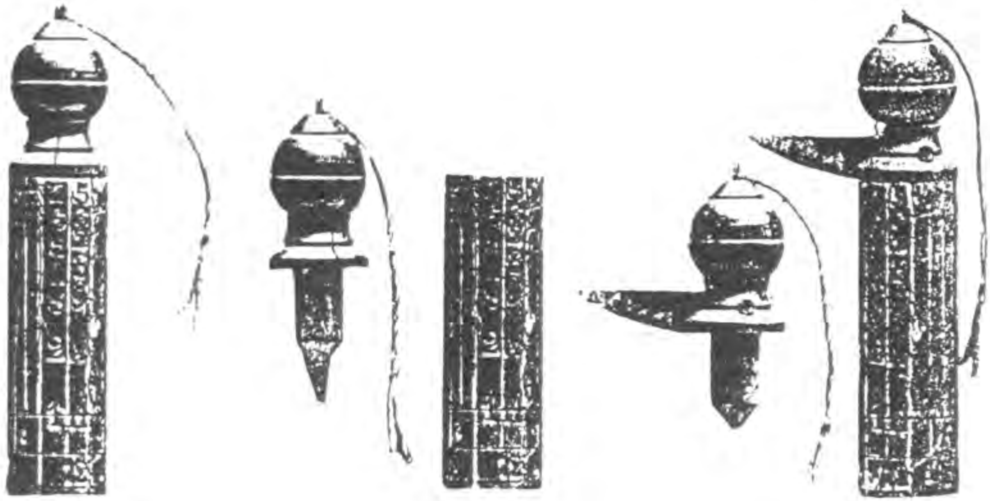
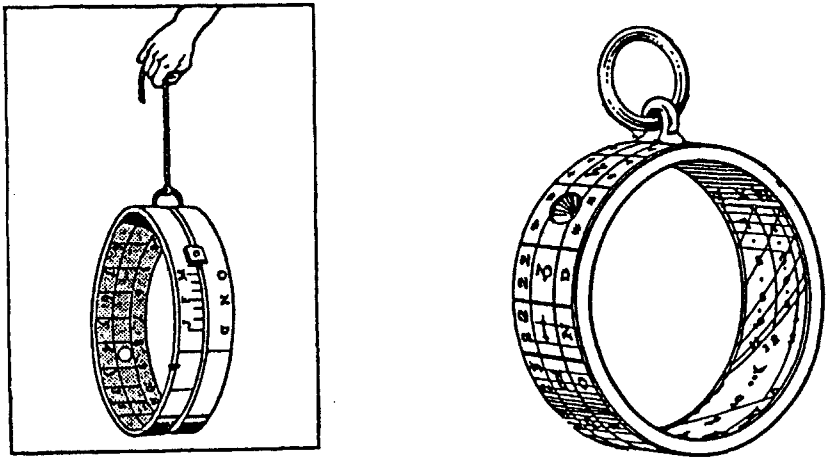


Fig. 11 The cylindrical sundial used in the Pyrénées (from Casanova's paper)

There are also European specimens of the ring dial. For the figure of English ring dial, given in H. Godfrey and R.T. Gunther's article in *The Encyclopaedia Britannica* (14th edition), see Fig. 12(a).[63] And for the figure of the ring dial in the Deutsches Museum, Munich, given in D.J. Price's article in *A History of Technology*, see Fig. 12(b).[64] D.J. Price wrote that the ring dial is "probably not so old as the 'chilindre'".[65] It is not clear to me whether the ring dial mentioned by H.Diels (see Fig. 5) can be traced back to earlier period or not.

According to the above descriptions, the Arabic cylindrical sundial is similar to the Indian *kaśā-yantra*, and Arabic record can be traced back earlier. When Rāmacandra and Hema described the *kaśā-yantra*, the astrolabe had already been known. And also, we have seen that a loan word from Persian, "*cābuka*", occurs in the *Kaśā-yantra* of Hema. Therefore, these authors must have had certain knowledge of Islamic astronomy. So, it is possible to suppose the Islamic influence on the Indian *kaśā-yantra*. However, we should also keep in mind that there is no definite evidence of the Islamic influence on the *kaśā-yantra*.

Although the *kaśā-yantra* itself first appeared in Sanskrit literatures in the 15th century, we have a material which suggests the earlier Indian tradition of the vertical sundial with a horizontal gnomon. We should remember that the vertical sundial with



(a) (b)

Fig. 12 European ring dials
(a) from The Encyclopaedia Britannica, 14th ed. (b) from A History of Technology.

a horizontal gnomon has been described in the *Vṛddha-vasiṣṭha-siddhānta* (III. 61-62)[66] as follows.

अथ दिनदलचिह्नां लम्बितां यष्टिकां वा
तदुपरितनशङ्कुं यष्टिकार्कांशतुल्यम् ।
विरचय तदधऽऽधो (-धोऽधो?) ऽङ्कं विलोमप्रभाभि-
र्दिनगतघटिकैष्याः शङ्कुभा यत्र लग्ना ॥ ६१ ॥
अर्काहतोन्नतज्या नतजीवापप्ता ऽङ्गुलादिका वामम् ।
छाया त्रिज्यार्कहता नतजीवाप्ता च कर्णः स्यात् ॥ ६२ ॥

"Now, a vertical rod (*yaṣṭi*) which has marks of [*ghaṭikās* of] a half-day is made. At its top is a gnomon (*śaṅku*), whose length is one twelfth of the rod. Its below is marked for the reverse shadow. The mark where the gnomon-shadow falls indicates the *ghaṭikās* elapsed or to elapse.

The R-sine of the [sun's] altitude is multiplied by 12, and divided by the R-sine of the zenith distance. The result is [the length of] the shadow in reverse in terms of *aṅgulas*.

The Radius is multiplied by 12, and divided by the R-sine of the zenith distance. The result is the hypotenuse".

The "reverse shadow" means the vertical shadow of the horizontal gnomon, in contrast with the ordinary horizontal shadow of a vertical gnomon.

This text can be explained as follows. (See Fig. 2). Let Z be the R-sine of the zenith distance of the sun, and A the R-sine of the altitude of the sun. Then we have the following proportions.

12: shadow = Z : A, and

hypotenuse : 12 = R : Z.

Therefore, we have

shadow = 12 x A/Z, and
hypotenuse = 12 x R/Z.

These are the formulae given in the text. Unfortunately, the date of the composition of the *Vṛddha-vasiṣṭha-siddhānta* has not been known. At least, we will be able to suppose that the vertical sundial with a horizontal gnomon, whose astronomical principle is the same as that of the *kaśā-yantra*, was used sometime during the Classical Siddhānta period. So, it was theoretically possible to invent the cylindrical sundial in India.

We do not have enough source material to investigate the origin of the *kaśā-yantra* definitely, but it will be justified in saying that it could only be produced during the Delhi Sultanate period when Indian astronomers were quite active in improving their science by studying newly introduced Islamic astronomy as well as their own traditional Hindu astronomy.

According to S. D. Sharma,[67] H.J.J. Winter,[68] and S. R. Sarma,[69] some specimens of South Asian cylindrical sundial are still extent. So, it must have been popular for some time.

ACKNOWLEDGEMENT

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I am grateful to the authorities of the following libraries who allowed me to consult manuscripts. (Their abbreviations used in this paper are shown within brackets).

(Ānandāśrama)	Ānandāśrama, Pune.
(AS Bombay)	The Asiatic Society of Bombay, Bombay.
(AS Calcutta)	The Asiatic Society, Calcutta.
(Baroda)	Oriental Institute, M.S. University, Baroda.

(Benares)	Sarasvati Bhavan, Sampurnanand Sanskrit University, Varanasi.
(Bombay U)	The Library of the University of Bombay, Bombay.
(BORI)	Bhandarkar Oriental Research Institute, Pune.
(LDI)	LD Institute, Ahmedabad.
(SOI)	Scindia Oriental Institute, Vikram University, Ujjain.
(VVRI)	Vishveshvaranand Vishva Bandhu Institute of Sanskrit and Indological Studies, Punjab University, Hoshiarpur.

I would like to express my sincere thanks to the Coordinator Dr. A. K. Bag, of History of Science Programme of Indian National Science Academy, for his arrangement for the publication of this paper, and to the referee of the *Indian Journal of History of Science* for his valuable comments.

References

1. See Ôhashi, Yukio: "Development of Astronomical Observation in Vedic and Post-Vedic India", *Indian Journal of History of Science*, **28**(3), 1993, pp. 185-251.
2. See Ôhashi, Yukio: "Astronomical Instruments in Classical Siddhāntas", *Indian Journal of History of Science*, **29**(2), 1994, pp. 155-313.
3. Ôhashi, Yukio: "Early History of the Astrolabe in India", *Indian Journal of History of Science*, **32**(3), 1997, pp. 199-295.
4. Dikshit, S.B. : *Bharatiya Jyotish Sastra*, translated into English by R.V. Vaidya, Part II, Positional Astronomy Centre, India Meteorological Department, Calcutta (1981), p. 232.
5. Sharma, Shakti Dhara (ed. and tr.): *Pratoda Yantra (Chabuka Instrument) by Sh. Ganesha Daivajna (A Gnomonic Whip-Shaped device to know time)*, Martand Bhavan, P.O. Kurali (Ropar) Punjab (1982). I am grateful to Dr. S.D. Sharma who kindly presented this book to me.
6. Ôhashi, Yukio: "Astronomical Instruments of Bhāskara II and After", in Subbarayappa, B.V. and S.R.N. Murthy (eds.): *Scientific Heritage of India* (Proceedings of a National Seminar, September 19-21, 1986, Bangalore), The Mythic Society, Bangalore (1988), pp. 19-23. When I wrote this paper, I did not know the date of the *Yantra-prakāśa* of Rāmacandra, which is still earlier. I first reported the date of the *Yantra-prakāśa* of Rāmacandra in Ohashi, Yukio: "Sanskrit Texts on Astronomical Instruments during the Delhi Sultanate and Mughal Periods". *Studies in History of Medicine and Science*, Vols. X-XI, 1986-1987, (actually published in 1991), pp. 165-181.
7. See my previous paper (*IJHS*, **32**(3), 1997) pp. 289-290.
8. Raghavan Nambiyar (ed.): *An Alphabetical List of Manuscripts in the Oriental Institute Baroda*, Vol. II, Baroda (1950), pp. 1164-1165.
9. Pingree, D.: *Census of the Exact Sciences in Sanskrit*, Series A, Vol. 2, American Philosophical Society, Philadelphia (1971), pp. 81-82.

10. See *The Wealth of India, Raw Materials*, Vol. III, Council of Scientific and Industrial Research, New Delhi (1952), pp. 7-12.
11. See my previous paper (*IJHS*, 32(3), 1997) p. 277.
12. Ibid, p. 278.
13. For Hindu method of the calculation of sine and arcsine, see B.B. Datta and A.N. Singh (revised by K.S. Shukla): "Hindu Trigonometry", *Indian Journal of History of Science*, **18**(1), 1983, pp. 39-108; especially pp. 90-99 where Brahmagupta's method of interpolation is explained.
14. See ibid. p. 93.
15. For the rationale, see ibid., pp. 96-98.
16. For the clepsydra, see my paper (*IJHS*, **29**(2), 1994), pp. 273-279.
17. See my previous paper (*IJHS*, 32(3), 1997), p. 216 ff.
18. For circular instruments in classical Siddhāntas, see my paper (*IJHS*, **29**(2), 1994), pp. 236-242. For the figures of the ring dial in Mughal miniatures and the actual specimens of the ring dial manufacutred for Sawai Jai Singh, see Sarma, Sreeramula Rajeswara: "Astronomical Instruments in Mughal Miniatures", *Studien zur Indologie and Iranistik*, Band 16/17, 1992, pp. 235-276.
19. Dvivedī, Sudhākara, *Gaṇaka-taraṅginī*, Benares (1892), reprinted: Hemant Sanskrit Seires Office, Varanasi (1986), pp. 54-59; Dikshit, S.B., op. cit. (English tr.), Part II, pp. 130-139; and Pingree, D., *Census* (op. cit.), A-2, pp. 94-106.
20. Dikshit, S.B., op. cit. (English tr.), Part II, p. 128.
21. Pingree, D., *Census* (op. cit.), A-2, pp. 65 and 94.
22. The English translation by R.V. Vaidya (Part II, p. 232) reads "13 chapters" in place of "13 verses", but it must be mistake. The Marathi original (AD 1896) of this Dikshit's work and the Hindi translation by Jhārakhaṇḍī rightly reads "13 śloka". (Regarding the reading of the Marathi original, I owe to Prof. K.S. Shukla's information. Regarding the Hindi translation, see Śivānātha Jhārakhaṇḍī (tr.): *Bhāratīya Jyotiṣa*, Hindī Samiti, Uttar Pradesh Śāsan, Lucknow (1957), third impression (1975), p. 462). Indeed, the *Pratoda-yantra* itself consists of 13 verses.
23. Dikshit, S.B., op. cit. (English tr.), Part II, p. 232.
24. Pingree, D., *Census* (op. cit.), A-2, p. 106.
25. Pingree, David : *Jyotiḥśāstra, Astral and Mathematical Literature*, Otto Harrassowitz, Wiesbaden (1981), p. 53.
26. See note 5.
27. Usually, the date of the *Graha-lāghava* is said to be Śaka 1442 (AD 1520). See the *Graha-lāghava* (I. 8).
28. Page "c" of the Introduction of S.D. Sharma's edition, op. cit.
29. Ibid., page "d".
30. I reported this fact in Ôhashi, Yukio: "A Note on some Sanskrit Manuscripts on Astronomical Instruments", in Swarup, G., A.K. Bag, and K.S. Shukla eds.: *History of Oriental Astronomy*, Cambridge University Press, Cambridge (1987), pp. 191-195.

31. Bombay U 375 (Account no. 833); Benares 35298, and 35702; AS Bombay 245 (BD 298/21); BORI 43 of 1898/99; SOI 9416; LDI 7041 (Accession no. 4038); Ānandāśrama 6673. (Benares 37502 contains Munīśvara's version also).
32. Benares 36922 and SOI 9421 (text only); and Baroda 9429, and AS Bombay 288 (BD 62) (with commentary).
33. Benares 34353, 35074, 35630, and 36676; Baroda 3190; BORI 546 of 1899/1915; SOI 9414; ,and VVRI 4731; and also Benares 35702. Benares 35702 contains Gaṇeśa's original version of the *Pratoda-yantra* also.
34. Benares 34999, and BORI 189 of 1883/84.
35. Benares 36676, 35630, and 34353; Baroda 3190; and VVRI 4731.
36. The *Siddhānta-tattva-viveka*, with notes by Sudhākara Dvivedī, has been edited by Muralīdhara Jhā, and published in5 fasciculi in Benares (1924-1935).
37. See Bahura, G.N. ed.: *Catalogue of Manuscripts in the Maharaja of Jaipur Museum*, Maharaja of Jaipur Museum Trust, Jaipur (1971), pp. 58-59.
38. Ghori, S.A. Khan "Development of Zij Literature in India", in Sen, S.N. and K.S. Shukla eds.: *History of Astronomy in India*, New Delhi (1985) 21-48; pp. 34-36.
39. Dikshit, S.B., op. cit., Part II, p. 162.
40. Dvivedī, Sudhākara: *Gaṇaka-taraṅginī*, (rep. 1986), pp. 91-92; Dikshit, S.B., op. cit., Part II, pp. 162-164; Pingree, D., *Census*, A-2, pp. 21-23.
41. Dvivedī, Sudhākara, *Gaṇaka-taraṅginī*, (rep. 1986), pp. 94-95; Dikshit, S.B., op. cit., Part II, pp. 165-166; Pingree, D., *Census*, A-3, pp. 173-174.
42. I have used AS Bombay 264 (BD 311) (*Yantrādhyāya* only), and BORI 206 of A. 1883/84.
43. The verses 74-75 of AS Bombay 264 (BD 311) are in folio 8 a-b, and the vss. 146-147 of BORI 206 of A. 1883/84 are in f. 47a.
44. Dvivedī, Sudhākara, *Gaṇaka-taraṅginī*, (rep. 1986), pp. 85-88; Dikshit, S.B., op. cit., Part II, pp. 161-162; and Pingree, D., *Census*, A-4, pp. 436-441.
45. Its *Pūrvārdha* has been published in 3 parts: the first two parts were edited by Gopīnātha Kavirāja, Benares (1932-1935), and the third part was edited by Mīṭhālāḷa Ojhā, Sampurnanand Sanskrit Vishvavidyalaya, Varanasi (1978).
46. I have used Benares 36922, and SOI 9421 (text only); and Barods 9429, and AS Bombay 288 (BD 62) (with auto-commentary).
47. See note 5.
48. It seems that the figure in the front cover of S.D. Sharma's book has been taken from the *Paṇḍitāśrama* (Āṣāḍha, Śukla 1, Monday, Vikrama Saṁ. 1969) mentioned in p. 9 of his book.
49. Vitruvius (translated by Morris Hicky Morgan): *The Ten Books on Architecture*, Harvard University Press (1914), reprinted: Dover Publications, New York (1960), p. 273.
50. Diels, Hermann: *Antike Technik*, Dritte Auflage, Verlag B.G. Teubner, Leipzig und Berlin (1924), p. 185.

51. Ibid., p. 191.
52. Godfrey, Hugh, and Robert Theodore Gunther: "Dial and Dialling", in *The Encyclopaedia Britannica*, Fourteenth Edition, Volume 7, The Encyclopaedia Britannica Company, London (1929), 310-314; p. 313.
53. Fig. 7(a) is from Price, Derek J.: "Precision Instruments: To 1500", in Singer, Charles et. al. (eds.): *A History of Technology*, Volume III, Oxford University Press, Oxford (1957), 582-619, p. 597. Fig. 7(b) is based on the photograph in Goudsmit, Samuel A. and Robert Claiborne (translated into Japanese under the supervision of Ono, Ken'ichi): *Toki no hanashi* (A story of time, in Japanese), Time Life International, Tokyo (1968), p. 131. The photograph of the two sides of its reproduction in the Hellmut-Kienzle Uhren-Museum at Schwenningen, Germany, is shown in Waugh, Albert E.: *Sundials*, Dover Publications, New York (1973), p. 166.
54. Casanova, Paul: "La Montre du Sultan Noûr ad Dîn", *Syria*, Tome IV, Paris (1923), 282-299; Planche XLV.
55. Godfrey, H. and R.T. Gunther, op. cit., p. 313.
56. Sédillot, Jean-Jacques et Louis-Amélie Sédillot: *Traité des instruments astronomique des Arabes*, Paris (1834), reprinted: Institut für Geschiche der Arabisch-Islamischen Wissenschaften, Frankfurt am Main (1984).
57. Ibid., Planche IX. For its description, see ibid., pp. 433-437. The figure in Arabic manuscript is in al-Marrākushī (= Abul-Ḥasan): *Comprehensive Collection of Principles and Objectives in the Science of Timekeeping* (photo-offset printing of ms. "Ahmet III (No. 3343)"), Part 1, Institut für Geschichte der Arabisch-Islamischen Wissenschaften, Frankfurt am Main (1984), p. 234. The figure in another Arabic manuscript is shown in Casanova, P., op. cit., Planche XLVI.
58. Robinson, F.N. (ed.): *The Complete Works of Geoffrey Chaucer*, Second edition, Oxford University Press, Oxford (1974), p. 158, (lines VII 204-207).
59. Price, D.J., op. cit., p. 598.
60. Godfrey, H. and R.T. Gunther, op. cit., p. 312.
61. Price, D.J., op. cit., p. 597.
62. Casanova, P., op. cit., Planche XLVII.
63. Godfrey, H. and R.T. Gunther, op. cit., p. 313.
64. Price, D.J., op. cit., p. 597.
65. Ibid., p. 598.
66. Dvivedī, Vindhyeśvarī Prasād (ed.): *Jyautiṣa-siddhānta-saṁgraha*, fasciculus 2, Benares (1917), p. 30. For the theory of gnomon in the Classical Siddhānta period, see Ôhashi, Yukio: "Astronomical Instruments in Classical Siddhāntas", *Indian Journal of History of Science*, 29(2), 1994, 155-313; pp. 168-196.
67. See note 5, pp.12-13.
68. Winter, H. J. J.: " A Shepherd's time-stick, Nāgarī inscribed", *Physis*, Anno VI, Fasc. 4, 1964, pp.377-384.
69. Sarma, Sreeramula Rajeswara : " Indian Astronomical and Time-measuring Instruments", *Indian Journal of History of Science* 29(4), 1994, 507-528; pp.515-516.

Fig. 1. The Kaśā-yantra
Fig. 2. The horizontal gnomon
Fig. 3. The Valaya-yantra
Fig. 4. The front cover of S.D. Sharma's edition of the Pratoda-yantra
Fig. 5. The traveller's dial found in Forbach (from Diels: Antike Technik)
Fig. 6. The Roman hanging dial found in Portici (from Diels: Antike Technik)
Fig. 7. The Saxon sundial found at Canterbury Cathedral
(a) from A History of Technology
(b) rough sketch based on the photograph in Goudsmit and Claiborne's book
Fig. 8. The Syrian sundial made for Nūr al-Dīn (from Casanova's paper)
Fig. 9. Abul-Ḥasan's development of cylindrical sundial (from Sédillot's translation)
Fig. 10. European cylindrical sundials
(a) from The Encyclopaedia Britannica, 14th ed.
(b) from A History of Technology.
Fig. 11. The cylindrical sundial used in the Pyrénées (from Casanova's paper)
Fig. 12. European ring dials
(a) from The Encyclopaedia Britannica, 14th ed.
(b) from A History of Technology.

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