INDIAN JOURNAL OF HISTORY OF SCIENCE

No. 2 June 1998 Vol. 33

SUPPLEMENT

TANTRASAṂGRAHA OF NĪLAKAṆṬHA SOMAYĀJI

(Sanskrit, English translation and exposition in terms of modern Mathematics)

Text by
K.V. SARMA

with English translation by
V. S. NARASIMHAN

INDIAN NATIONAL SCIENCE ACADEMY
NEW DELHI

नीलकण्ठसोमयाजि विरचितः

तन्त्रसंग्रहः

अथ तृतीयोऽध्यायः

छायाप्रकरणम्

[शङ्कुस्थापनम्]

शिलातलेऽपि वा भूमौ समायां मण्डलं लिखेत् ।
तन्मध्ये स्थापयेच्छङ्कुं कल्पितं द्वादशाङ्गुलम् ॥ १ ॥

[पूर्वापररेखा]

तच्छायाग्रं स्पृशेद्यत्र वृत्ते[1] पूर्वापराह्णयोः ।
तत्र बिन्दू निधातव्यौ वृत्ते पूर्वापराभिधौ ॥ २ ॥
भेदात् पूर्वापरक्रान्त्योश्छायाकर्णाङ्गुलाहतात् ।
लम्बकाप्तं पूर्वबिन्दोर्नीत्वा कार्योऽत्र सोऽयनात् ॥ ३ ॥

[याम्योत्तरादिरेखाः]

मध्यं कृत्वा तयोर्बिन्द्वोस्तुल्ये वृत्ते समालिखेत् ।
तत्संश्लेषोत्थमत्स्येन ज्ञेये याम्योत्तरे दिशौ ॥ ४ ॥
तद्वृत्तमध्यमत्स्येन पूर्वापरदिशावपि ।
दिङ्मध्यमत्स्यसंसाध्याश्चतस्रो विदिशोऽपि च ॥ ५ ॥

[अधऊर्ध्वदिगवगमनम्]

अधऊर्ध्वदिशौ ज्ञेये लम्बकेनैव नान्यथा ॥ ६a ॥

[विषुवच्छाया]

एकसूत्र[2]गता च्छाया यस्मिन्नह्न्युदयास्तयोः ॥ ६b ॥
मध्याह्ने विषुवाख्यः स्यात् कालस्तस्मिन् दिने यतः ।
तस्मात् तद्दिनमध्याह्नच्छाया वैषुवती मता ॥ ७ ॥

[छायाशङ्कुकर्णानां सम्बन्धः]

तच्छङ्कुवर्गसंयोगमूलं कर्णोऽस्य वर्गतः ।
त्यक्त्वा शङ्कुकृतिं, मूलं छायाशङ्कुर्विपर्ययात् ॥ ८ ॥

---

1. A. वृत्तं   2. B_2 एकत्सूत्र; $C_{2\_6.8\_9}$ एतत्सूत्र

ज्ञेयो दोःकोटिकर्णेषु द्वाभ्यामन्योऽखिलेष्वपि ॥ ९a ॥

[अक्षो लम्बश्च]

छायां तां त्रिज्यया हत्वा स्वकर्णेन हरेत्, फलम् ॥ ९b ॥
अक्षजीवा, तथा शङ्कुं कृत्वा लम्बकमानयेत् ॥ १०a ॥

[भगोले अक्षज्या लम्बज्या च]

51770
अक्षज्यार्कगतिघ्नाप्ता खस्वराद्र्येकसायकैः ॥ १०b ॥
फलोनमक्षचापं स्यादर्कबिम्बार्धसंयुतम् ।
स्फुटं, तज्ज्याक्षजीवापि, तस्याः कोटिश्च लम्बकः ॥ ११ ॥

[सममण्डलं उन्मण्डलं अग्रा च]

पूर्वापरायता रेखा प्रोच्यते सममण्डलम् ।
रेखा प्राच्यपरा साध्या विषुवद्भाग्रगा तथा ॥ १२ ॥
उन्मण्डलं च विषुवन्मण्डलं साभिधीयते ।
इष्टच्छायाग्रतद्रेखाविवरं त्वग्रसंज्ञितम् ॥ १३ ॥

[लङ्कोदयप्राणाः स्वदेशराश्युदयप्राणाश्च]

राश्यन्तापक्रमैः कोटिः प्राणाः प्राग्वच्चरासवः ।
प्राणान् लङ्कोदयान् प्राहुः स्वोदयाश्चरसंस्कृताः ॥ १४ ॥
चरमाद्यन्त्ययोः शोध्यं पदयो, र्योज्यमन्ययोः ।
एवंकृतास्तु विश्लिष्टा राशीनामुदयासवः ॥ १५ ॥
ओजयोस्तु क्रमेणैव, युग्मयोरुत्क्रमेण च ॥ १६a ॥

[इष्टशङ्कुः छाया च]

प्राक्कपाले गतान् प्राणान्, गम्यान् मध्यदिनात्[1] परम् ॥ १६b ॥
विन्यस्यार्कचरप्राणाः शोध्या भानावुदग्गते ।
योज्या दक्षिणगे, तेभ्यो जीवा ग्राह्या यथोदितम् ॥ १७ ॥
व्यस्तं कृत्वा चरज्यां च द्युज्याघ्नां त्रिज्यया हरेत् ।
लम्बकघ्नात् फलात् त्रिज्याहृतः शङ्कुर्विवस्वतः ॥ १८ ॥
तत्त्रिज्याकृतिविश्लेषान्मूलं छाया महत्यपि ।
[2]छायायास्त्र्यङ्ग[3]नागाप्त[4] लिप्ताव्यासार्धतस्त्यजेत् ॥ १९ ॥

---

1. A. मध्यन्दिनात्
2. A. B. C9 insert lines 21b and 22a before this.
3. C. त्र्यग
4. A. नागाप्ता

शिष्टेन शङ्कुमाहत्य त्रिज्याप्तं त्यज्यतामिह[1] ।
छायाया,[2] श्छाययाऽऽहत्य त्रिज्याप्तं शेषतोऽपि च ॥ २० ॥
क्षिपेच्छङ्कौ[3] सुसूक्ष्मोऽयं शङ्कुश्च महती प्रभा ।
छायां द्वादशभिर्हत्वा शङ्कुभक्तेष्टशङ्कुभा[4] ॥ २१ ॥

[महाशङ्कोः गतगन्तव्यप्राणाः]

शङ्कुच्छाये त्रिजीवाघ्ने महत्यौ कर्णसंहृते ।
लम्बकाक्षज्ययोः[5] स्वर्णमन्योन्योत्थफलं[6] यथा ॥ २२ ॥
तथा नृच्छाययोः कार्यं विपरीतप्रभाविधौ ।
व्यासार्धघ्नात् ततः शङ्कोर्लम्बकाप्तं त्रिजीवया[7] ॥ २३ ॥
हत्वा द्युज्याविभक्ते तच्चरज्या स्वर्णमेव च ।
याम्योदग्गोलयोस्तस्य चापे व्यस्तं चरासवः ॥ २४ ॥
संस्कार्या गतगम्यास्ते पूर्वापरकपालयोः ॥ २५a ॥

[मध्यन्दिनच्छाया]

क्रान्त्यक्षचापयोगाच्च भेदाद्वा याम्यसौम्ययोः ॥ २५b ॥
जीवा मध्यन्दिनच्छाया, ततो वार्कस्फुटं नयेत् ॥ २६a ॥

[मध्यन्दिनच्छायया अर्कस्फुटः]

मध्यार्कनतभागेभ्यः स्वाक्षभागान् विशोधयेत् ॥ २६b ॥
शङ्कोरुदग्गता भा चेद् याम्यक्रान्तिर्हि शिष्यते ।
स्वाक्षभागान्नताश्चोना नतांस्तर्हि विशोधयेत् ॥ २७ ॥
उदक्क्रान्तिस्तदा शिष्टा[8] नत्यक्षयुतिरन्यदा ।
तज्ज्या त्रिज्याहता भक्ता क्रान्त्या परमया रवेः ॥ २८ ॥
दोर्ज्या, तच्चापमेव स्यात् सौम्ये गोलेऽयनेऽपि च ।
रवि, स्तत्रायने भिन्ने राशिषट्कं तदूनितम् ॥ २९ ॥
याम्ये[9] गोलेऽयने चापि राशिषट्कयुतं रविः ।
तदूनं मण्डलं भानुर्याम्यस्थे चोत्तरायणे ॥ ३० ॥

[अयनचलनम्]

करणागतसूर्यस्य छायानीतस्य चान्तरम् ।
आयनं चलनं ज्ञेयं तात्कालिकमिदं स्फुटम् ॥ ३१ ॥

---

1. A. B. योजयेदिह 2. A. छायायां 3. A. B. शङ्कोस्त्यजेत्
4. A. C_1. शङ्कुभां 5. A. B. कार्य for स्वर्ण 6. A. न्योत्थं फलं
7. B. लम्बकाप्तत्रिजीवया 8. A. शिष्टं 9. C. सौम्ये for याम्ये

छायार्कादधिकेऽन्यस्मिन् शोध्यं, योज्यं विपर्यये ।
उदग्विषुवदादित्वसिद्ध्ये करणागते ॥ ३२ ॥
मेषादिके ग्रहे कार्यमंशादिकमिदं खलु ।
वृद्धिः क्षयश्च दिव्याब्दैः पञ्चभिः स्याद् धनर्णयोः ॥ ३३ ॥
दशांशोनाब्दतुल्या स्याद् गतिस्तस्य कलात्मिका ।
सप्तविंशतिभागान्तं चलनं चापनक्रयोः॥ ३४ ॥
सिद्धान्तेषूदितं, तस्य छाययापि विनिर्णयः ॥ ३५a ॥

[नत्यपक्रमाभ्याम् अक्षः]

क्रान्त्यर्कनतिभेदोऽक्षो याम्ये गोले युतिः पुनः ॥ ३५b ॥
छायायामपि सौम्येऽर्केऽप्यन्यथा स्यात्तदन्तरम् ॥ ३६a ॥

[छायया दिगवगमनम्]

सायनार्कभुजाजीवा परमक्रान्तिताडिता ॥ ३६b ॥
लम्बकाप्ताग्रजीवा स्याच्छायाकर्णहता हृता ।
त्रिज्ययाग्राङ्गुलं याम्ये विपुवद्भायुतं भुजा ॥ ३७ ॥
सौम्याथ सौम्यगोलेऽपि न्यूनमग्राङ्गुलं यदि ।
शोधयेद् विषुवद्भायाः सौम्यो बाहुस्तदापि च ॥ ३८ ॥
विषुवद्भां त्यजेत् तस्माद् रवावुत्तरगेऽधिकात् ।
याम्य एव तदा बाहुस्तच्छायाकृतिभेदतः ॥ ३९ ॥
मूलं कोटिः, श्रुतिः, छाया त्रिभिस्त्र्यश्रं भवेदिदम् ।
भ्रामयित्वाऽथ तत् त्र्यश्रं यावच्छायानुगा श्रुतिः ॥ ४० ॥
कोट्या पूर्वापरे ज्ञेये, बाहुना दक्षिणोत्तरे ॥ ४१a ॥

[छायाभ्रमणवृत्तपरिलेखः]

छायाभ्रमणमप्येवं ज्ञेयमिष्टदिनोद्भवम् ॥ ४१b ॥
इष्टकालोद्भवां छायां बाहुं कोटिं[1] च पूर्ववत् ।
तत्तुल्याभिः शलाकाभिस्तिसृभिस्त्रिभुजं तथा ॥ ४२ ॥
कृत्वा, पूर्वापरां कोटिं वृत्तमध्याद् यथादिशम् ।
कृत्वा, बाहुं च बाहोश्च छायायाश्चाग्रयोर्युतौ ॥ ४३ ॥
बिन्दुं कृत्वाऽपराह्णेऽपि बिन्दुं तत्र प्रकल्पयेत् ।
मध्यच्छायाशिरस्यन्यस्तृतीयो बिन्दुरिष्यते ॥ ४४ ॥

1. A. B. बाहुकोटि

लिखेद् वृत्तत्रयं, तेन[1] यथा मत्स्यद्वयं भवेत् ।
तन्मत्स्यमध्यगे सूत्रे प्रसार्यैवं तयोर्युतिः ॥ ४५ ॥
दृश्यते यत्र तन्मध्यं वृत्तं[2] बिन्दुस्पृगालिखेत् ।
छाया तन्नेमिगा तस्मिन् दिने स्यात् सर्वदापि च ॥ ४६ ॥

[प्रकारान्तरेण छायाभुजानयनम्]

अक्षज्याघ्नान्महाशङ्कोः शङ्क्वग्रं लम्बकाहृतम् ।
सर्वदा दक्षिणं तद्धि, योज्यमर्काग्रयापि तत् ॥ ४७ ॥
याम्ये गोले महाबाहुः, सौम्ये चाग्रद्वयान्तरम् ।
अधिकेऽत्रापि शङ्क्वग्रे याम्यः स्यादन्यथोत्तरः ॥ ४८ ॥
छायाकर्णहतः सोऽपि त्रिज्याभक्तोऽङ्गुलात्मकः ।
विपरीतदिगप्येष पूर्वानीतसमोऽपि च ॥ ४९ ॥
द्वादशघ्नोऽथवा बाहुः शङ्कुना महता हृतः ।
अङ्गुलात्मकमेवं वा छायाभ्यामथवा नयेत् ॥ ५० ॥

[सम (मण्डल) शङ्कुः]

अक्षज्योना यदा क्रान्तिः सौम्या, तां त्रिज्यया हताम् ।
अक्षज्यया विभज्याप्तः शङ्कुः स्यात् सममण्डले ॥ ५१ ॥

[समशङ्कुना अर्कस्फुटः]

अक्षज्याघ्नः परक्रान्त्या[3] हृतः शङ्कुः स दोर्गुणः[4] ।
तच्चापमेव भानुः स्यात् चक्रार्धं वा तदूनितम् ॥ ५२ ॥

[समशङ्कोरङ्गुलात्मकः कर्णः]

लम्बाक्षज्ये विषुवद्भार्कघ्ने क्रान्तिजीवया भक्ते ।
सममण्डलगे भानौ कर्णौ तावङ्गुलात्मकौ स्पष्टौ ॥ ५३ ॥

[प्रकारान्तरेण समशङ्कुकर्णः]

मध्यच्छाया यदा मध्ये विषुवत्समरेखयोः ।
तन्मध्याह्नभवः कर्णो विषुवच्छायया हतः ॥ ५४ ॥
मध्याह्नाग्राङ्गुलैर्भक्तः कर्णः स्यात् सममण्डले ॥ ५५a ॥

---

1. A. C_{10} तेषु for तेन
2. B. om. वृत्तं
3. C_1. पराक्रान्त्या
4. A. स्वदोर्गुणः

[समशङ्कुना गतैष्यप्राणाः]

सममण्डलशङ्कुर्लम्बघ्नस्त्रिज्यया हृतः ॥ ५५b ॥
उन्मण्डला द्युवृत्तज्या, त्रिज्याघ्ना द्युज्यया हृता ।
तच्चापं चरचापाढ्यं गतैष्यासव एव हि ॥ ५६ ॥

[समशङ्कुना नतप्राणाः]

लम्बघ्नः समशङ्कुः स द्युज्याभक्तोऽथ तत्कृतिम् ।
त्यक्त्वा त्रिज्याकृतेर्मूलं चापितं हि नतासवः ॥ ५७ ॥

[प्रकारान्तरेण नतप्राणाः]

सममण्डलगा छाया त्रिज्याघ्ना द्युज्यया हृता ।
चापिता वा नतप्राणाः कोट्या वा सर्वदा तथा ॥ ५८ ॥

[समशङ्कोः क्षितिज्या]

अक्षज्याघ्नौ समौ शङ्कू त्रिज्यालम्बकभाजितौ ।
क्रान्त्यर्काग्रे, तयोः कृत्योर्भेदमूलं क्षितेर्गुणः ॥ ५९ ॥

[दशप्रश्नाः]

इह शङ्कु-नत-क्रान्ति-दिगग्रा-ऽक्षेषु पञ्चसु ।
द्वयोर्द्वयोरानयनं दशधा स्यात् परैस्त्रिभिः ॥ ६० ॥
सशङ्कवो नतक्रान्तिदिगक्षाः सनतास्तथा ।
अपक्रमदिगग्राक्षा दिगक्षौ क्रान्तिसंयुतौ ॥ ६१ ॥
दिगक्षाविति नीयन्ते द्वन्द्वीभूयेतरैस्त्रिभिः ॥ ६२a ॥

[प्रश्नः १. अपक्रमा-ऽशाग्रा-ऽक्षै शङ्कु-नत्यौ]

आशाग्रा लम्बकाभ्यस्ता त्रिज्याभक्ता च कोटिका ॥ ६२b ॥
भुजाक्षज्या तयोर्वर्गयोगमूलं श्रुतिर्हरः ।
क्रान्त्यक्षवर्गौ तद्वर्गात् त्यक्त्वा कोट्यौ तयोः पदे[1] ॥ ६३ ॥
कुर्यात् क्रान्त्यक्षयोर्घातं कोट्योर्घातं तथा परम् ।
सौम्ये गोले तयोर्योगात् भेदाद् याम्ये तु घातयोः ॥ ६४ ॥
आद्यघातेऽधिके सौम्ये योगभेदद्वयादपि ।
त्रिज्याघ्नाद् हारवर्गाप्तः शङ्कुहिरष्टदिगुद्भवः ॥ ६५ ॥

1. A. om. पदे

छाया तत्कोटिराशाग्राकोटिघ्ना सा द्युजीवया ।
भक्ता नतज्या क्रान्त्यक्षदिगग्राभिर्भवेदिति ॥ ६६ ॥
क्रान्त्यक्षघाते तत्कोट्योर्घाताद् याम्येऽधिके सति ।
नेष्टः[1] शङ्कुर्भवेत् सौम्ये हाराच्चापक्रमेऽधिके ॥ ६७ ॥

[प्रश्नः २. नताशाग्राक्षैः शङ्क्वपक्रमः]

नतलम्बकयोर्घातात् त्रिज्याप्तं तत् स्वदेशजम् ।
स्वदेशनतकोट्याप्तं नताक्षज्यावधात्तु यत् ॥ ६८ ॥
तदाशाग्रावधे कोट्योस्तयोर्घातं क्षिपेदथ[2] ।
शोधयेद् दक्षिणाग्रायां त्रिज्यया च ततो हरेत् ॥ ६९ ॥
लब्धात् स्वनतकोटिघ्नात् पृथक् त्रिज्याप्तवर्गितम् ।
युक्त स्वनतवर्गेण तन्मूलेन हृतं फलम् ॥ ७० ॥
पृथक्कृताद् भवेच्छङ्कुः, छाया तत्कोटिका भवेत् ।
छायाग्रकोटिसंवर्गाद् द्युज्या लब्धा नतज्यया ॥ ७१ ॥
नतज्याद्युज्ययोस्तद्वत् छायाकोटित्रिजीवयोः ।
छायादिगग्रोकोट्योश्च घात एको भवेत् ततः ॥ ७२ ॥
द्वयोरेकेन विहृतस्तत्सम्बन्धीतरो भवेत् ।
द्युज्यात्रिजीवयोर्वर्गभेदमूलमपक्रमः ॥ ७३ ॥

[प्रश्नः ३. नतापक्रमाक्षैः शङ्क्वाशाग्रे]

नतकोट्या हता द्युज्या विभक्ता त्रिभजीवया ।
सौम्ययाम्यदिशोर्भूज्यायुतोना लम्बकाहता ॥ ७४ ॥
त्रिज्याप्ता शङ्कुराशाग्रा, कोटिर्द्युज्या च पूर्ववत् ॥ ७५a ॥

[प्रश्नः ४. नतक्रान्त्याशाग्राभिः शङ्क्वक्षौ]

छायां नीत्वाथ तत्कोटिद्युज्यावर्गान्तरात् पदम् ॥ ७५b ॥
तच्छायाबाहुघातो यः शङ्कुक्रान्त्योर्वधोऽपि यः ।
क्रान्त्यग्रयोस्तुल्यदिशोस्तयोर्भेदोऽन्यथा युतिः ॥ ७६ ॥
उन्मण्डलक्षितिजयोरन्तरेऽर्के च तद्युतिः ।
तद्धतां विभजेत् त्रिज्यां तच्छायाकोटिवर्गयोः ॥ ७७ ॥
अन्तरेण भवेदक्षो नताद्यैर्विदितैस्त्रिभिः ॥ ७८a ॥

---

1. A. नेष्टशङ्कुः   2. $C_{7,9,10}$ क्षिपेदुदक्

[प्रश्नः ५. शङ्क्वाशाग्राक्षैः नतापक्रमौ]

अक्षशङ्क्वोर्वधो यश्च यश्च भाबाहुलम्बयोः ॥ ७८b ॥
[1]सौम्ययाम्यस्थिते भानौ तयोर्योगान्तराद् ततः ।
क्रान्तिस्त्रिज्याहता प्राग्वन्नतज्यां च समानयेत् ॥ ७९ ॥

[प्रश्नः ६. शङ्क्वपक्रमाक्षैः नताशाग्रे]

त्रिज्यापक्रमघातो यो यश्च शङ्क्वक्षयोर्वधः ।
तयोर्योगान्तरं यत्तु गोलयोर्याम्यसौम्ययोः ॥ ८० ॥
भाबाहुर्लम्बकाप्तोऽस्मात् त्रिज्याघ्नाद् भाहतेष्टदिक् ॥ ८१a ॥

[प्रश्नः ७. शङ्क्वपक्रमाशाग्राभिः नताक्षौ]

वर्गान्तरपदं यत् स्यात् छायाकोटिद्युजीवयोः ॥ ८१b ॥
तच्छायाबाहुयोगो यःशङ्कुक्रान्त्यैक्यवर्गतः ।
तेनाप्तं यत् फलं तस्मिन्नेव तत् स्वमृणं पृथक् ॥ ८२ ॥
तयोरल्पहता त्रिज्या महताऽऽप्ताक्षमौर्विका ॥ ८३a ॥

[प्रश्नः ८. शङ्कुताक्षैः अपक्रमाशाग्रे]

त्रिज्याहताक्षशङ्कू स्वनतकोट्योद्धृतौ पृथक् ॥ ८३b ॥
ये तत्कोट्यौ च तत्त्रिज्यावर्गभेदपदीकृते[2] ।
मिथः कोटिघ्नयोर्योगाद् याम्ये सौम्येऽन्तरात् तयोः ॥ ८४ ॥
त्रिज्यया विहता द्युज्या क्रान्त्याशाग्रे तु पूर्ववत् ।
नतमण्डलदृश्यार्धमध्यतः सौम्ययाम्यता ॥ ८५ ॥

[प्रश्नौ ९-१०. अपरैस्त्रिभिः क्रान्त्यक्षौ, आशाग्राक्षौ च]

दिगग्रायास्तु तत्कोटिस्तच्छायाघाततो हृता ।
नतज्यया भवेद् द्युज्या, तद्भुजा क्रान्तिरेव हि ॥ ८६ ॥
द्युज्यानतज्ययोर्घातादग्राकोटिः प्रभाहृता ।
अक्षः प्राग्वदिति प्रश्नदशकोत्तरमीरितम् ॥ ८७ ॥

[इष्टदिक्छाया]

दिगग्रा विहता यद्वा तत्कोटिघ्ना पलप्रभा ।
तत्कोटिका तयोः कृत्योर्योगमूलं स्वदृग्गुणः ॥ ८८ ॥

---

1. A. B. C_9 स्फुटार्के सौम्ययाम्यस्थे   2. $C_{1\text{-}8.10}$ पदीकृतौ

शङ्कुदृग्गुणयोः कृत्योः छायाकर्णो युतेः पदम् ।
शङ्कुच्छाये त्रिजीवाघ्ने छायाकर्णेहृते स्फुटे ॥ ८९ ॥
दृग्गुणाभिहतक्रान्तेरक्षज्याप्तो ह्यपक्रमः ।
क्रान्तिदृग्गुणयोः कोटिस्त्रिज्यावर्गान्तरात् पदम् ॥ ९० ॥
मिथः कोटिहतत्रिज्याभक्तयोः[1] क्रान्तिदृग्ज्ययोः ।
तयोर्योगान्तरं छायागोलयोर्याम्यसौम्ययोः ॥ ९१ ॥

[कोणशङ्कुच्छाया]

भुजाऽक्षो, लम्बवर्गार्धमूलं कोटिः, श्रुतिस्तयोः ।
हारः, क्रान्तिघ्नकोट्योश्च दोःश्रुत्योः क्रान्तिहारयोः ॥ ९२ ॥
कोटिघ्नाक्षस्य चाप्तैक्यं याम्ये भेद उदक्प्रभा ।
अक्षकोट्यधिकायां तु क्रान्त्यां योगोऽप्युदक्प्रभा ॥ ९३ ॥
क्रान्त्यक्षयोश्च तत्कोट्योर्वधाद् भेदयुती नरः ।
तद्वद् विरुदगन्यत्राप्यभावः कोणयोर्द्वयोः ॥ ९४ ॥
अर्कघ्ने माश्रुती शङ्कुभक्ते ते अङ्गुलात्मिके ॥ ९५a ॥

[प्राग्लग्नम्]

संस्कृतायनभानूत्थराशिगन्तव्यलिप्तिकाः ॥ ९५b ॥
तद्राशिस्वोदयप्राणहता राशिकलाहृताः ।
असवो राशिशेषस्य गतासुभ्यस्त्यजेच्च तान् ॥ ९६ ॥
उत्तरोत्तरराशीनां प्राणाः शोध्याश्च शेषतः ।
पूरयित्वा रवे राशिं क्षिपेद् राशींश्च तावतः ॥ ९७ ॥
विशुद्धा यावतां प्राणाः शेषात्[2] त्रिंशद्गुणात् पुनः ।
तदूर्ध्वराशिमानाप्तकान् भागान् क्षिप्त्वा रवौ तथा ॥ ९८ ॥
षष्टिघ्नाच्च पुनः शेषात्[3] तन्मानाप्तकला अपि ।
एवं प्राग्लग्नमानेयम्, अस्तलग्नं तु षड्भयुक् ॥ ९९ ॥
व्यत्ययेनायनं कार्यं मेषादित्वप्रसिद्धये ॥ १००a ॥

[प्राग्लग्नस्य स्थूलता]

एकस्मिन्नपि राशौ तु क्रमात् कालो हि भिद्यते ॥ १००b ॥
तेन त्रैराशिकं नात्र कर्तुं युक्तं यतस्ततः ।
एवमानीतलग्नस्य स्थूलतैव न सूक्ष्मता ॥ १०१ ॥

1. A. B. लब्धयोः for भक्तयोः 2. C. शेषाः 3. B. शेषाः

[काललग्नम्]

सायनार्कभुजाप्राणा: प्राग्वत् स्वचरसंस्कृता: ।
काललग्नं तदेवाद्ये,[1] द्वितीये[2] तु तदूनितम् ॥ १०२ ॥
राशिषट्कं पदेऽन्यस्मिस्तद्युतं चरमे पुन: ।
तदूनं मण्डलं लग्नकाल: स्यादुदये रवे: ॥ १०३ ॥
द्युगतप्राणसंयुक्त:[3] कालो[4] विषुवदादिक:[5] ॥ १०४ a ॥

[दृक्क्षेप:]

अन्त्यद्युज्याहताक्षाद् यत् त्रिज्याप्तं यश्च लम्बक: ॥ १०४ b ॥
काललग्नोत्थकोटिघ्न: [8452] करार्थाब्ध्युरगैर्हृत: ।
दृक्क्षेपस्तद्भिदैक्यं च काले कर्किमृगादिके ॥ १०५ ॥
विश्लेषे लम्बजाधिक्ये सौम्यो याम्योऽन्यदा सदा ।
तत्त्रिज्याकृतिविश्लेषान्मूलं दृक्क्षेपकोटिका ॥ १०६ ॥

[दृक्क्षेपलग्नम्]

मध्याह्नाद्वा नतप्राणा[6] निशीथाद्वोन्नतासव: ।
एतद्बाणोनिता त्रिज्या चरज्याढ्या नता यदि ॥ १०७ ॥
उन्नताश्चेच्चरज्योना गोले याम्ये विपर्ययात् ।
द्युज्या लम्बकघातघ्ना त्रिज्याप्ता च पुनर्हृता ॥ १०८ ॥
कोट्या दृक्क्षेपजीवाया लब्धचापं रवौ क्षिपेत् ।
तल्लग्नं प्राक्कपाले स्यान्निशि चेत् तद्विवर्जितम् ॥ १०९ ॥
प्रत्यग्गतेऽस्तलग्नं स्याद् व्यस्तमेव दिवानिशो: ।
प्राक्पश्चाल्लग्नयोर्मध्यं लग्नं दृक्क्षेपसंज्ञितम् ॥ ११० ॥

[मध्यलग्नम्]

काललग्नं त्रिराश्यूनं मध्यकालस्तत: पुन: ।
लिप्ताप्राणान्तरं नीत्वा तद्दोश्चापे तु योजयेत् ॥ १११ ॥
ततश्चासून् नयेत् प्राग्वत् तल्लिप्तान्तरमुद्धरेत् ।
कालदोर्धनुषि क्षेप्यं तत: प्राणकलान्तरम् ॥ ११२ ॥

---

1. C. तदैवाद्ये　2. C_1 द्वितीयं　3. B. संयुक्त-
4. C. कालौ (C_1 काले)　5. A. दादित:; C_1. दादिकम्　6. C_{10} मध्याह्नात् प्राङ्नतप्राणा

कालदोर्धनुषि क्षिप्त्वा तच्चापमविशेषयेत् ।
मध्यलग्नं तदेव स्यात् तत्काले प्रथमे पदे ॥ ११३ ॥
द्वितीयादिषु च प्राग्वन्मध्यलग्नमिहानयेत् ॥ ११४ a ॥

[अविशेषं विना मध्यलग्नानयनम्]

अविशेषं[1] विना मध्यलग्नमानीयते यथा ॥ ११४ b ॥
मध्यकालस्य कोटिज्या परमापक्रमाहता ।
त्रिज्यालब्धकृतिं त्यक्त्वा कालकोटित्रिजीवयोः ॥ ११५ ॥
वर्गाभ्यां शिष्टमूले द्वे कोटिज्या स्याद् द्विमौर्व्यपि ।
कोटिज्याद्युज्ययोर्घाताद् द्युज्यावाप्तं तु चापितम् ॥ ११६ ॥
कालासवो मध्यलग्नभुजा तद्धीनमत्रयम् ।
पदव्यवस्था सुगमैवाद्यमध्य[2] विलग्नवत् ॥ ११७ ॥

[॥ इति तन्त्रसंग्रहे छायाप्रकरणं नाम तृतीयोऽध्यायः ॥]

* * *

1. B. does not have the verses 114b-17. 2. C: सुगमैवान्यमध्य

CHAPTER - III
Chāyā Prakaraṇam (Gnomic Shadow)

Fixing the Gnomon

1. Either on a plane stone slab or on a uniform place on the earth, draw a circle. Fix a gnomon of 12 inches at its centre.

East-West Line

2. During the forenoon and afternoon mark the points on the circle when the tip of the shadow just grazes the circumference. These (points where the tip goes out and enters into the circle) are termed East and West points.

3. Multiply the hypotenuse of the shadow by the difference between the R Sines of the declinations (determined at those instances when the tip of the shadow leaves or enters into the circle) in the forenoon and afternoon. Divide the result by the R Cosine of the latitude. The East point is to be obtained by marking the correction due to the motion of the *ayana*.

Note : The correction for drawing the east-west line, the deviation in *aṅgulas* of the east-point is given as

$$\frac{R \text{ Sin } \delta_1 - R \text{ Sin } \delta_2}{R \text{ Cos } \phi} \times \textit{karṇa}$$

See also *Siddhānta Śiromaṇi*, Arka Somayāji pp 231-234.

South-North and other lines

4. With these two points (east and west) as centres draw two equal cricles. The south and north directions are to be known by the fish that is formed by the intersections of the circles.

5. By the fish formed out of the two circles (with, the north and south points as centres), east-west directions are also known. From the pairs of fish (formed) in between two directions the four-directions are known.

Note : *Matsya-karṇa* is the geometrical construction in the form of fish shape for finding the perpendicular bisector of a given line-segment. The method is exactly similar to that being followed in current text-books on geometry. The intersecting arcs suggest the form of a fish shape and hence the term *matsya-karṇa*.

To fix the vertical line

6a. The above and below vertical directions (Zenith and Nadir) are known by the plumb-line and by no other means.

Equinoctial midday shadow :

6b, 7a On the day during which at sun-rise and sun-set, the extremities of the shadow fall on a line (parallel to the east-west line), the noon-time is termed as *viṣuvat*. Hence the noon-shadow on that day is (considered to be) *viṣuvat-chāyā* or equinoctial shadow.

Note : According to Gregorian calendar, equinoctial day falls on 21 March and 23 September. It would be a good exercise for the students to verify whether the locus of the extremity of the shadow is a line parallel to the east-west line on these days. This would also establish that the practice followed in celebrating *viṣuvat dina* should not be on *meṣa saṃkrānti* day.

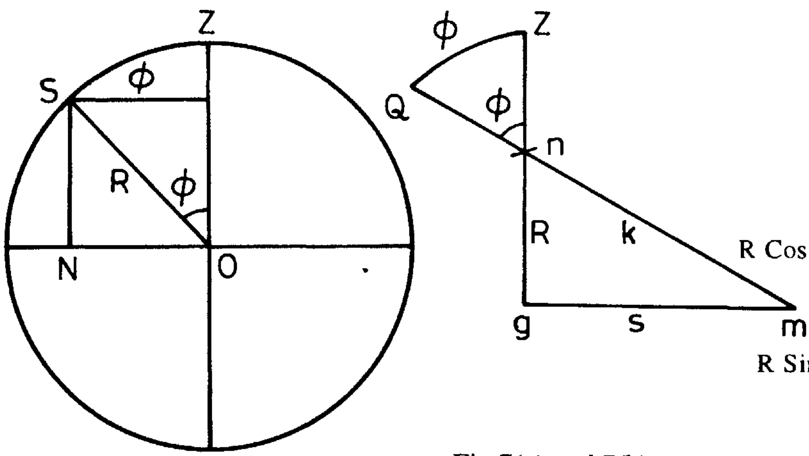
Relations among the shadow, Gnomon and Hypotenuse:

8. The square root of the sum of the squares of that (equinoctical shadow) and the gnomon is the hypotenuse. From its square subtract the square of the gnomon. Then its square root is the shadow. The gnomon is obtained by the reverse process.

9a. It is to be understood that out of the three-base, vertical and hypotenuse (*koṭi, doḥ karaṇa*) - in all cases from any two, the other (the third) is known.

To find the latitude and co-latitude. (R Sin ø R Cos ø)

9b, 10a. The (equinoctial) shadow multiplied by R and divided by its own hypotenuse (defined in *śloka* 8) gives *akśajyā* (or R Sin ø) as the result. Then repeating the process with the gnomon *lambaka* (or R Cos ø) is obtained [Fig.7(a) and 7(b)].



$$\frac{SN}{12} = \frac{NO}{s} = \frac{R}{k}$$

$$R \text{ Cos } ø = SN = 12. \frac{R}{k}$$

$$R \text{ Sin } ø = NO = s. \frac{R}{k}$$

Fig.7(a) and 7(b)

Note: To get R Sin ø, R Cos ø from *Dṛggola* to the *Bhagola* (from the Celestial sphere to Zodiacal sphere)

10b, 11. *Akṣajyā* (R Sin ø) is multiplied by the true daily motion of the Sun (in minutes) and divided by 51,770. This value is subtracted from the arc length of the latitude and half the angular diameter of the sun (in minutes) is added. The result is the true value of that (*akṣa cāpa*,). Its R Sine is the true R Sin ø. Its complement gives R cos ø.

Lines termed samamaṇḍala, unmaṇḍala are defined:

12, 13. The east-west line is termed as *samamaṇḍala*. Draw a parallel line (at a distance of) the extremity of the equinoctial shadow in the east-west direction. This is called *unmaṇḍala*, as well as *viṣuvanmaṇḍala*. The distance between the tip of the given day's noon-shadow and *viṣuvanmaṇḍala* is called the *agrā*.

Prāṇas of the rising of the signs at Laṅkā and at any place:

14. As stated earlier from the declinations at the end of every *rāśi* and (their) *koṭis*, the rising time in *prāṇas* and ascensional differences in *prāṇas* (are to be calculated). These rising times are called *Laṅkodaya prāṇas*. The *prāṇas* at *Lankodaya* corrected for the *caraprāṇas* (of each sign are the *prāṇas* required for) the rising of *rāśi* at the desired place.

Method of cara-saṃskāra

15. The *cara* is to be subtracted (from that of *Laṅkodaya*) in the first and last quarters; in the others it should be added.

15b, 16a. (The *Laṅkodaya prāṇas*) are obtained in the same order in the odd quarters and in the reverse order in the even quarters. Having calculated thus and kept separately these become the rising times in *asus* of the signs *meṣa* etc.

To find the śaṅku at the desired place:

16b, 17. When the sun is in the eastern hemisphere (find) the *prāṇas* that have elapsed from sunrise; (find also) the *prāṇas* that are yet to elapse till the sun set when the sun has crossed the meridian (and hence is in the western hemisphere). Keep them separately. When the sun is to the north of the ecliptic the *cara prāṇas* of the sun is to be subtracted (from the results kept separately). When the sun is to the south of the ecliptic, its *cara prāṇas* are to be calculated as stated earlier and its sine (*jyā*) obtained.

Note : The result obtained is in time units and it should be converted into arcs. *Nāḍis* are converted into degrees at 6 per *nāḍī* etc.

18. (To the sine of the resulting arc) the sine of the *cara* is applied in the reverse order. Multiply the result by *dyujyā* (R Cos δ) and divide by R. The result when multiplied by *lambaka* (R Cos ø) and divided by R is the *śaṅku* of the sun.

Note : The *śaṅku* given in śl. 18 is not the *Iṣṭaśaṅku*. It is taken as the *mahāśaṅku*, as śl. 19 talks of *mahāchāyā*.

19. The great shadow (*mahā-chāyā*) is the square root of the difference of the squares of R and the *śaṅku*.

Note : Even though the formula is given for *Iṣṭaśaṅku* in the previous *śloka* the commentator says, "*tatra labdho revēstātkālikē mahāśaṅkur bhavati*". Hence its complement is termed *Mahā-chāyā* in *śloka* 19a.

19b- 21. Dividing the shadow (the *mahāchāyā* of 19a by 863) subtract the result from the angular radius of the sun in minutes. (Let the result be kept in two places). (With the result at one place) the *śaṅku* is multiplied and then divided by R. Subtract the result from shadow. (The result is the accurate value of the great shadow, *mahāchāyā*). The result at the other place is multiplied by the great shadow and divided by R. The result is added to the *śaṅku*. Thus are obtained the accurate values of the *mahāśaṅku* and *mahāchāyā*. The great shadow multiplied by 12, and divided by the *mahāśaṅku* gives the desired shadow of the (12" *śaṅku*)

Note : Let r be angular radius of the sun in minutes.

Author first gets $x = \left(r - \frac{\text{mahāchāyā}}{863}\right)$

Accurate value of *mahāchāyā* = $\text{mahāchāyā} - x . \frac{\text{mahāśaṅku}}{R}$

Accurate value of *mahāśaṅku* = $\text{mahāśaṅku} + x . \frac{\text{mahāchāyā}}{R}$

Iṣṭa śaṅku = $\frac{\text{accurate mahāchāyā}}{\text{accurate mahāśaṅku}} \times 12$

Prāṇas that have elapsed or yet to elapse from great gnomon:

22a. The 12" *śaṅku* and its *chāyā* multiplied by R and divided by their hypotenuse are respectively the *mahāśaṅku* and *mahāchāyā*.

22b, 23a. Just as the addition or subtraction of the one with the other was given in the case of *lambaka* (R Cos ø) and *Akṣajyā* (R Sin ø), similarly in the case of the gnomon and the shadow, operations are to be done in the reverse order.

23b, 24, 25a. The 12" gnomon is multiplied by the angular radius (of the sun), then divided by *lambaka* (R Cos ø). The result is multiplied by R and divided by *dyujyā* (R Cos δ). The *carajyā* is added or subtracted according as the sun is in the southern or northern hemisphere. To the result converted in arcs, the *cara* correction is made in *asus*, in the reverse order. The two results give the (*prāṇas*) elapsed from the sun-rise or yet to elapse till the sun-set in the eastern and western hemisphere (in the forenoon and afternoon).

Finding the midday shadow using δ and ø.

25b-26a. From the sum or difference of the arcs of R Sin δ and R sin ø (according as the sun is) in the southern or northern hemisphere, the midday shadow is found from its R Sine. Then the true position of the sun may also be calculated (using that midday shadow)(Fig.8).

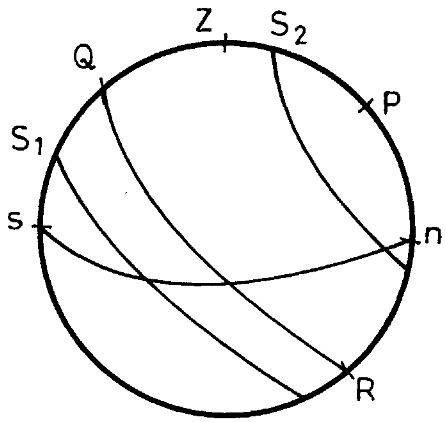


Fig. 8

Note : sun at S_1 (Southern) $z = ZS_1 = \phi + \delta$
Sun at S_2 (Northern) $z = ZS_2 = \delta - \phi$

Method for calculating the true position of the sun from the midday shadow

26b, 28a. Subtract the latitude in degrees from the zenith - distance (Z. d., *nata*) of the midday sun, if the shadow of the gnomon is in the northern direction, the southern declination (S.d.) is obtained. If the S.d. is less than the *nata* then subtract *nata* from it. The balance is the Northern declination. Otherwise (when the shadow is in the southern direction) it (the southern declination) is the sum of *nata* and *aṃśa*.

28b. The sine of the declination (R Sin δ) multiplied by R and divided by the maximum R Sine declination of the Sun (i.e. by R Sin ω), (gives the R Sine longitude of Sun).

Note : The result is equivalent to $\sin\lambda = \sin\delta / \sin\omega$. This follows by applying sine formula to $\Delta\gamma SD$. In Hindu Astronomy ω was taken to be 24°.

29. Its arc is the position of the sun with *ayanāṃśa*, if the *ayana* and hemispere are northern (if it is northern and it is *dakṣiṇāyana*) then the arc subtracted from six *rāśis* (is the position).

30. If the *gola* and *ayana* are both southern, then the arc added to six *rāśis* is the Sun. When it is *uttarāyana* and the sun is in the northern hemisphere, 360° minus the arc (is its position).

To find the precession of equinoxes (ayanacalana) at an instant

31. The difference between the true positions obtained from the procedure given in the texts) and the shadow is to be known as the true motion of the *ayana* upto the given instant (*ayana calana*).

Note : Commentator Śankara Vāriar explains '*karaṇāgata sūrya*' thus - the true position of the sun obtained from the mean sun at the instant.

32. This *ayanacalana* should be subtracted (from the value obtained by *karaṇa*), if the value obtained is greater than the one obtained from the shadow. Otherwise it should be added. This is to determine the true *viṣuvat* and the true status of the sun to the north of equator etc. from the calculated values.

33. This correction indeed (is to be done) for the *meṣadi* longitudes of the planets also. The increase and decrease of the *ayanacalana* takes place once in five divine years (5 x 360 years), both during when it is positive and when it is negative.

Note : In *Siddhānta Darpaṇa*, ślokas 17-18, Nīlakaṇṭha himself states thus; the conjunction (of the equinoxes) moves east and west by 27 degrees on each side. This increase and decrease (i.e. moving east and returning, then moving west and returning occurs regularly, (each increase or decrease taking place) once in five divine years (i.e. once in 1800 years) For five divine years, the *ayanāṃśa* is 27'. Hence for each civil year, it is $\frac{27 \times 60 \times 60''}{1800} = 54''$. The current value is 50.26".

34-35a. The value of the (*ayanacalana*) in minutes in one divine year is equal to the number of human years in it deficient by one-tenth of it, (i.e., 360-36 = 324). The movement of the winter-solstice (*uttarāyana*) between Dhanus and Makara is determined by the *Siddhānta* texts as equal to 27 degrees. This rate can be determined by the movement of the shadow also.

Note : *Śloka* 18 of *Śiddhānta Darpaṇa* declares that, "(We find) a moment when the two solstices were roughly in the middle of Sagittarius (Dhanus) and Gemini (Mithuna) respectively", *dhanus mithunayormadhye prayasastuvanaye ubhe'*. Commentator Śaṅkara Variar: It is declared in *Surya Śiddhānta, 'trimśatkṛtyā yuge bhāmśai cakram prāk parilambate'*.

To find latitude (of the place) from meridian Zenith-distance, and declination of the Sun

35b-36a. In the southern hemisphere, latitude is the sum of the declination and meridian zenith distance of the sun. If the (midday) shadow as well as the sun be in the northern direction, then their difference (is the latitude of the place).

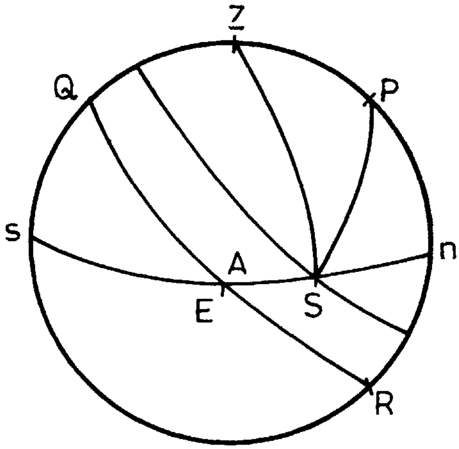
To fix the directions from the shadow at a desired place/time

36b-37a. Multiply the R Sine (*sāyana*) longitude of the sun by the R sine of maximum declination, (the result) divided by R Cosine of the latitude is the *agrajīva* (or *agra* or *arkāgra*).

$$\text{R Sin ES} = \text{R Sin A} = \textit{Agrajīva}$$

$$\text{R Sin ES} = \frac{\text{R Sin } \lambda \,.\, \text{R Sin } \omega}{\text{R Cos } \phi}$$

In ΔPZS, Sin δ = Cos ø Sin A, in γSD, Sin δ = Sin λ = Sin ω.
Hence the result for Sin A [Fig. 9(a) & 9(b)]



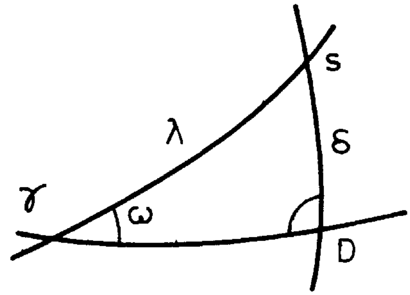


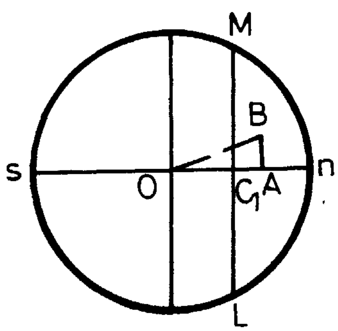
Fig. 9(a) Fig. 9(b)

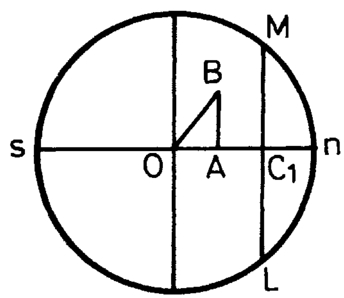
37. The *agrajīva* is multiplied by the hypotenuse of the shadow (in *aṅgulas*) and divided by R. The result is the *arga* in *aṅgulas*. $\frac{\text{R Sin A : k}}{\text{R}} = a$

(If the Sun is) in the southern hemisphere, this added with equinoctial shadow (s) becomes the *bhuja* of the northern shadow.

38. In the northern hemisphere itself, if the *agrāṅgula* (a) is less (than s) subtract it from the equinoctial shadow. Than the result is the *bhuja* in the northern direction.

39. When the sun is in the northern hemisphere, deduct the equinoctial shadow from the *agrā* which is greater, then that is the southern *bhuja*.





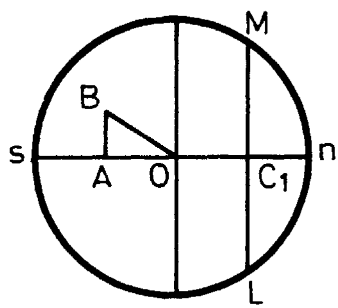


Fig. 10(a) Fig. 10(b) Fig. 10(c)

Note : OA = s + a (Fig.10(a))

= s - a (Fig 10(b))

= a - s (Fig 10(c))

OC_1 = s, AC_1 = a, *bhuja* is OA, along north-south direction, and *koṭi* is along east-west direction.

39-40, 41a. The square root of the difference between the squares of the shadow and *bhuja* is the *koṭi*, the shadow being the hypotenuse. With these three (*koṭi, bhuja, chāyā*) a triangle is formed. (Keep the triangle on the ground with the intersection of the *koṭi* and *karṇa* as centre). Rotating this triangle, till the hypotenuse follows the shadow, then the east-west direction is known by the *koṭi* and the south-north direction by the *bhuja*.

Method of drawing the locus of the extremity of the shadow

41b. The movement of the shadow that is caused on any desired day is to be known thus.

42. Obtain as explained earlier, the shadow, *bhuja* and *koṭi* that are formed at any desired time. With three sticks equal to them form a triangle.

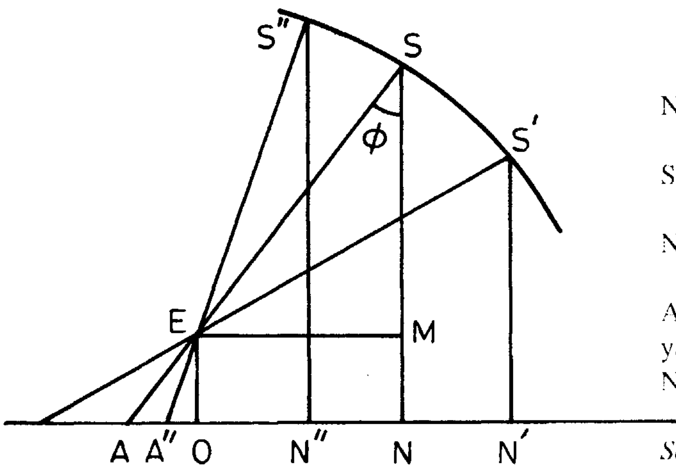
43-44. From the centre of the circle, having placed the *koṭi* along the east-west direction and having placed the *bāhu* also (along the north-south direction), having marked the point (of intersection) of the *bāhu* and the shadow, mark another point (similalry) in the afternoon also. The third point is the tip of the midday shadow.

45-46. Draw the three circles with three points as centres such that (at least) two *matsya* (fishes) are formed. In the middle of these two fishes, two strings are passed. Where their intersection is seen, there (lies) the centre of the circle. Draw the circle which passes through the three points. The locus (of the extremity) of the shadow becomes that (circle) on that day at all times.

Note : The locus of the extremity of the shadow being stated to be a circle is only an approximation. In the *Mahābhāskarīyam* of Bhāskara I (550-628 A.D.) ch. III verse 52, in the *Pañcasiddhāntika* ch. 14, ślokas 14-15, and in the *Śiṣyadhī vṛddhida* of Lalla ch 4, ślokas 43-46, we find that the locus is stated to be a circle. Arka Somayāji in his *Siddhānta Śiromaṇi* explains in detail the method to determine the locus (pp. 267-269)

It is interesting to note that this point is discussed in detail by the commentator of *Yukti Dīpika* as follows, (ch III, verse 246): "What has been said about the movement of the shadow is just a routine one, the movement of the tip of the shadow in a circle is not established. This is indicated here only because of following the earlier teachers" "*purvācāryānurodhena kevalam tadihoditam*'

Alternate method for finding the bhuja of the shadow



NA = SN. tan ø

SM = *mahāśaṅku*

NA = *śaṅku agra*

AO = *chāyā bāhu*
yāmye,*mahābāhu* is
NA + AA[1] = NA[1]

Soumye, mahābāhu is
NA - AA'' = NA'' (Fig. 11)

Fig. 11

47-48. The *mahā śaṅku* multiplied by R Sin ø and divided by R Cos ø is *śaṅku-agra* (NA). It is always south. If the sun is in the southern hemisphere, it should be added to the *arkāgra*. The result then is the *mahābāhu*. If in the northern hemisphere, the *mahābāhu* is the difference of the two *agras*. Here also if the *śaṅku agra* is greater (than the *arkāgra*) (the *mahābāhu*) becomes southern, otherwise northern. (This is for *viṣuvat* only).

49. That (the *mahābāhu*) multiplied by the hypotenuse of the shadow in *aṅgulas* and divided by R is (the *chyāyābāhu*) in *aṅgulas*. This will be in the opposite direction. It is also equal to the one obtained earlier.

50. Or else (*chāyābāhu*) is equal to the *mahābāhu* multiplied by twelve and divided by the *mahaśaṅku*. (i.e. $OA = \frac{OE \times NA}{NS}$)

The value in *aṅgulas* (of *chāyābāhu*) is obtained like wise or from the shadow.

To find the samaśaṅku (Sine altitude of the sun on the prime vertical) and the true longitude of the sun.

51. When R Sine of the northern declination of the sun is less than the R Sine of the latitude (R Sin δ < R Sin ø), that multiplied by R, divided by R Sin ø gives the value of the *śaṅku* (when the sun is) on the prime vertical.

Note : · S is the sun on the prime vertical.

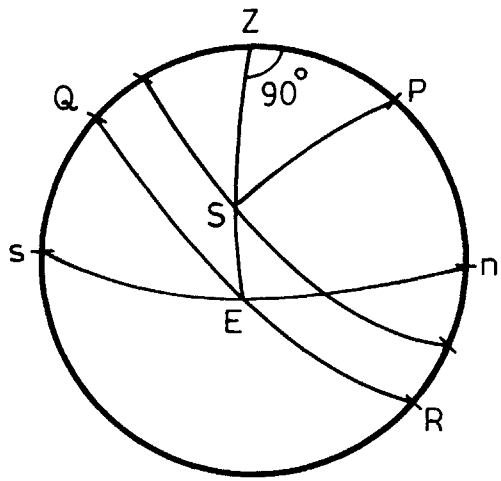


Fig.12

$$R \text{ Cos } Z S = \frac{R \text{ Sin } \delta . R}{R \text{ Sin } \phi}$$

(or) $\text{Cos } z = \frac{\text{Sin } \delta}{\text{Sin } \phi}$ which follows by applying cosine formula for Δ PZS Here R Cos ZS $\leq$ R Hence, the condition R Sin $\delta \leq$ R Sin ϕ (Fig. 12)

52. (The *samamandala śaṅku*) is multiplied by R Sin ϕ, and divided by the R Sine of maximum declination (R Sin w) gives the R Sine of the longitude (R Sin O). Its arc or that deficient from 180° is the *sāyana* longitude of the sun.

In $\Delta \gamma SD$, $\frac{R \text{ Sin } O}{R} = \frac{R \text{ Sin } \delta}{R \text{ Sin } \omega}$

$\therefore R \text{ Sin } O = R \text{ Cos } ZS \times R \text{ Sin } \phi \times \frac{1}{R \text{ Sin } \omega}$,
as stated in the *śloka*.

To find the hypotenuse of the samamaṇḍala śaṅku

53. Multiply separately the *lambaka* and *akṣajyā* by the equinoctials shadow 's' and 12. Divide both by R Sin δ. The two results give the true values in *aṅgulās* of the two hypotenuses when the sun is on the *samamaṇḍala*.

Note : *karṇa* $= \frac{R \text{ Cos } \phi . s}{R \text{ Sin} \delta} = \frac{R \text{ Sin} \phi . 12}{R \text{ Sin} \delta}$

To find the karṇa by an alternate method:

54, 55a. When the midday shadow lies between the *viṣuvat rekhā* (equinoctial line) and the *samarekhā* (east-west line) the hypotenuse of that midday shadow is multiplied by the equinoctial shadow 's', and is divided by the *agrā* in *aṅgulas* of that midday. The result is the *karṇa* in *aṅgulas* of the *samamaṇḍala*.

Note : Commentator Śaṅkara Vāriar states that 'the midday *agrā aṅgula* is obtained by subtracting the midday, shadow in aṅgulas from the equinoctial shadow in aṅgulas.' See *ślokas* 37-39 also.

To find the prāṇas elapsed or yet to elapse from samamaṇḍala śaṅku

55b-56. *Samamaṇḍala śaṅku* (R Cos ZL) is multiplied by the *lambaka*, and divided by R. The result is called *unmaṇḍala vṛttajyā*. (It is the R Sine of the arc of the diurnal circle measured upwards from the point of intersection of the diurnal circle and the *unmaṇḍala* circle PEP'W. Hence *unmaṇḍala vṛttajyā* is R Sin KL (Fig. 13)

This is multiplied by R and divided by *dyujyā*, R Cos δ. That arc, added with the *caracāpa* is indeed the *asus* (or *prāṇās*) that are gone (in the forenoon) and yet to elapse in the afternoon.

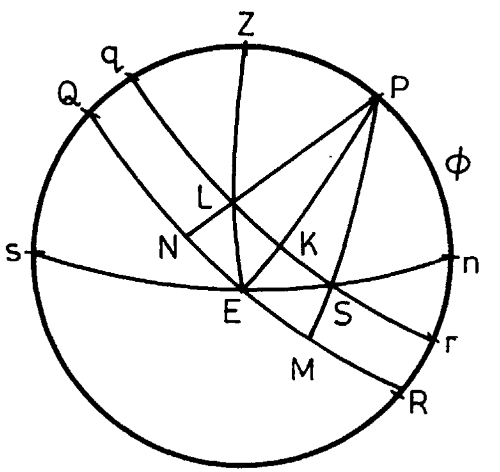


Fig. 13

$$R \cos ZL = \frac{R \operatorname{Sin}\delta .\, R}{R \operatorname{Sin} \phi} \quad (\text{śloka } 51)$$

$$\frac{R \operatorname{Sin} KL}{R \operatorname{Sin} EN} = \frac{R \operatorname{Cos} \delta .\, R \operatorname{Cos} \delta}{R}$$

$$R \operatorname{Sin} KL = \frac{R \operatorname{Cos} ZL .\, R \operatorname{Cos} \phi}{R}$$

∴ EN is obtained as indicated in the śloka, *prāṇas* that are gone = MPN = ME + EN.

In modern spherical Trig. KL = EN Cos δ

Unmaṇḍala is the great circle through P and E.

Unmaṇḍala Vṛttajyā is R Sin KL.

To find the nata (hour-angle) from Sama śaṅku.

57. *Samamaṇḍala śaṅku* is multiplied by *lambaka*, and then divided by *dyujyā*. Deduct the square of the result from the square of *trijyā*. Take the square root and convert into arcs. That indeed is the (hour-angle) nata in *asus*.

Note : The *śloka* gives $h = \sin^{-1} \sqrt{R^2 - x^2}$ where

$$x = \frac{\operatorname{Sin} \delta}{\operatorname{Sin} \phi} . \operatorname{Cos} \phi . \frac{1}{\operatorname{Cos} \delta} \quad \text{or} \quad x = \operatorname{Cos} h .$$

From the figure under śloka 56, we have in Δ PZL,

$\text{Cos ZL} = \text{Cos z} = \text{Sin } \phi \text{ Sin } \delta + \text{Cos } \phi \text{ Cos } \delta \text{ Cos h}.$

and $\text{Cos}(90\text{-}\delta) = \text{Sin } \delta = \text{Cos z. Sin } \phi$

hence $\frac{\text{Sin } \delta}{\text{Sin } \phi} - \text{Sin } \phi \,.\, \text{Sin } \delta = \text{Cos } \phi \text{ Cos } \delta \text{ Cos h}$

which, $\text{Cos h} = \frac{\text{Sin } \delta \text{ Cos}^2 \phi}{\text{Sin } \phi} \cdot \frac{1}{\text{Cos } \phi} \cdot \frac{1}{\text{Cos } \delta} = x$ implies,

as given in the *śloka*.

An alternate method for nata.

58. The *samamaṇḍala chāyā* (R Sin ZL) is muliplied by R and divided by *dyujyā*. (The result) converted into arcs is the *nata* in *prāṇas*. Otherwise also, (the *nata* is obtained) with the complement (*koṭi*) of the (great) shadow then,

Note : It gives $h = \text{Sin}^{-1} \left\{ \frac{\text{R . Sin ZL . R}}{\text{R Cos } \delta} \right\}$ which follows by applying sine formula to ΔPZL.

Otherwise, suggests finding R Cos ZL from R Sin ZL and then calculating h from R Cos ZL, as per *śloka* 57.

To find Kṣitijyā (R Sin SK) from samaśaṅku

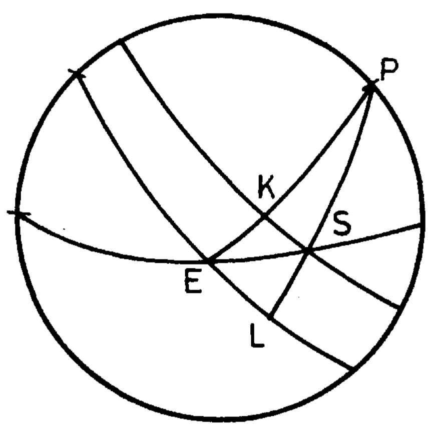


Fig. 14

59. Two *samaśaṅkus* are multiplied by *akṣajyā* (and kept separately). Each is divided separately by R and *lambaka*. The two results are respectively *kranti* and *arkāgra*. The square root of the difference of their squares is *kṣitijyā*.

Note : We thus have, $R \operatorname{Sin} \delta = \frac{R \operatorname{Cos} ZL . R \operatorname{Sin} \phi}{R}$ and

arkāgra $= \frac{R \operatorname{Cos} ZL. R \operatorname{Sin} \phi}{R \operatorname{Cos} \phi}$ we know $\operatorname{Cos} ZL = \frac{\operatorname{Sin} \delta}{\operatorname{Sin} \phi}$

Hence kṣitijyā = $\operatorname{Sin} SK = \sqrt{\operatorname{Sin}^2 \delta - \frac{\operatorname{Sin}^2 \delta}{\operatorname{Cos}^2 \phi}} = \frac{\operatorname{Sin} \delta \operatorname{Sin} \phi}{\operatorname{Cos} \phi}$

Using Sin EL = Tan ø, Tan δ, and sin SK = sin EL. Cos δ (Fig. 14), we get the result.

Ten Problems, Daśapraśnā:

60. Here out of the five, *śaṅku* (R Cos z), *nata* (R Sin h) *krānti* (R Sin δ), *dikagra* (R sin a) and *akṣa* (R sin ø) obtaining any two of them given the other three gives rise to ten different possibilities.

61. *Nata, krānti, dikagra* and *akṣa* alongwith the *śaṅku* and similarly with the nata (take) *apakrama, dikagra* and *akṣa. Dikagra* and *akṣa* are taken with *krāntī*.

62a. *Dik* (*agra*) and *akṣa* are taken as pairs. These are obtained from the other three.

Note : These ten problems are verified using modern formulae. R.C. Gupta, M.I.T. Institute, Mesra, Ranchi has also given the solutions to these problems. (*Indian Journal of History of Science*, **9**(1). 1974 pp. 86-99). K.V. Sarma and S. Hariharan, state that the rationale for these problems are documented in *Yuktibhāsā* and the *Yuktidīpika*, a commentary on *Tantra saṅgraha*. (*Yuktibhāsā* of Jyeṣtadeva, *IJHS*. **26**(2) 1991, page 196)

Given Sin δ , Sin a, Sin ø to find Cos Z, sin h.

62b. The *Āśagrā* (R Sin a) multiplied by *lambaka* (R Cos ø) and divided by R is the koṭi (Sin a Cos ø)

63a. R Sin ø is the *bhujā*. The hypotenuse is the square root of the sum of their squares. It is the divisor (K).

$$K = \{(\operatorname{Sin} a \operatorname{Cos} \phi)^2 + \operatorname{Sin}^2 \phi\}^{\frac{1}{2}}$$

64. Obtain the product of the *krānti* and *akṣa* (A = Sin δ. Sin ø) and that of the two *koṭis*. $B = \sqrt{(K^2 - \operatorname{Sin}^2 \delta)(K^2 - \operatorname{Sin}^2 \phi)}$ of these two products, their sum (when the sun is) in the northern hemisphere and their difference in the southern (hemisphere is taken).

65. If the first product is greater then the second (A>B) in the northern hemisphere, both the sum and the difference, are multiplied by R and divided by the square of the Divisor (K). The result gives the Śaṅku that is formed in the desired direction.

Śaṅkara Vāriar states that $\frac{A+B}{K^2}$, $\frac{A-B}{K^2}$ both are values of *śaṅku*. The first will be to the north of east-west line and the second to the south of it.

66a. The shadow is its complement (*koṭi*) *ślokas* give

$$\text{Cos } z = (\text{Sin } \delta . \text{Sin } \phi \pm \sqrt{K^2 - \text{Sin}^2\delta} . \sqrt{K^2 - \text{Sin}^2\phi} + K^2$$

where $K^2 = (\text{Sin a Cos } \phi)^2 + \text{Sin}^2 \phi$

When a quadratic equation in x is given as follows,
a Cos x + b Sin x = c, then

$$\text{Cos } x = \frac{c.a \pm \sqrt{a^2 + b^2 - a^2} . \sqrt{a^2 + b^2 - c^2}}{(a^2 + b^2)}$$

$$\text{Sin } x = \frac{c.b \pm \sqrt{a^2 + b^2 - b^2} . \sqrt{a^2 + b^2 - c^2}}{(a^2 + b^2)}$$

we can prove that, in Δ PZS,

Sin ø . Cos z + Cos ø . Cos a . Sin z = Sin δ

$$\text{Hence Cos } z = \frac{\text{Sin } \phi . \text{ Sin } \delta \pm \text{Cos } \phi \ \text{ Cos a } \{ \text{Sin}^2\phi + \text{Cos}^2\phi \text{ Cos}^2 a - \text{Sin}^2\delta\}^{\frac{1}{2}}}{\text{Sin}^2\phi + (\text{Cos } \phi \text{ Cos a})^2}$$

Which is the same as given in 62b-65.

66. .... that (the shadow) multiplied by the complement (koṭi) of the *āśāgra* (R Sin a) and divided by dyujyā (R Cos δ) is the sine of the hour angle. Thus it is formed from the *krānti*, *akṣa* and *dikāgra* (δ, ø, a). It is given that, $\text{Sin h} = \frac{\text{Sin z . Cos a}}{\text{Cos } \delta}$. This follows by applying sine formula to ΔPZS.

67. When the sun is in the southern hemisphere, if the product of *krānti* and *akṣa* is greater than the product of the two *koṭis*, (i.e. A>B. śl. 64), the result (given above) does not become the *śaṅku*. (So also) in the northern hemisphere if (R Sin δ) *apakrama* is greater then the divisor.

Note : Śankara Vāriar offers the comment for this śloka 67. Under this problem he gives 4 examples. The values calculated using Log tables tally to a great extent with the answers given by the commentator. In all he gives 40 such examples under *Daśa praśna*. Problem 2 says, given h, a, ø to find z and δ.

68a. From the product of the *nata* (Sin h) and *lambaka* (R Cos ø), the *nata* of the locaity (*svadeśa nata*) is obtained by dividing it by R.

The *svadeśanata* is 'R Sin ZM, where ZM is drawn perpendicular to PS from Z. This could be verified by sine formula applied to triangle PZM.

68. The product of *nata* and *akṣajyā* is divided by the *koṭi* of the *svadeśanata*. The result is *yat*, (यत), something say (x).

Note : $\therefore x = \frac{\text{Sin h . Sin } \phi}{\text{Cos ZM}}$

In Δ PZM

$\frac{\text{Sin h}}{\text{Sin ZM}} = \frac{\text{Sin } 90^\circ}{\text{Sin } (90 - \phi)}$ $\therefore$ Sin h . Cos ø = Sin ZM. (*śvadeśu nata*)

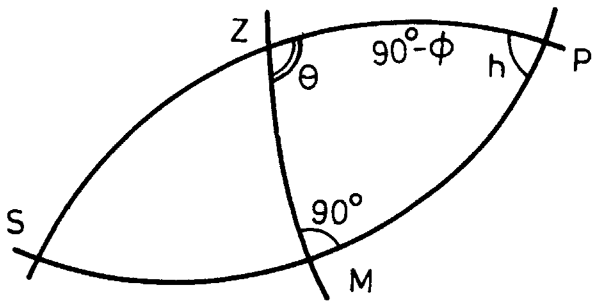


Fig. 15

It can be proved that '*yat*' of the *śloka* is R Cos θ, by applying the formula Sin C. Cos a = Cos A Sin B + Sin A. Cos B. Cos C where A = 90°, B = θ and C = h.

69. That (R Cos θ) is multiplied by *āśāgra* (R Sin a). (To the result) the product of their complements (Rsin θ . RCos a) is added if (the *āśāgra* is) north and subtracted if it is South. Then divide by R.

we thus get $\frac{1}{R} . \{R \text{ Cos } \theta . R \text{ Sin a} \pm R \text{ Sin } \theta . R \text{ Cos a}\} = K$

70. From that which is obtained, multiplying by the *svanatakoṭi* and dividing by R, keep it separately.

We get $\frac{K . R \text{ Cos ZM}}{R} = x.$

70b, 71a. Its square is added to the square of the *svanata*. By the square root, of the result that which is kept separate is divided. The result becomes the *śaṅku*. The shadow becomes its complement.

Thus $R. \text{ Cos } z = \frac{x}{\{x^2 + (R \text{ Sin ZM})^2\}^{\frac{1}{2}}}$

i.e. $\text{Cos } z = \frac{\text{Cos ZM} . \{\text{Cos } \theta \text{ Sin a} \pm \text{Sin } \theta . \text{Cos a}\}}{\{\text{Cos}^2 \text{ ZM } (\text{Cos}\theta \text{ Sin a} \pm \text{Sin } \theta \text{ Cos a})^2 + \text{Sin}^2 \text{ ZM}\}^{\frac{1}{2}}}$

$$= \frac{\text{Cos ZM . Sin}(a \pm \theta)}{\{\text{Cos ZM Sin}(a \pm \theta)^2 + \text{Sin}^2 \text{ZM}\}^{\frac{1}{2}}}$$

In Δ ZSM(fig. 16(a)) if ψ = SZM we have $\theta + \psi$ = PZS = 90 —a

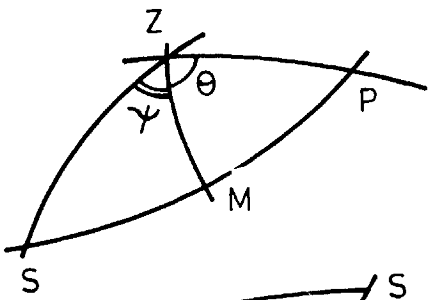


Fig. 16(a)

$\therefore$ $a + \theta = 90 - \psi$

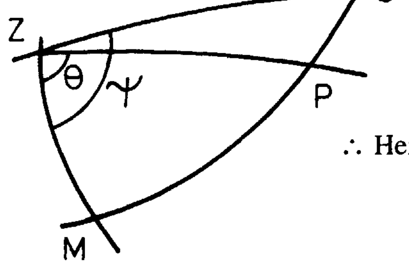
In fig. 16(b) $\psi - \theta = 90 - a$

$\therefore$ $a - \theta = 90 - \psi$

Hence $a \pm \theta = 90 - \psi$

Hence Sin $(a \pm \theta)$ = Cos ψ

Cos ψ = Tan ZM Cot Z, from Δ SZM.



$$\therefore \text{ Hence } = \frac{\text{Sin ZM. Cot z}}{\{\text{Sin}^2 \text{ZM} (\text{Cot}^2 z+1\}^{\frac{1}{2}}} = \frac{\text{Cot z}}{\text{Cosec}^2 z} = \text{Cos z}$$

Fig. 16(b)

71b. The product of the shadow and the complement of the *agra* (i.e. Sin z. Cos a) (is taken). *Dyujyā* (R Cos δ) is obtained from the *Natajyā*.

Śaṅkara Vāriar clarifies that the above product when divided by the natajyā results in *dyujyā*. Hence, $\frac{\text{Sin z . Cos a}}{\text{Sin h}}$ = Cos δ (this gives δ)

72-73. (The Product) of *natajyā* and *dyujyā* (R Sin h. R Cos δ) and in the same way (the product) of the complement of the shadow (Sin h. Cos δ) and R and the product of the shadow and the complement of the *digagra* become the same. Hence out of the two, dividing one by an element, the other element related with it is got. *Apakrama* (R Sin δ) is the square root of the difference of the squares of R and *dyujyā*.

Problem 3 says, Given h, δ, ϕ to find z and a.

74, 75a. The *dyujyā* is multiplied by the complement of the *natajyā* and divided by R. To the result is either added or subtracted the *Bhūjyā*, according as (the declination) is northern or southern. (The result) multiplied by the *lambaka* and divided by R, is the *śaṅku*. The complement of the *āśāgra* (is found) as explained earlier by (using the value) of *dyujyā*.

Note: Ch. II. śl. 27 defines *kṣitijyā* or *bhūjyā* as $\left\{\frac{\text{Sin } \phi \, . \, \text{Sin } \delta}{\text{Cos } \phi} + \text{Cos } \delta \text{ Cos h}\right\} . \text{ Cos } \phi$

By using cosine formula in ΔPZS, the result can be got.

"dyujyā ca" in 75a should be taken as *"dyujyayā ca"* to make the sense clear.

Problem 4 says, Given h, δ, a to find z and ø

75b. Obtain the shadow. Then find the square root of the difference of the squares of its *koṭi* (*chāyā-koṭi*) and that of *dyujyā*.

Note : It states: "The product of Sin h and Cos δ is divided separately by Sin a and R. The results give shadow and its *koṭi*: Since Sin h . Cos δ = Sin z. Cos a = *chāyākoṭi* in modern notation, the above statement is true.

We now get $\{\text{Cos}^2 \delta \text{-(Sin h . Cos } \delta)^2\}^{\frac{1}{2}} = \text{Cos } \delta \text{ . Cos h.} = x$

75b. That (x) multiplied by the shadow - arm. (*chāyā-bāhu* = Sin. z. Sin a) and also the product of *śaṅku* and *krānti* (Cos z. Sin δ) (are taken as two values यः)

76a. If the declination and the directional amplitude are in the same direction their difference, otherwise their sum (is to be taken),

76b. If the sum is between the *unmaṇḍala* (6 O' Clock circle) and the horizon, their sum (is to be taken).

77-78a. That (sum or difference) is to be divided R and divided by the difference between the squares of R and the *chāyā-koṭi*. The result becomes the sine of latitude, obtained from the three (data) given earlier as *nata* etc.

Note: Here $\text{Sin } \phi = \frac{\text{Cos z . Sin } \delta \pm \text{Sin z . Sin a . Cos } \delta \text{ Cos h}}{1 - (\text{Sin}^2 \text{ z . Cos a})^2}$

In Δ PZS, cos z = Sin ø . Sin δ + Cos h Cos δ . Cos ø

$$c = b \, . \, \text{Sin } \phi + a \, . \, \text{Cos } \phi$$

$$\therefore \text{Sin } \phi = c \, . \, b \pm \sqrt{a^2+b^2-c^2} \, . \, \sqrt{a^2+b^2-b^2} + (a^2+b^2) \text{ gives}$$

$$\text{Sin } \phi = \frac{\text{Cos Z . Sin } \delta \pm \{(\text{Cos h Cos } \delta)^2 - \text{Cos}^2 z\}^{\frac{1}{2}} . (\text{Cos h Cos } \delta) + \text{Sin}^2 \delta}{\text{Sin}^2 \delta + (\text{Cos h Cos } \delta)^2}$$

$$= \frac{\text{Cos z . Sin} \delta \pm (1 - \text{Sin}^2 \text{h}) \text{ Cos } \delta + \text{Sin}^2 \delta - \text{Cos}^2 \text{z . Cos h Cos } \delta}{\text{Sin}^2 \delta + \text{Cos}^2 \delta \, (1 - \text{Sin}^2 \text{h})}$$

$$= \frac{\text{Cos z . Sin} \delta \pm \{\text{Sin}^2 \text{z . Sin}^2 \text{z . Cos}^2 \text{a}\}^{\frac{1}{2}} . (\text{Cos h Cos } \delta)}{1 - (\text{Cos } \delta \text{ . Sin h})^2}$$

$$= \frac{\text{Cos z Sin } \delta \pm \text{Sin z . Sin a Cos h Cos } \delta}{1 - (\text{Sin z . Sin a})^2}$$

Problem 5 says, Given z, a, ø to find h and δ.

78b-79. That which is the product of *akṣa* and *śaṅku* (R Sin ø. R Cos z) and that which is the product of the shadow-arm and *lambaka* ($\frac{\text{R Sin z . R Sin a}}{\text{R}}$. R Cos ø), their sum or differece according as the sun is in northern or southern direction (are taken).

Then (the result) divided by R gives *krānti* (R Sin δ). Calculate the sine of the hour-angle (R Sin h) as described earlier.

Note : Hence Sin δ = Sin ø Cos z ± Sin z . Sin a Cos ø, a result that follows directly by applying cosine formula for Δ PZS. To find Sin h, use the result, Sin h. Cos δ = Sin z . Cos a.

Problem 6 says: given z, δ, ø, to find h and a

80-81 a. That which is the product of R and sine of declination and that which is the product of *śaṅku* and *akṣa* are taken (A = R . R Sin δ, R = R Cos Z . R Sin ø). Their sum or difference according as the sun is on the arc southern or northern. That when divided by the *lambaka* (R Cos ø) is the shadow-arm. From this multiplied by R and divided by the shadow, the *dikagra* as desired (is found).

Note: It is given $\text{Sin a} = \frac{(\text{Sin } \delta \pm \text{Cos z . Sin } \text{ø})}{\text{Cos ø}} \times \frac{1}{\text{Sin z}}$

i.e. shadow-arm, (Sin z . Sin a) = (Sin δ ± Cos z . Sin ø) ÷ Cos ø . This follows by applying cosine formula to Δ PZS.

Problem 7 says: Given z, δ, a, to find h and ø.

81b-82. That which (say, A) is the square root of the difference of the squares of *chāyā-koṭi* and *dyujyā* (is taken). To that (A) the shadow-arm is added. By this result (say, B) divide the square of the sum of *śaṅku* and *krānti* . To that itself (say, C) that (B) is separately added or subtracted. Of these two, the lesser is multi plied by R and divided by the greater, (gives) the sine of latitude.

$$\text{i). } A = \left[(\text{R Cos } \delta)^2 - \left\{\frac{\text{R Sin z . R Cos a}}{\text{R}}\right\}^2\right]^{\frac{1}{2}}$$

$$\text{ii). } A + \frac{\text{R Sin z . R Sin a}}{\text{R}} = B$$

$$\text{iii). } C = \frac{(\text{R Cos z} + \text{R Sin } \delta)^2}{B}$$

$$\text{iv). R Sin ø} = \frac{(C-B)R}{(C+B)}$$

In modern notation $A = \{\text{Cos}^2 \delta - (\text{Sin h Cos } \delta)^2\}^{\frac{1}{2}}$

$= \text{Cos } \delta \text{ Cos h}$, since

$\text{Sin z. Cos a} = \text{Sin h . Cos } \delta$.

$\therefore B = (\text{Cos h Cos } \delta + \text{Sin z . Sin a})$

$$\therefore \text{Sin } \phi = \frac{(\text{Cos z} + \text{Sin } \delta)^2}{B} - B \div \frac{(\text{Cos z} + \text{Sin } \delta)^2}{B} + B$$

$$= \frac{(\text{Cos z} + \text{Sin } \delta)^2 - (\text{Cos h . Cos } \delta + \text{Sin z . Sin a})^2}{(\text{Cos z} + \text{Sin } \delta)^2 + (\text{Cos h . Cos } \delta + \text{Sin z . Sin a})^2} \text{ — I}$$

Consider the following triangles : Δ SZM and Δ SMP (Fig. 17)

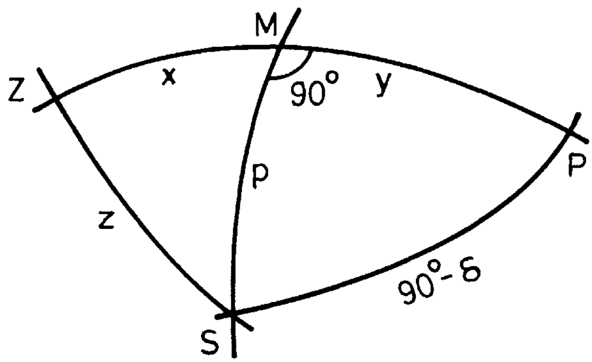


Fig .17

In Δ ZMS, $\text{Cos z} = \text{Cos p . Cos x}$

In Δ PMS, $\text{Sin } \delta = \text{Cos p . Cos y}$ $\therefore$ $\text{Cos z} + \text{Sin } \delta = \text{Cos p (Cos x + Cos y)}$ —II

In Δ ABC, $\text{Sin a Cos c} = \text{Cos c Sin b} - \text{Sin c. Cos b . Cos A}$

In Δ PMS, $\text{Sin } (90 - \delta) \text{ Cos h} = \text{Cos p. Sin y} - \text{Sin p. Cos y. Cos } 90°$.

$\text{Cos } \delta \text{ Cos h} = \text{Cos p. Sin y}$

In Δ ZMS, $\text{Sin z . Cos } (90\text{-a}) = \text{Cos p. Sin x}$

$\text{Sin z . Sin a} = \text{Cos p . Sin x}$

$\therefore \text{Cos h Cos } \delta + \text{Sin z Sin a} = \text{Cos p (Sin x + Sin y)}$ —III

From II and III, R.H.S. of I is $= \dfrac{(\text{Cos x} + \text{Cos y})^2 - (\text{Sin x} + \text{Sin y})^2}{(\text{Cos x} + \text{Cos y})^2 + (\text{Sin x} + \text{Sin y})^2}$

$$= \frac{\text{Cos } 2x + \text{Cos } 2y + 2 \text{ Cos}(x+y)}{2+2 \text{ Cos } (x-y)}$$

$$= \frac{2 \text{ Cos } (x+y)\,(1+ \text{Cos } (x-y))}{2[1 + \text{Cos } (x-y)]}$$

$$= \text{Cos } (x+y) = \text{Cos } (90° - \phi) = \text{Sin } \phi$$

8 th problem says : Given z, h, ϕ to find δ and a

83 b. *Akṣa* and *śaṅku* both are multiplied by R . Both are divided with by *svadeśa natakoṭi* separately.

Note: *Svadeśanaṭa* is R sin ZM, where ZM is perpendicular to PS.

(III. 68a), R Sin ZM = R Sin h. R Cos ϕ ÷ R . Therefore its *koṭi* R Cos ZM

$$= \{ R^2 - \frac{(\text{Sin h R Cos } \phi)^2}{R} \}^{\frac{1}{2}} \text{ or } \{ 1 - \text{Sin}^2 \text{h Cos}^2 \phi \}^{\frac{1}{2}}$$ in modern notation.

We then have $\sin \alpha = \frac{\text{Sin } \phi}{\text{Cos ZM}}$, $\text{Sin } \beta = \frac{\text{Cos z}}{\text{Cos ZM}}$.

$$\text{Cos}^2 \text{ZM} = 1 - \text{Sin}^2 \text{h Cos}^2 \phi = 1 - (1 - \text{Cos}^2 \text{h}) \text{Cos}^2 \phi$$

$$= 1 - \text{Cos}^2 \phi + \text{Cos}^2 \text{h Cos}^2 \phi$$

$$\text{Cos}^2 \text{ZM} = \text{Sin}^2 \phi \, . + \text{Cos}^2 \text{h Cos}^2 \phi$$

84-85a Those which are (Sin α, Sin β) and their complements that are the square roots of the differenece of squares of R and those, are multiplied mutually(crossly) by their complements. Of these (Sin α, Cos β and Sin β Cos α) that which is their sum, when the sun is in the southern direction or their difference when when sun is in the northen direction. Their (value) being divided by R is the *dyujā* (R Cos δ). Sine of declination and that of amplitude(are obtained) as before.

85b. The southern and northen declination are determined from the meridian crossing of the sun

We thus have $\text{R Cos } \delta = \frac{1}{R} (\text{R Sin } \alpha \, . \text{ R Cos } \beta \pm \text{R Cos } \alpha \, . \text{ R Sin } \beta)$

$$\text{Now} \quad \text{Cos}^2 \alpha = 1 - \frac{\text{Sin}^2 \phi}{\text{Cos ZM}} = \frac{\text{Cos}^2 \text{ZM. Sin}^2 \phi}{\text{Cos}^2 \text{ZM}} = \frac{\text{Cos}^2 \text{h Cos}^2 \phi}{\text{Cos}^2 \text{ZM}}$$

$$\text{Cos}^2 \beta = 1 - \frac{\text{Cos}^2 z}{\text{Cos}^2 \text{ZM}} = \frac{1 - \text{Sin}^2 \text{ZM} - \text{Cos}^2 z}{\text{Cos}^2 \text{ZM}}$$

$$= \frac{\text{Sin}^2 z - \text{Sin}^2 h \,\text{Cos}^2 \phi}{\text{Cos}^2 ZM.}$$

$$\text{Cos}\,\delta = \frac{\text{Sin}\,\phi}{\text{Cos}\,ZM} \cdot \frac{\sqrt{\text{Sin}^2 z - \text{Sin}^2 h\,\text{Cos}^2\phi}}{\text{Cos}\,ZM} \pm \frac{\text{Cos}\,h\,\text{Cos}\,\phi\,.\,\text{Cos}\,z}{\text{Cos}\,ZM\,.\,\text{Cos}\,ZM}$$

$$= \frac{\text{Sin}\,\phi\sqrt{\text{Sin}^2 z - \text{Sin}^2 h\,.\,\text{Cos}^2\phi} + \text{Cos}\,h\,\text{Cos}\,\phi\,.\,\text{Cos}\,z}{\{\text{Cos}\,ZM\}^2} \quad \text{—I}$$

From the result, Cos δ . Cos ϕ . Cos h + Sin . Sin ϕ = Cos z

if a Cos δ + b Sin δ = c, we have cos δ = c . a $\pm$ b $\sqrt{a^2 + b^2 - c^2} \div (a^2 + b^2)$

$$= \frac{\text{Cos}\,z\,.\,\text{Cos}\,\phi\,\text{Cos}\,h \pm \text{Sin}\,\phi\sqrt{\text{Cos}^2\phi\,\text{Cos}^2 h + \text{Sin}^2\phi - \text{Cos}^2 z}}{\text{Cos}^2\phi\,\text{Cos}^2 h + \text{Sin}^2\phi} \quad \text{—II}$$

That I and II are the same could be easily verified.

9th problem says : Given z, h, a to find δ and ϕ .

86. From R sin a, find its complement (R Cos a). The product of this and the shadow (R Sin z), divided by *natajyā (R Sin h)* becomes *dyujyā (R Cos δ)*. It s arm indeed is the *krānti (R Sin δ)*.

$\frac{\text{R Cos a . R Sin z}}{\text{R Sin h}}$ = R Cos δ is a result already given(see III 71 b)

10th Problem says : Given z, h, δ to find a and ϕ

87. When the product of *dyujyā* and *natajyā* is divided by the shadow, the complement of *digagrā* is obtained. *Akṣa* is obtained as described earlier. Thus the answers for the ten problems are described.

Note : R Cos a = $\frac{\text{R Cos d . R Sin h}}{\text{R Sin z.}}$

"The *akṣa* is found from the problem '*Chayam Netva—śloka* 75b. Problem 4" thus states the comentator.

An alternate method to find the shadow at a desired direction.

88a. The equinoctial shadow multiplied by the complement of the *digagrā* and divided by it, is the complement of that (the equinoctial shadow).

Note : Equinoctial shadow s = 12 . $\frac{\text{R Sin}\,\phi}{\text{R Cos}\,\phi}$

∴ It's *koṭi is* $\frac{s \times R \cos a}{R \sin a}$

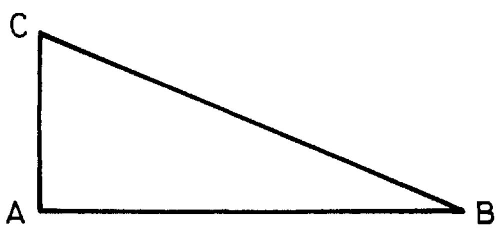
88b. .... the square root of these two is the *svadṛgguṇa*.

$$\{s^2 + s^2 \cot^2 a\}^{\frac{1}{2}} = s \cos a \text{ } svadṛgguṇa.$$

89 a. The square root of the sum of the squares of the gnomon (12 inches) and *dṛgguṇa* is the hypotenuse—of the shadow (*chāyā-karṇa*).

Note : *Chāyā-karṇa* $=\{144 + (s \operatorname{Cosec} a)\}^{\frac{1}{2}} = k$ or

$$k = \frac{12}{\sin a}\sqrt{\sin^2 a + \tan^2 \phi} = \frac{12}{\sin a \cos \phi}\sqrt{\sin^2 a \cos^2 \phi + \sin^2 \phi}$$



AC = 12
BC = *karṇa*
AB = s Cosec a

Fig. 18

89b. Both the gnomon and the shadow are multiplied by R and divided by the *chāyākarṇa* = k
(Both the results give) their true values

Note: Śankara Vāriar states that *śaṅku* and *dṛgguṇa* are to be multiplied by R. Obviously the shadow—*chāyā* —is the *svadṛgguṇa* from triangle ABC.

90 a. The *krānti* (R Sin δ) multiiplied by the (true) *dṛgguṇa* and divided by the *akṣa jyā* (R Sin ø) gives the *apakarma* .

Note: Here *apakarma* is not the usual R and δ. The commentator says it is अपक्रमाधीनः छायाखण्डः ॥

Hence this is taken as $R \sin \alpha = R \sin \delta \cdot \frac{(s \operatorname{Cosec} a) R}{k} \times \frac{1}{R \sin \phi}$

$$\therefore \sin \delta = \frac{\sin \delta \cdot 12. \sin \phi}{k \cos \phi \cdot \sin a} \ldots\ldots \frac{1}{\sin \phi}$$

90 b. Of these two *krānti* (*chāyākhanda*,, R Sin α) and the (true) *dṛgguṇa*, their complements are the square-root of the difference between R^2 and their squares.

Note : True *Dṛgguṇa* = $\text{Sin}\,\theta = \dfrac{12.\ \text{Sin}\,\phi}{k.\ \text{Sin a Cos}\,\phi}$

91. When these two-*krānti* and true *dṛgguna*—are multiplied crossly by their complements, and then divided by R, their sum of difference is taken according as the sun, is in the southern or northern hemisphere, is the shadow.

Note : $\text{Sin z} = \text{Sin}\,\alpha\ \text{Cos}\,\phi \pm \text{Cos}\,\alpha\ \text{Sin}\,\phi$ —I

Since $\text{Sin}\,\phi.\ \text{Cos z} + (\text{Cos}\,\phi\ .\ \text{Sin a}).\ \text{Sin z} = \text{Sin}\,\delta$, (or)

We have $\text{Sin z} = \dfrac{c.b \pm \sqrt{a^2 + b^2 - c^2}}{a^2 + b^2}$ —II

R. H. S. of I and II should be proved to be equal.

We have

$$\text{Sin}\,\alpha = \frac{12.c}{kb}\ \therefore \text{Cos}\,\alpha = \frac{\sqrt{k^2b^2 - 144\,c^2}}{kb} = \frac{12}{kb}\sqrt{a^2 + b^2 - c^2} \text{ and } kb = 12\sqrt{a^2+b^2}$$

$$\text{Sin}\,\phi = \frac{12.a}{kb},\ \text{Cos}\,\theta = \frac{\sqrt{k^2b^2 - 144\,a^2}}{kb} = \frac{12b}{kb} = \frac{12}{k.}$$

It now follows that, $\text{Sin}\,\alpha.\ \text{Cos}\,\theta \pm \text{Cos}\,\alpha\ \text{Sin}\,\theta$

$$= \frac{144\,c}{k^2\,b} \pm \frac{144\,a}{k^2\,b^2}\sqrt{a^2+b^2 - c^2}$$

$$= \frac{144}{k^2.\,b^2}\{c.b \pm a\sqrt{a^2 + b^2 - c^2}\}$$

$$= \frac{1}{a^2+b^2}\left\{c.b \pm a\sqrt{a^2 + b^2 - c^2}\right\}$$

$= \text{Sin z}$.

An alternate method to find *koṇa* , *śaṅku* and shadow.

koṇa , *śaṅku* —when the sun is on the verticale with angle PZS being 45 °. Hence *agrā*, a = 45 °

From śl. 91, $\text{Sin } z = \dfrac{c.b \pm a\sqrt{a^2 + b^2 - c^2}}{a^2 + b^2}$

Where a = Sin ø, b = Cos ø. Sin a, c = Sin δ

Putting a = 45 : we get $\text{Sin } z = \dfrac{\text{Sin } \delta \dfrac{\text{Cos } \phi}{\sqrt{2}} \pm \text{Sin } \phi \sqrt{\text{Sin}^2\phi + \dfrac{\text{Cos}^2\phi}{2} \text{Sin}^2 \delta}}{\left\{\text{Sin}^2 \phi + \dfrac{\text{Cos}^2\phi}{2}\right\}}$

This result is given in the following ślokas.

92a. The arm is *aksa* (R Sin ø). The *koṭi* is the square-root of half the square of *lambaka* ($R \sqrt{\dfrac{\cos^2 \phi}{2}}$). Their hypotenuse is the divisor (i.e. $\text{Sin}^2\phi + \dfrac{\text{Cos}^2}{2}$ in modern notation).

92b. (Take) the product of *krānti* and *koṭi* ($A = \text{Sin } \delta . \dfrac{\text{Cos } \phi}{2}$). (Take) also the product of the *akṣas* and the *koṭi* of the *krānti* and the hypotenuse, the divisor B is as follows:

$$B = \text{Sin } \phi \; \{\text{Sin}^2\phi + \frac{\text{Cos}^2\phi}{2} - \text{Sin}^2\delta\}^{\frac{1}{2}}$$

Note : The commentator makes this passage clear by stating:
tasya karṇasya apakarmasya ca vargāntara mūlam tat koṭi . 1

93a. Of these (A and B) sum when sun is in the southern and difference in the northen is the shadow.

$$\therefore \text{Shadow Sin } z = \frac{A \pm B}{\text{divisor}}$$

93b. If the complement of *akṣa* (R Cos ø) is more than that of the *krānti* (R cos δ), then, the shadow in the northern hemisphere is their sum.

(com.: He says : सौम्य गोले तद्योगतोऽप्यल्या कोणच्छाया स्यात् ।)

Thus there are two such *Koṇa chāyās*.

94. The gnomon (*nara*) is the difference or sum of the (mutual) products of *krānti* and *akṣa* and their complements. In the same manner two (*koṇa śaṅku* are obtained) in the northern (hemisphere). But the formation of two *koṇa śaṅku* does not exist in the other (southern hemisphere)

95a. The shadow and the hypotenuse are multiplied by 12 and divided by (their) gnomons (*śaṅku*). The two are in *aṅgulas*.

Rising point on the ecliptic at the east, *prāglagna*.

95b. From the position of the sun corrected for the *āyana calana*, the minutes to be elapsed in the particular *rāśi* (are calculated).

96a. That is multiplied by the rising-*prāṇas* related to that *rāśi*; (the result) is divided by the minutes of that *rāśi*.

96b. That is then subtracted from the minutes that have elapsed from rising to the instant that is desired. The result is the balance in *asus* to elapse in that *rāśi*; that should be diminished from the *asus* that are gone (from the rising time to the desired instant).

97. Subtracting from the remainder, the *prāṇās* of the signs that follow one after another, completing the sign of the sun, again add those - *rāśis*.

98. The remaining of the *prāṇas* (after finding various *rāśis*) is multiplied by 30°, and divided by the *prāṇas* at rising of that sign immediately following, the result so obtained is added to the sun then.

99. The result again is multiplied by 60, and divided by the same quantity (*svadeṣa udaya prāṇa*). Thus the rising point of tjhe ecliptic in minutes is to be obtained. The setting point is (got by) adding 6 (signs).

100a. The *ayana* correction is done in the reverse way to get the *nirāyana lagna* in that well-known meṣa etc. form. The approximate value of the above process is indicated.

100b-101. In each sign, the (rising) time varies steadily. Since the rule of three cannot be applied for that reason, the orient ecliptic point obtained thus, is only approximate and not accurate.

To obtain the accurate (subtle) value, he first explains the method to determine *kāla lagna* (ecliptic point at a given time).

102a. Obtain as described earlier, the arm in *prāṇas* of the *sāyana arka* (position of sun with precession taking into account) and after making the correction for the cara also.

Commentator states, 'the R Sin λ of the sun, is multiplied by R Cos w and divided R Cos δ; the result converted into arcs gives the arm of the *sāyanarka*'.

$$R \text{ Sin } \lambda = \frac{R \text{ Sin } \lambda \text{ . } R \text{ Cos } \omega}{R \text{ Cos } \delta}$$

This result is given earlier (Ref: Ch. II. śl. 26)

102b. In the first quarter, *kālalagna* is, the result of subtracting that (*svacara*) from the *sāyanarka bhuja;* in the second quarter, *svacara prāṇa* is subtracted from six *rāśis* (i.e. 180°.); in the other quarter (the third) add the sum of *svacara* and *sayanarka bhuja* to six *rāśis*; again in the last quarter (fourth), the value deficint of that (*cara* subtracted from *sayanarka*) from a circle (12 *rāśis*, 360°) becomes the time of the ecliptic point of the rising sun.

104a. For the *prāṇas* that have elapsed also, the corresponding degrees and minutes are to be found for the *sāyana lagna* from *viṣuvat* etc.

Dṛk Kṣepa: The sine of the Zenith distance of Nonagesimal.

104b-105. The complement of the last sine (R Cosine of the maximum obliquity) is multilied by R Sin ø, and that divided by R (is kept separate; x). That which is the product of the *lambaka* (R Cos ø) and the complement of the ecliptic-point at that time is divided by 8452 (is also kept separately). Their difference or sum is the sine of *dṛk kṣepa*, according as the time (of the *kālalagna*) is in Karkaṭa (cancer) or Mṛga (Capricorn).

Note : *Dṛkṣepa* or nonagesimal is the point V on the ecliptic(fig. 19) 90° from the point of intersection of the horizon and the ecliptic. It is also called *vitṛbhā*. It could be proved that the point ZV is the verticle and hence ZVA is 90° (see *Siddhanta Śiromani*, pp. 410–411, Arka Somayāji).

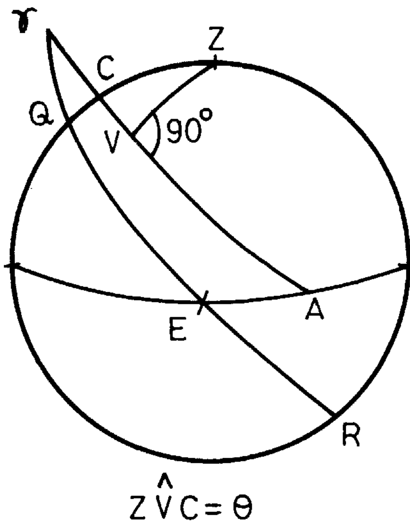


Fig. 19

Now $x = \text{Cos}\ \omega\ \text{Sin}\ ø$.

$y = \text{Cos}\ ø\ .\ \text{Cos}\ r\ E\ .\ \text{Sin}\ \omega$

Since $8452 = \dfrac{R}{R\ \text{Sin}\ \omega}$

ślokas give ,

$\text{Sin}\ ZV = \text{Cos}\ \omega\ \text{Sin}\ ø \pm \text{Cos}\ ø\ \text{Cos}\ \gamma E.\ \sin\ \omega$—I

Now in Δ ZVC, $\text{Sin}\ ZV = \text{Sin}\ ø\ .\ \text{Sin}\ ZC$

$= \text{Sin}\ ø\ .\ \text{Sin}\ (ø \pm QC)$

$= \text{Sin}\ ø\ (\text{Sin}\ ø\ .\ \text{Cos}\ QC \pm \text{Cos}\ ø\ \text{Sin}\ QC)$

In Δ YQC using $\text{Cos}\ A = — \text{Cos}\ B.\ \text{Cos}\ C + \text{Sin}\ B\ \text{Sin}\ C\ .\ \text{Cos}\ A$

$\text{Cos}\ W = — \text{Cos}\ ø\ \text{Cos}\ 90° + \text{Sin}\ ø\ \text{Sin}\ 90°\ \text{Cos}\ QC$

$\therefore\ \text{Sin}\ ZV = \text{Sin}\ ø\ \text{Cos}\ \omega \pm \text{Cos}\ ø\ .\ \text{Sin}\ ø\ \text{Sin}\ QC$

$= \text{Sin}\ ø\ \text{Cos}\ \omega \pm \text{Cos}\ ø\ .\ \text{Sin}\ \omega\ \text{Sin}\ YC$

$= \text{Sin}\ ø\ \text{Cos}\ \omega \pm \text{Cos}\ ø\ .\ \text{Sin}\ \omega\ (-\ \text{Cos}\ YE)$, which is I

106. In case of difference, if the (*lagna phala*) from R cos ø is greater than that obtained from R sin ø, then the sine of Nonagesimal is in the northern direction;

otherwise it is always southern. That square subtracted from R^2 and the square root of the result obtained thus is the complement of the *dṛk kṣepa*.

Dṛkkṣepa Lagna :

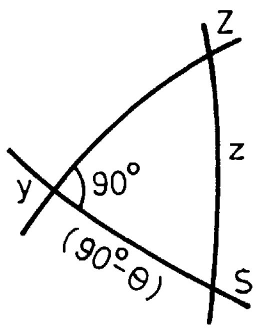


Fig. 20(a)

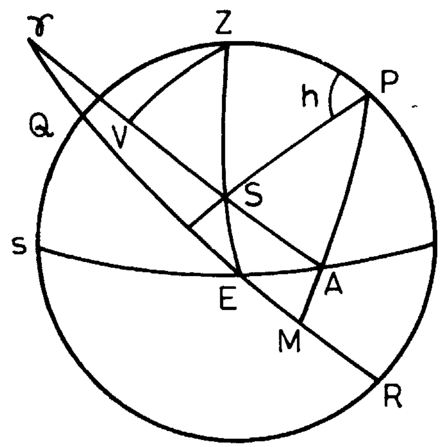


Fig. 20(b)

107 a. Either the *prāṇas* elapsed or yet to elapse from the midday(are subtracted). (The balance is the *naṭa* obtained from the midday). In the night, it is the *prāṇa* yet to elapse (*unnata*).

107 b. If that is the *nata*, then the result of subtracting the *bāṇa* (*utkramajyā*, R—R Cos h) from R is added to the *carajyā*. (i.e., we get R Cos h + R Cos h + R Sin EM) ; If it is *unnata*, (the result) deficient from *carajyā* (is taken) (i.e. R Sin EN - R Cos h). (This is for the northern hemisphere), For the southern hemisphere, the (reverse) process(is to be followed).

108.b. This multiplied by the product of *dyujyā* and *lambaka*.

109. Let x = (R Sin EM $\pm$ R Cos h). R Cos ø . R Cos δ

This is divided by R. Once again divided by the complement of *nṛk kṣepajyā* (i.e. x ÷ R. R cos ZV). To arc that is obtained, add the sun (longitude of S i.e. rS). That is the *lagna* in the eastern hemisphere. For the night it is (the result) reduced from the sun.

110. In the western hemisphere it is the setting point of the ecliptic. For the day and night reverse the process. The middle of the eastern and western *lagnas* is known as *dṛkkṣepa lagna*.

Note : If A is *prāk-lagna* and A' *paścāt-lagna*, then AV = VA = 90° where V is the nonagesimal. (see *Siddhānta Siromani* p. 411).

The author gives $\gamma A = \gamma S + \text{Sin}^{-1}\left\{\frac{X}{\text{Cos ZV}}\right\} = \gamma S + \phi$ (say)

To show that, SA = θ , (or) $\text{Sin}\,\theta = \frac{1}{\text{Cos ZV}}\left\{\text{Sin EM} \pm \text{Cos h}\right\}.\ \text{Cos}\,\phi\ \text{Cos}\,\delta$ —I

Since $\text{Sin EM} = \frac{\text{Sin}\,\phi\ .\ \text{Sin}\,\delta}{\text{Cos}\,\phi\ .\ \text{Cos}\,\delta}$ (Refer II, 27b-II, 28a)

and $\text{Cos z} = \text{Sin}\,\phi\ \ \text{Sin}\,\delta + \text{Cos}\,\phi\ \text{Cos}\,\delta\ \text{Cos h}$

I becomes $\text{Sin}\,\phi = \frac{1}{\text{Cos ZV}}\left\{\text{Sin}\,\phi\ \text{Sin}\,\delta + \text{Cos z} - \text{Sin}\,\phi\ \text{Sin}\,\delta\right\}$

$\text{Sin}\,\phi\ .\ \text{Cos ZV} = \text{Cos z}$, if we take + sign only —II

But in Δ ZVS, $\text{Cos z} = \text{Cos}\,(90° - \phi).\ \text{Cos ZV}$

$= \text{Sin}\,\phi\ .\ \text{Cos ZV}$ which is —III

To find the Madhya- lagna, Meridian Ecliptic point:

111a. The *kālalagna* (ecliptic point at the given time) deficient by three *rāśis* is the *madhya kāla* (i. e. it is the portion of the equator from the point of intersection of the equator and meridian. Commentator Śankara Vāriar. It is γQ).

111 b. From that once again obtain the difference in *prāṇas* and add with that arc of the longitude.

Note: (Commentator makes clear this abstruse *śloka*): " From the value of γQ *madhyakāla*, obtain the R Sine of the longitude, R Sin RC. Then by the rule that states 'multiplying by the R Sin of 24° , before finding R Sin δ. R Cos δ find the difference in minutes of the longitude in *prāṇas* and add it to the arc of the longitude of the *madhya kāla*".

112. Then obtain the *asūs* as before and calculate the difference in seconds(*liptā*). This should be added to the arc of the longitude of the (*madhya*) *kāla*. From that (calculate) the difference in *prāṇas*.

113. 114 a. Adding this to the arc of the longitude of (*madhya*) *kāla*, the arc is calculated by iteration. That itself is the meridian ecliptic point (*madhya lagna*)., in the first quarter, at that instant. For the II onwards, the *madhya lagna* is obtained as before.

Note : Com: for the II, III, IV quarter. The process mentioned in *śloka* 102 b - 103 is to be followed.

The method of finding *madhya lagana*, without the above process of successive approximation(iteration) is given:

114. 115b. The *madhya lagna* is obtained without the process of iteration, as in the manner mentioned, The R Cosine of *madhya kāla*(R Cos γQ)(is multiplied) by the R sine of Maximum declination (R sin W).

Note : Com: The product is divided by R. The result obtained is the R Sine of the declination of *kāla koṭi*. (*kāla koṭi apakramajyā*. $\frac{\text{R Cos } \gamma \text{ Q . R Sin } \omega}{\text{R}} = x$

115 b. 116 a. The square of the result is subtracted from the squares of *kāla koṭa* and R. The two square roots that are left over, are the *koṭijyā* and *dvimaurvī*.

koṭijyā = $\{(\text{R Cos } \gamma\text{Q})^2 - x^2\}^{\frac{1}{2}}$

dvimaurvī = $R^2 - x^2$

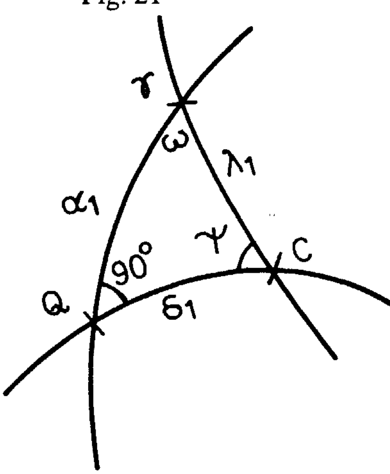
116 b. 117 The *koṭijya* is multiplied by *dyujyā* and divided by the *dyujyā*. The result is converted into arcs. That time in *asus* is the meridian ecliptic point (*madhya- lagna*). This subtracted from 3 *rāśis* is the arm of the *lagna*. The process with respect to each quarter is easily followed(as for the other *kāla lagnas*).

Note : '*Kotijyadyujayorghātād*' seems to be wrong. It should be *trijyayo* : Śankara Vāriar writes : *tatas tatrādyam trijayyā nihatya.......*

Consider Δ γQC(fig. 21) the śloka declares:

$$\frac{\{(\text{R Cos } \alpha_1)^2 - \left(\frac{\text{R Cos } \alpha_1 \text{ R Sin } \omega}{\text{R}}\right)^2\}^{\frac{1}{2}} \cdot \frac{1}{\text{R}}}{[\text{R}^2 - \{\text{R Cos } \alpha_1 \text{ R Sin } \omega\}^2]^{\frac{1}{2}}} = \text{Sin } \phi \text{ a (say)}$$

Fig. 21



Then $\lambda_1 = 90° - \phi$ or $\cos \lambda_1 = \text{Sin } \theta$

To prove $\text{Cos}^2 \lambda_1 . (1 - \text{Cos}^2 \alpha_1 \text{ Sin}^2 \omega)$

$= \text{Cos}^2 \alpha_1 - \text{Cos}^2 \omega$—(I)

Proof: $\text{Cos } \lambda_1 = \text{Cos } \alpha_1 . \text{Cos } \delta_1$ from Δ γQC

also $\text{Cos } \omega = \text{Sin } \psi \text{ Cos } \delta_1$

$\text{Cos } \psi = \text{Sin } \omega \text{ Cos } \alpha_1$

$\therefore$ I becomes, $\text{Cos}^2 \alpha_1 . \text{Cos}^2 \delta_1 (1 - \text{Cos}^2 \alpha_1 \text{ Sin}^2 \omega)$

$= \text{Cos}^2 \alpha_1 \text{ Cos}^2 \omega$

or $(1 - \text{Cos}^2 \alpha_1 \text{ Sin}^2 \omega) \text{ Cos}^2 \delta_1 = \text{Sin}^2 \psi \text{ Cos}^2 \delta_1$

or $\text{Cos}^2 \psi = \text{Cos}^2 \alpha_1 \text{ Sin}^2 \omega$,

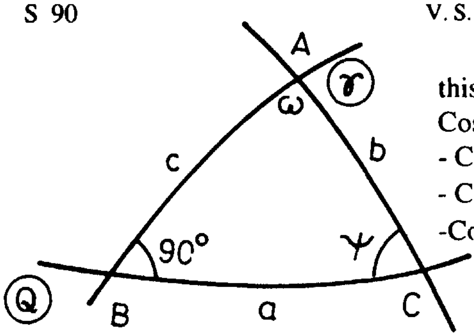


Fig. 22

this is true since it follows from fig. 22 that:

Cos a = Cos b Cos c + Sin b Sin c Cos A

- Cos A = + Cos B cos C Cos C - Sin B Sin Cos a

- Cos ω = Cos Cos 90° - Sin ψ Sin 90° Cos δ_1

-Cos ψ = Cos 90° Cos ω - Sin 90° Sin ω . Cos α_1

End of Chapter III.