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SOME MEDIEVAL ARITHMETICAL TABLES

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European travellers to India in the seventeenth century were much impressed by the Indian merchant's ability to do mental calculations with great speed. The secret of this ability lies naturally in the number of multiplication and other arithmetical tables he had committed to memory in childhood. This paper investigates some early references to such training and introduces a set of Prakrit tables of multiplication, squares, square-roots, cubes and cube-roots.

Key Words : *gaṇanā, lipi, saṃkhyā,* Mental calculation, multiplication tables, Table of cubes, Table of cube-roots, Tables of interest, Table of squares, Table of square-roots.

1.1 European travellers to India in the seventeenth century were much impressed by the Indian merchant's ability to perform mental calculations with great speed. The French jeweller Jean-Baptiste Tavernier, who visited India several times and in 1665 appraised Aurangzeb's state jewels including the famous *Koh-i-noor,* remarks that

> They (sc. the Banias) accustom their children at an early age to shun slothfulness, ... teach them arithmetic which they learn perfectly, using for it neither pen nor counters, but the memory alone, so that in a moment they will do a sum, however difficult it might be.[1]

1.2 J. Ovington visited India some three decades later and observed likewise that

> The *Bannias*, by the Strength of his Brain only, will sum up his Accounts with equal Exactness, and quicker Dispatch than the readiest Arithmetician can with his Pen.[2]

1.3 Therefore, as early as 1677 the East India Company felt it necessary to offer rewards for learning arithmetic from the Banias. The first number of the *Madras Notes and Extracts* records that

> In their letter to Fort St. George, 15th May 1677, the Court offer 20 pounds reward to any of our servants or soldiers as shall be able to speak, write, and translate the Banian Language, and to learn their arithmetic.[3]

1.4 The secret of the Indian merchant's ability to calculate fast mentally lies naturally in the number of multiplication and other arithmetical tables he had committed to memory in childhood. About the tables customarily memorised by children in Gujarat at the beginning of this century, the *Gazatteer of the Bombay Presidency* reports as follows:

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> .... the Vánia boy commits to memory a number of very elaborate tables. These tables, of which there are no fewer than twenty, contain among others two sets for whole numbers, one table of units up to ten multiplied as high as forty times: the other for numbers eleven to twenty multiplied by eleven to twenty times. There are fractional tables giving the results of multiplying $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $1\frac{1}{4}$, $1\frac{1}{2}$, $2\frac{1}{2}$, and $3\frac{1}{2}$ into units from one to one hundred; interest tables showing, at the monthly rate of one per cent on sums of from Re. 1 to Rs. 1000, the amount due for each quarter of a month; tables of the squares of all numbers from one to one hundred, and a set of technical rules for finding the price of a part from the price of the whole.[4]

2.1 But earlier notices of the mathematical training are totally wanting. There are no doubt occasional references to the education of children, especially the children of the Vaiśyas, but these do not reveal much as the following examples will show.

2.2 In the *Divyāvadāna* (ca. 2nd century A.D.), Koṭikarṇa, son of a merchant *(gṛhapati)* was taught the following:

> When he became big, he was taught |successively| writing *(lipi)*, counting *(saṃkhyā)*, calculation/arithmetic *(gaṇanā)*, assay and exchange of coins *(mudrā)*, money-lending *(uddhāra)*, deposit (*nyāsa, nikṣepa*[5]), evaluation of commodities *(vastuparīkṣā)* and examination of gems *(ratnaparīkṣā)*.[6]

To this list of the subjects studied, other stories in the same collection add some more items, viz. the evaluation of elephants *(hasti-parīkṣā)*, horses *(aśva-parīkṣā)*, male and female slaves *(kumāra-parīkṣā, kumārī-parīkṣā)*, clothes *(vastra-parīkṣā)*, and timber *(dāru-parīkṣā)* — thus altogether the evaluation of eight varieties of saleable commodities *(aṣṭasu parīkṣāsu)*.[7] It is, however, not certain how exactly *saṃkhyā* and *gaṇanā* on the one hand, and *nyāsa* and nikṣepa on the other, differed from one another, nor do we know what the instruction in *saṃkhyā* and *gaṇanā* entailed.

2.3 *Gaṇana* as a career is mentioned in the story of Upāli in the *Mahāvagga* of the *Vinayapiṭaka*.[8] Upāli's doting parents ponder over the career which the dear son should take up so that he could lead a comfortable life. *Lekha* is out of the question for it would tire his little fingers;[9] *gaṇana* is dismissed because it would strain his poor chest;[10] *rūpa* is not advisable either as it would hurt his eyes.[11] So they choose monkhood as the most comfortable career for the dear Upāli.

Now *lekha* is certainly writing, i.e. the profession of the scribe, but what are *gaṇana* and *rūpa* which strain respectively the chest and the eyes? The Pali Text Society's Pali Dictionary explains *gaṇana* in this passage as "the art of counting, arithmetic as a study and profession, forbidden to bhikkus."[12] On *rūpa* there is

no unanimity of opinion: it might mean "drawing, or arts and crafts," or "weights and measures" (according to Mrs Rhys Davids) or "money-changing" (according to W. Hardy).[13]

3.1 Because *gaṇana* is said to cause pain to the chest, Gaurishanker Hiralal Ojha conjectured that the word may denote multiplication tables, for to recite them aloud would indeed strain the chest.[14] In support of this view, Ojha cited Khāravela's Hathigumpha inscription of the first century B.C., which describes the monarch as *lekha-rūpa-gaṇanā-vavahāra-vidhi-visārada*. But there is no certainity that the word *gaṇanā* in this expression meant precisely the multiplication tables. Indeed, K. P. Jayaswal and R. D. Banerji, who edited this inscription, interpret the word quite differently.[15] Comparing it with analogous expressions in Kauṭilya's *Arthaśāstra*, they assert that *lekha* is not mere writing but royal correspondence, *rūpa* currency and *gaṇanā* accountancy or finance.

3.2 Be that as it may, the inescapable conclusion is that we know little about the history of the multiplication and other tables which children must have learnt by rote through all the ages.[16] Of course, those of us who did not go to convent schools may still know the multiplication tables in modern Indian languages. But there is no clue what their earlier forms must have been in the various Middle Indo-Aryan dialects or in Sanskrit, nor do we know the names by which they may have been known. There is not a single mention of these in any Sanskrit mathematical text or commentary. This total lack of mention does not, of course, mean that multiplication tables did not exist at all; it just means that such tables were thought to be too trivial to be recorded. But for the history of mathematics in India, especially history of mathematical training, their importance needs no justification.

3.3 In Hindi, multiplication tables are called *pahāḍe* or *pahāṛe*, but it is not known why they are called so. Sudhākara Dvidedī, a great scholar of Indian mathematics and astronomy, traces the word *pahāṛa* backwards up to Tulasīdāsa (1532-1624), who attributes a profound metaphor to the Table of Nine. The sum of the digits in each multiple of nine is always nine. For example,

$3 \times 9 = 27$; $2 + 7 = 9$
$7 \times 9 = 63$; $6 + 3 = 9$ and so on.

Therefore, says Tulasīdāsa, just as Nine is inherent in all its multiples, so is Rāma ever present in all his manifestations.[17] Beyond this, we do not know how the tables may have been worded in Tulsīdāsa's times.

4.1 In these circumstances, I was fortunate enough to discover the only specimens of some Middle Indo-Aryan or Prakrit arithmetical tables some time ago. The

Ganitasārasaṃgraha composed by Mahāvīra in the ninth century in what is present-day Karnātaka is well known to the students of Sanskrit mathematics. Towards the end of the eleventh century, this text was rendered into Telugu by Pāvulūri Mallana.[18] This Telugu rendering is perhaps the first ever translation of a scientific text in India.[19] A commentary on this Telugu translation occasionally cites from the Prakrit arithmetical tables. Unfortunately nothing is known about the author of this commentary nor of its time. But there are reasons to believe that these Prakrit tables were in use in Āndhra-Karṇāṭaka region some time in the early medieval period. The commentary refers to these tables as *magga,* from Sanskrit *mārga*, meaning paradigm or table. The word still survives in Kannada as *maggi.*

4.2 There are four tables in the commentary: of squares, square-roots, cubes and cube-roots. There is no full multiplication table. A few fragments of the Table of Three are found scattered in various passages which I rearranged.[20] I give below the tables exactly as the they are found, without attempting to correct them phonetically.

(i) Table of Squares

ekkasa vargo ekka

biyyasa vargo cāri

tiyyasa vargo navvā

cārisa vargo ṣoḷā

pamcasa vargo paṇuvīsā

chāyasa vargo chatrīsā

sattasa vargo navvetāḷā

aṭṭasa vargo causaṣṭi

navvasa vargo ekāsīti

sunnasa vargo sunnā.[21]

(ii) Table of Square-Roots

ekkasa vargomūlo ekkā

cārisa vargomūlo binni

navvasa vargomūlo tinni

ṣoḷasa vargomūlo cāri

paṇṇavisa vargomūlo paṃca

chattisa vargomūlo cāhā

navvetāḷasa vargomūlo satta

cauṣaṣṭisa vargomūlo aṭṭa

ekāsīṭisa vargomūlo navva

sunnasa vargomūlo sunna.[22]

(iii) Table of Cubes

ekkasa ghanno ekkā

biyyasa ghanno aṭṭā

tiyyasa ghanno sattāvīsā

cārisa ghanno cauṣaṣṭi

paṃcasa ghanno paṇavīsottara-ekaśataṃ

cāyasa ghanno ṣolottara-binniśataṃ

sattasa ghanno tretāḷottara-tinniśataṃ

aṭṭasa ghanno binnidaśottara-paṃcasataṃ

navvasa ghanno navabinnottara-sattaśataṃ

sunnasa ghanno sunnā. [23]

(iv) Table of Cube-Roots

ekkasa ghannomūlo ekkā

aṭṭasa ghannomūlo binni

sattavisa ghannomūlo tinni

causaṣṭisa ghannomūlo cāri

paṇuvīsottara-ekkaśata ghannomūlo paṃca

soḷāhottara-binniśata ghannomūlo cāhā

tretālottara-tinniśata ghannomūlo sattā

binnidaśottara-paṃcaśata ghannomūlo aṭṭā

navabinnottara-saptaśata ghannomūlo navvā

sunnasa ghannomūlo sunna.[24]

(v) Table of Three

bi tiyyā cāhā

tiṃ tiyyā navvā

cari tiyyā bārā

sapta tiyyaṃ yakkāvīsā

bārā tiyyā chattrisā. [25]

4.3 These are quite clearly mnemonic tables children learnt by heart. The peculiar phonology with elongated vowels (e. g. *vargomūlo, ghannomūlo, navvetāḷā*) makes these tables suitable for recitation in a sing-song fashion. Interestingly enough the *ekona-* or *ūna*-form of the numerals 19, 29, ... 79, as it obtains in north-Indian languages (e. g. *ekonaviṃśati* or *unnīs*), does not occur here. Instead, we have here formations like *navvetāḷa* for 49 and *navabinni* (or more correctly *navavīsa*) for 29 which are more akin to the Dravidian numerals. The fact that the commentary quotes from these tables in the middle of Telugu passages, anticipating thus the knowledge of these tables, indicates that at some point of time after the eleventh century, somewhere in the Telugu-speaking areas, children memorised these tables.

5.1 Similar tables should exist in all the regional languages of India.[26] There is an urgent necessity that historians of mathematics record these soon. Otherwise they would be irretrievably lost.

Notes And References

1 Jean-Baptiste Tavernier, *Travels in India*, tr. V. Ball, second edition, ed. William Crooke, London 1925, II, p. 144.

2 J. Ovington, *A Voyage to Surat in the Year 1689*, ed. H. W. Rawlinson, London 1929, reprint: New Delhi 1994, p. 166.

3 Cited in Henry Yule and A. C. Burnell, *Hobson-Jobson*, new edition by William Crooke, second edition, Delhi 1968, s.v. Banyan, p. 64.

4 *Gazetteer of the Bombay Presidency*, Volume IX, Part I: Gujarat, Population, Hindus, Bombay 1901; reprinted as, *Hindu Castes and Tribes of Gujarat*, compiled by Bhimbhai Kirparam, ed. James M. Campbell, Gurgaon 1988, vol. 1, p. 80.

5 *Nyāsa* and *nikṣepa* are two different kinds of deposits. On the various views concerning the exact meaning of these technical terms, see P. V. Kane, *History of Dharmaśāstra (Ancient & Medieval Religious and Civil Law)*, vol. III, Poona 1946, pp 454-460.

6 *Divyāvadāna*, ed. P. L. Vaidya, Darbhanga 1959, p. 2: *sa yadā mahān saṃvṛttas tadā lipyāṃ upanyastaḥ saṃkhyāyāṃ gaṇanāyāṃ mudrāyām uddhāre nyāse nikṣepe vastuparīkṣāyāṃ ratnaparīkṣāyam. so' ṣṭasu parīkṣāsūdghaṭako vācakaḥ paṇḍitaḥ paṭupracāraḥ saṃvṛttaḥ.*

7 Ibid, pp. 16 (Pūrṇāvadāna), 35 (Maitreyāvadāna), 63 (Supriyāvadāna), 287 (Sudhanakumārāvadāna).

8 *The Mahāvagga*, ed. Bhikku J. Kashyap, Nalanda 1956, pp. 80-81.

9 Ibid, p. 80: *sa ce kho Upāli lekhaṃ sikkhissati anguliyo dukkhā bhavissanti.*

10 Ibid, p. 80: *sa ce kho Upāli gaṇanaṃ sikkhissati urassa dukkho bhavissati.*

11 Ibid, p. 81: *sa ce kho Upāli rūpaṃ sikkissati akkhīni dukkhā bhavissanti.*

12 T. W. Rhys Davids and William Stede, *The Pali Text Society's Pali Dictionary*, part III, London 1922, p. 70.

13 Ibid, part VII, London 1924, p. 32.

14 *Bhyāratīya Prācīna Lipimālā*, Delhi 1894, Reprint: Delhi 1959, p. 6.

15 K. P. Jayaswal and R. D. Banerji. "The Hathigumpha Inscription of Khāravela," *Epigraphia Indica*, 20, 71-89, 1929-30, esp. p. 86: "(who) had mastered (royal) correspondence, currency, finance, civil and religious laws..."

16 On the history of the multiplication tables in Europe, see Smith, David Eugene, *History of Mathematics*, Dover Edition of 1958, undated reprint, New York, Vol. II, pp. 123-128, esp. p. 127: "For tables to be committed to memory it was sufficient, in the days of the medieval abacus, to go only to 5 x 10; even 4 x 9 was far enough for practical purposes." On Islamic sexagesimal multiplication tables, see King, David A., "On Medieval Islamic Multiplication Tables," *Historia Mathematica*, 1, 317-323, 1974; idem, "Supplementary Notes on Medieval Islamic Multiplication Tables," ibid, 6 405-417, 1979.

17 Tulasīdāsa, *Sattasaī*:

Tulasī Rāma sancha karu tyāgu sakala upacāra /
jaise ghaṭatana anka nava nava ke likhata pahāra //

cited by Sudhākara Dvivedī, *Gaṇita kā Itihāsa*, part I, Benares 1910, p. 81.

18 Only a very small portion of this highly valuable text is published so far under the title *Sārasaṃgrahagaṇitamu*, Pāvulūri Mallana praṇītamu, ed. Veṭūri Prabhākara Śāstrī, part 1, Tirupati 1952. On this translation, see Sreeramula Rajeswara Sarma, "Mathematical Literature in Telugu: An Overview," *Sri Venkateswara University Oriental Journal*, **28**, 86-95, 1985; and also the paper mentioned in the note below.

19 Cf. Sreeramula Rajeswara Sarma, "Pāvulūriganitamu: the first Telugu Work on Mathematics," *Studien zur Indologie und Iranistik*, **13-14**, 163-176, 1987.

20 Ibid, pp. 173-174.

21 "One's square [is] one, ... nine's square [is] eightyone, zero's square [is] zero," *Ekkasa* from Sanskrit *ekasya* is the genitive form of *ekka* (= one).

22 "One's square-root [is] one, four's square-root [is] two, ... eightyone's square-root [is] nine, zero's square-root [is] zero."

23 "One's cube [is] one, ... nine's cube [is] twentynine beyond seven hundred, zero's cube [is] zero,"

24 "One's cube-root [is] one, ... twentynine beyond seven hundred ['s] cube-root [is] nine, zero's cube-root [is] zero,"

25 *Tiyya* or *tiya*, from Sanskrit *trika*, means "thrice"; see R. Pischel, *A Grammar of the Prakrit Languages*, tr. from German by Subhadra Jha, second revised edition, Delhi 1981, p. 377. Accordingly the passage translates as follows: "Two thrice [makes] six, three thrice [makes] nine, four thrice [makes] twelve, seven thrice [makes] twentyone, twelve thrice [makes] thirtysix."

26 I have heard of Śubhaṅkara's tables in Old Bengali, but do not know if these are available in print.