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THE METHOD FOR FINDING OUT THE NUMBER OF MOONS AND THEIR FAMILIES IN THE TILOYAPAṆṆATTĪ

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Yativṛṣabhācārya (5th century A.D.) composed a text on the Jaina Cosmology, cosmogony and cosmography, known as the *Tiloyapaṇṇattī* (Knowledge of three universes), in nine chapters in Śaurasenī Prakrit. Its seventh chapter deals with astronomy and at the end what is given is the method of finding the total number of the astral bodies in the families of all the moons in the universe. This account has been denoted as the universe-line (*jagaśreṇī*) squared and divided by the product of the square of 256 *aṅgulas* and numerate. The universe-line (*jagaśreṇī*) and the finger (*aṅgula*) represent the cardinal of the set of points (*pradeśas*) contained in their lengths, respectively. The point is defined as the space occupied by the ultimate particle (*paramāṇu*). This requires the calculation of the logarithm of the set of points contained in the rope (*rāju*) or the seventh part of the universe-line (*jagaśreṇī*) which is related with the number of islands and seas extending to an innumerate stretch from the Sumeru to the last sea on the flat middle universe.

Key Words : *Aṅgula, jagaśreṇī, rāju, pradeśa, rāśi, ardhaccheda, saṅkhyāta, yojana,* Lavaṇa Sea, Kālodaka Sea, Puṣkarārdha Island, *Samīkaraṇa,* Svayambhūramaṇa Sea, *palya,* Mānuṣottara, *pracaya,* gaccha, Vāruṇīvara, *Saṅkhyāta, guṇyamāna-rāśi, madhyadhana, ādidhana, uttaradhana.*

INTRODUCTION

In the Jaina cosmography, the middle universe is a flat earth, naturally made up of concentric rings which are alternately islands and seas, with the base of the Sumeru mountain at the centre, and extending to the end of a universe in the middle, which has a diameter of one *rāju*. The first ring surrounds the Jambu Island which has a diameter of one lakh *yojanas*. The Jambu Island has 2 moons, 2 suns, 176 planets, 56 constellations, 13,3950 crore-squared unstable stars and 36 stable stars. The Lavaṇa Sea is surrounded by another ring which makes its width as 2 lakh *yojanas*. As per arrangements, the Lavaṇa Sea has 4 moons, 4 suns, 352 planets, 112 constellations, 2,67,900 crore-squared unstable stars and 139 stable stars. Similarly, the next is the Dhātakīkhaṇḍa Island surrounded by the ring which makes its width as 12 moons, 12 suns, 1056 planets, 336 constellations, 8,03,700 crore-squared unstable stars and 1,010 stable stars. From the above it is evident that the moons get in excess of the double of its preceding ring at an interval of one lakh *yojanas*. Adding to these the respective astral bodies of the Kālodaka Sea and

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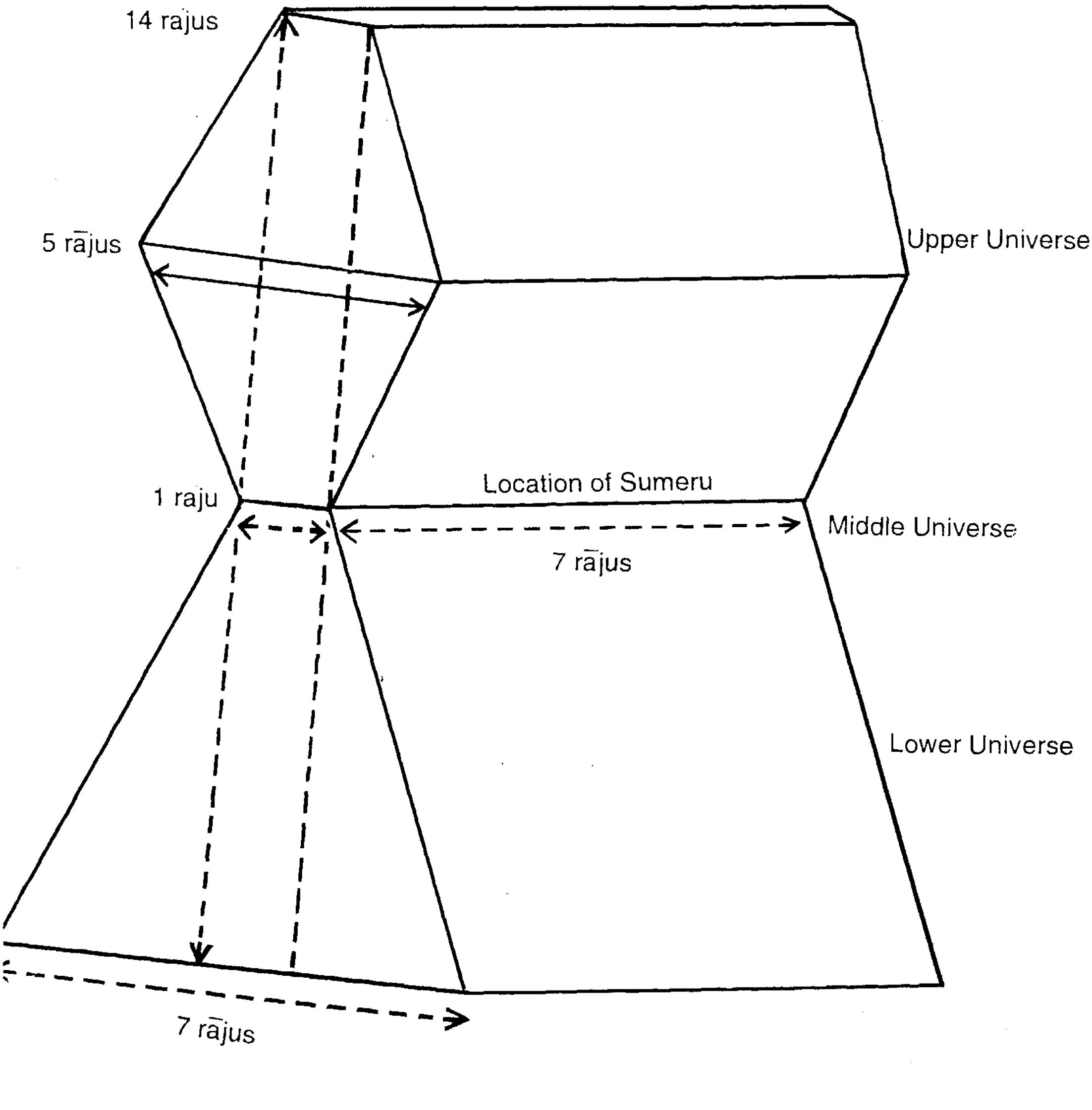

14 rajus
5 rājus
Upper Universe
Location of Sumeru
1 raju
Middle Universe
7 rājus
Lower Universe
7 rājus

Fig. 1

the subsequent Puṣkarārdha Island, the total of the bodies upto the human-universe (*Mānuṣa-Loka*) are given by 132 moons, 132 suns, 11,616 planets, 3,696 constellations, 88,40,700 crore-squared unstable stars and 95535 stable stars.[1]

After the human universe at a further distance of 50,000 *yojanas* there again starts the counting of the rings and the contents of the astral bodies therein; every further ring being at a distance of the lakh *yojanas*, the last ring being at 50,000 *yojanas* preceding the extreme shore of the Svaymbhūramaṇa Sea. The actual number of these rings is :[2]

(world-line/1400000) – 23. ... (1.1)

The author of the *Tiloyapaṇṇattī* then proceeds from the first ring of the Puṣkarārdha Island which contains 144 moons and 144 suns. The first ring of the Puṣkaravara Sea contains 288 moons and 288 suns. In this way the astral bodies, moons and suns become double of those lying in the first ring of its preceding island or sea as the case may be, for the first ring of the succeeding ring. The formula for finding out the number of the moons or the suns in the first ring of an island or a sea[3] is as follows :

Number of moons or suns in the first ring of an island or sea
= (the width of the island or sea × 9)/(100000) ... (1.2)

The number of the rings in the second, third, fourth and so on increases in an arithmetical progression with a common difference of 4. Thus, they are respectively given by 148, 152, 156, 160, etc. The method in this way consists of finding out the sum of such series of moons etc. lying in various rings of the island or sea.

The adjoining figure of the universe (*Loka*) consists of the Lower, Middle and Upper types of universe. The scale is 1 c.m. = 2 *rājus*. The middle universe contains the horizontal-mobile-bios universe, 1 square *rāju* in area and one lakh *yojana* thick from the base of the Sumeru, which is like a celestial axis of one lakh *yojana* high.

The astral bodies revolve round the Sumeru leaving a distance of 1,121 *yojanas*, over the volume of space given by the product of a square *rāju* and a height of 110 *yojanas*. The set of all the astral bodies has been given as[4]

$(jagaśreṇī)^2/(256\ pramāṇāṅgulas)^2$ ... (1.3)

SYMBOLIC AND CONCEPTUAL EXPOSITION OF THE METHOD

The *rāju* or the *jagaśreṇī* is a cosmic measure of a distance containing innumerate number of points. We shall denote them by R and L. Similarly the small unit of length is *aṅgula* containing also yet another measure of an existential point-set, hereafter denoted by F. The measures of R, L, F, the existential sets are given through the construction-set, the *palya,* hereafter denoted by P, the measure of which has already been evaluated in various articles and books.[5] The existential sets are thus the point sets defined in terms of the *palya*, a construction instant-set, as follows :[6]

$$L = (F^3)^{(\log_2 P/\text{innumerate})} \qquad \text{... (2.1)}$$

and

$$F = (P)^{(\log_2 P)} \qquad \text{... (2.2)}$$

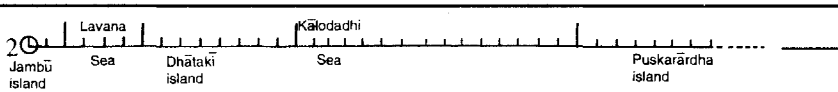
where F denotes the set of space-points contained in a width of a standard Finger (*Pramāṇa-aṅgula*) well defined in the *Tiloyapaṇṇattī*.[7] The operator $\log_2$ is called the *ardhaccheda*, and various similar such relations and the rules of logarithms have been given in the Jaina School of Mathematics.[8]

The following relations have been obtained from the above two equations (2.1) and (2.2), and used in the application of the method.

NAME	RELATION IN SYMBOLIC EXPRESSION
ardhaccheda of 100000 *yojanas*	$\log_2$ (100000) or 17
aṅgulas of one *yojana*	768000
ardhaccheda of a *yojana* till *aṅgula* is obtained	19
saṅkhyāta ardhaccheda	(17 + 19 + 1)
ardhaccheda of *Jambu* island width till one space-point is obtained	$\log_2 p \log_2 p + 17 + 19 + 1$
ardhaccheda of a *palya*	$\log_2 p$
aṅgula or square of *ardhaccheda* of a *palya*	F or $\log_2 p \log_2 p$
ardhaccheda of a *jagaśreṇī*	$(\log_2 p \log_2 p \log_2 p\ (3))/A$
ardhaccheda of a *rāju*	$[(\log_2 p)/A] \log_2 p \log_2 p.\ (3) - 3$

where A denotes the innumerate number.

While in the calculation of the number of moons etc. there was a controversy in the period of Vīrasenācārya whether the value of $\log_2(L/7)$ be taken as the sum of $\log_2$(Jambu island) + 1 or $\log_2$(Jambū island) + *saṅkhyāta*, it may be resolved as follows :[9]

Fig. 2


The above line denotes a part of the universe-line or L, starting from the centre of the middle universe, denoting the successive rings of islands and seas as described earlier. It is known that the width of the Jambu Island is 1,00,000 *yojanas*. Let it be taken as 1 unit. Then the widths of the successive rings of Lavaṇa Sea, Dhātakī Island, Kālodadhi Sea, Puṣkara Island and so on may be denoted respectively as 2, 4, 8, 16 etc. units.

Now in a family of a moon there are one sun, 88 planets, 28 constellations and 66975000,000,000,00 stars. There are 2 moons in the Jambu Island, 4 moons in the Lavaṇa Sea, 12 moons in the Dhātakī Island, 42 moons in the Kālodaka Sea, 72 moons in the Puṣkarārdha Island on the rear side of the Mānuṣottara mountain, 144 moons in the first row of the Mānuṣottara. These are along with their families. This first row is 50000 *yojanas* ahead of the Mānuṣottara, where the number of moons increases with 4 as the common difference and the first term is 144, in arithmetical progression. Thus the number of moons in them are 148, 152, 156, etc. respectively. Now in the inner subring of the sea, there are 288 moons. Here also the rings are situated successively one lakh *yojanas* ahead of each, where the number of moons increases as before with a common difference of 4. Ahead of this sea there is the island in whose first ring or subring, there are 288×2 moons. There are 64 subrings, increasing successively in an arithmetical progression with common difference of 4, with the first term as 288×2.

In the text, the data of the moons for the first three islands and two seas is not taken into account. The author starts with the third (Puṣkaravara) sea which has the number of subrings as 32. In the first subring there are 288 moons forming the first term, 32 is number of terms or *gaccha,* and 4 is the common difference for finding the total moons in this sea. For the fourth island the number of terms is 64, first term is 288×2, common difference (*pracaya*) is 4. Similarly for the fifth (Vāruṇīvara) sea the number of terms is 128, the first term is 1152 and the common difference is 4.

Thus the following formula may be used for finding out the sums of moons in every one of the islands or seas :

$$\text{Sun} = (n/2)\ [2a + (n-1)d], \quad \ldots (2.3)$$

where n is the number of terms, a is the first term, d is the common difference. Thus we have

The sum of all the moons in the third sea is

$$= (32/2)\ [2\times288 + (32-1)\times4]$$
$$= 32\times288 + (32-1)\times64, \quad \ldots (2.4)$$

The sum of all the moons in the fourth island is

$$= (64/2)\ [2^2\times288 + (64-1)\times4]$$
$$= 64\times2\times288 + (64-1)\times64\times2,$$

The sum of all the moons in the fifth sea is

$= (128/2)\ [2^3 \times 288 + (128-1) \times 4]$

$= 64 \times 2^3 \times 288 + (128-1) \times 64 \times 2^2,$ ... (2.6)

This process is continued for n–5 islands-seas as the first 5 islands-seas have been omitted in the account. The sum of the moons for the n–5 islands-seas is

$= 64 \times 288\ (1/2 + 2 + 2^3 + 2^5 +$ upto (n–5) terms
$+ (64)2\ (1/2 + 2 + 2^3 + 2^5 +$ upto (n–5) terms
$- 64\ (1 + 2 + 2^2 + 2^3 + 2^5 +$ upto (n–5) terms ... (2.7)

The above are the geometrical series for summation of which the formula is

$\text{sum} = [a(r^n - 1)]/(r-1),$ ... (2.8)

where a is the first term, r is the common ratio and n is the number of terms.

Hence the sum of the expression in series of (2.7) is

$(64\ (288\ (1/2\ (4^{(n-5)} - 1))))/(4-1) - 1\ (1\ (2^{(n-5)} - 1))/(2-1)$
$+\ \ 64\ (1/2\ (4^{(n-5)} - 1)/(4-1)),$ ... (2.9)

or it is

$64\ (176/3)\ (2n-5)2 - (2)(n-5) - 57(2/3)$ ... (2.10)

Hence the total of all the astral bodies of the moons with their families is given by

$(669750{,}000{,}000{,}001{,}17)(64(176/3)(2^{n-5})^2 - (2)^{n-5} - 52(2/3)$
+ (the moons and their families of the first five islands-seas) ... (2.11)

In the above expression the term $(2^{(n-5)})^2$ is important to note.

In order to get the measure of the $\log_2$ *(Rāju)*, the following formula is given :

$\log_2$ *(rāju)* = n + (1 or S) + $\log_2$ (Jambu Island diameter point-set) ... (2.12)

where S denotes a numerate number (*saṅkhyāta*). Now the diameter of Jambu Island is 100000 yojanas which is the same as converted into $100000 \times 6 \times 2 \times 2 \times 2 \times 2 \times 2000 \times 4$ standard fingers (*pramāṇāṅgulas*). One *pramāṇāṅgula* contains 500 *utsedhāṅgulas*. The logarithm of a linear finger (*sūcyaṅgula*) to the base two is given by $(\log_2 P)^2$, where P is the *palyopama* instant-set as a construction set. Here one trail (*āvali*) is an instant-set which contains minimal yoked innumerate (*Jaghanya yukta asaṁkhyāta*) instants. Hence the *pramāṇāṅgula* is an innumerate set which being above the maximal numerate (*Utkṛṣṭa saṅkhyāta*) transgresses the limit of the subject of the Omniscript (*Śrutakevalī*). Thus if we take the width of the Jambu island to be at most $2^{(40)}$ *pramāṇāṅgulas,* we have

$\log 2\ (L/7) = n + (1 \text{ or } S) + \log_2 (2^{(40)}$ *pramāṇāṅgulas*)
$= n + (1 \text{ or } S) + 40$ *pramāṇāṅgulas* ... (2.13)

In case we take 1 in place of S then at most we have

$$n-5 = (\log_2 (L/7) - \log_2 (2)^{40} \textit{pramāṇāṅgulas})$$

$$= (\log_2 \frac{L/7}{2^{40} \textit{ pramāṇāṅgulas}}) \quad \ldots (2.14)$$

In this way on substituting the value of n–5 from (2.14) in (2.11) we get the total number of the astral bodies as follows :

$$(66975000{,}000{,}000{,}001{,}17)\ (64\ (176/3)\,(\frac{L/7}{2^{40}\ Pra.})^2-(\frac{L/7}{2^{40}\ Pra.})-57\frac{2}{3}))\ ..\ (2.15)$$

It is evident that the terms $(L/7)/(2^{40}$ *Pra.*) and 57 2/3 are insignificant in comparison with the first term. Thus, in order to get $(256)^2$ in the denominator of the first term we can not do with the expression $(2)^{80}$, because its coefficient given by $((176/3) \times 64 \times 66975000{,}000{,}000{,}001{,}17)$ has the number of its logarithm to the base two ought to have been 16 in excess, there are only 80 – 77 or only 3 logarithms remain in the exponent of 2 in the denominator. Even if division by 49 is required, still then 5 more will be added as logarithms and in this way in place of 16 only 8 will in the exponent of 2. Hence it is proper to take 5 in place of 1. Further whatever terms are to be subtracted, there will be increase in the denominator. In this calculation the set of moons and astral family members are insignificant for the first five islands-seas.

Exposition in the Tiloyapaṇṇattī and the Trilokasāra

In order to find out the set of moons and their families, in the innumerate islands-seas the number of the islands and seas is required.[10] For this purpose the logarithm to base two of the *rāju* (*ardhaccheda* of *rāju*) is calculated till the Jambu island is reached. Then logarithm of Jambu Island's diameter to the base two is found out.

Beginning with the centre of the Sumeru mountain, the distance upto the end of the Svayabhūramaṇa Sea, is half a *rāju,* hence the first bisection-point or the first count of $\log_2$ R falls on the centre of the Sumeru. The next bisection point falls in the Svayambhūramaṇa Sea, 75,000 *yojanas* beyond the circumference of the Svayambhūramaṇa Island. The third bisection point falls in the Svayambhūramaṇa Island. This process of bisecting the distance is carried on till the Lavaṇa Sea is reached where it is found that two bisection points lie therein. The boundary of the Jambu Island is 50,000 *yojanas* from the centre of Sumeru. The second bisection point of the Lavaṇa sea is 50,000 *yojanas* beyond this boundary hence its diṣtance from the Sumeru-centre is 100000 *yojanas.* Now $\log_2$ (100000) may be approximately taken to be 17. The remaining is one *yojana* which consists of 768000 *aṅgulas.* The logarithm of this to the base two is 19, and one *aṅgula* remains. Then the logarithm of *aṅgula* is the square of the logarithm of *palya* to the base two. This is in excess of 19+17+1 or 37 which has also been called as numerate (*Saṅkhyāta*).

According to the *Tiloyapaṇṇattī*, ch. 1, v. 131 and the *Trilokasāra*, v. 108, the *ardhaccheda* of *jagaśreṇī* or 7 *rājus* is given by

$$\frac{(\textit{ardhaccheda of palya})}{\textit{asaṅkhyāta}} \quad ((\textit{ardhaccheda of palya})^2 + 37) \times 3 \qquad \ldots (3.1)$$

or

$$\frac{\log_2 P}{A} \quad \times ((\log_2 P)^2 + 37) \times 3 \qquad \ldots (3.2)$$

If 3 is subtracted from the above amount, then the logarithm of *Rāju* to the base two is obtained as

$$\log2\ (L/7) \quad \frac{\log_2 P}{A} \times ((\log_2 P)^2 + 37) \times 3 - 3 \qquad \ldots (3.3)$$

From the above expression the number of all islands-seas can be obtained by subtracting from it or (3.3) the quantity given by numerate plus the square of the *ardhaccheda* of *palya*.

Now we shall give special explanation of the verses of the *Tiloyapaṇṇattī*[11] for finding out the total number of moons with their families :

> "*eto caṁdaṇa saparivāraṇamaṇayaṇa-vihāṇaṁ vattaissamo/taṁ jahā- ... puṇo ekkammi bimbammi tappaugga-saṅkhejja-jīva atthitti taṃ saṅkhejja-rūvehiṃ guṇidesiṃ savva-joisiya-jīva-rāsi-parimāṇama hodi/taṃ cedaṃ-* ($\overline{\overline{4}}$)/(65536)".

The explanation of the above may be given in different steps as follows :

1. The number of terms (*gaccha*) involved on taking the third sea as initial and proceeding upto the Svayambhūramaṇa sea –

The number of moons of the first five islands-seas have been described already, hence leaving them apart the *gaccha* for the remaining seas and islands is as follows :

serial no.	Sea and Island	number of terms
3rd	Puṣkaravara Sea	32
4th	Vāruṇīvara Island	64
5th	Vāruṇīvara Sea	128
6th	Kṣīravara Island	256
7th	Kṣīravara Sea	512

In accordance with the above distribution the number of terms form a geometric progression with two as common ratio upto the Svayambhūramaṇa sea.

2. The first term in a geometric progression is called a *guṇyamāna rāśi*. This represents the number of moons in the first ring of an island or a sea. For example, there are 32 rings in the Puṣkaravara Sea hence the number of terms (*gaccha*) is 32. In the first ring of the sea there are 288 moons which is therefore the first term or *guṇyamāna rāśi*. In the fourth island there are 64 rings. Its first ring has 576 moons hence the number of terms is 64 and the first term (*guṇyamāna rāśi*) is 576. Thus there are these two which become twice for the successive ones (island or sea), on the first ring.

The author then divides all the first terms (*guṇyamāna rāśis*) by 288 and the quotients so obtained are multiplied by their own *gaccha*. The *gacchas* then form a geometric sequence with the common-ratio as 4. Thus the following table has been obtained :

No.	Sea or Island	Number of terms/Divisor	Quotient	Quotient × *gaccha*	Four times *gaccha*
3rd	Puṣkaravara Sea	288/288	1	1 × 32	32
4th	Vāruṇīvara Island	576/288	2	2 × 64	126
5th	Vāruṇīvara Sea	1152/288	4	4 × 128	512
6th	Kṣīravara Island	2304/288	8	8 × 256	2048
7th	Kṣīravara Sea	4608/288	16	16 × 512	8192

The *pracaya* is the common dfference of an arithmetical progression or regression For example, in the third sea there are 32 rings (or subrings). There are 288 moons on its first ring. Due to increase by the common difference 4 the moons on the second, third, etc. rings are given by 292, 296, 300, etc. till 572 moons on the last ring are obtained. Further on the first ring of the fourth island this number becomes 576.

Here the concept of the middle sum (*madhyadhana*) has been used. Whatever is the quantity as the increase over the middle number of the number of terms corresponding to the sum is called the middle sum. As the number of terms becomes double successively, so also does this middle sum go on doubling. The following table shows that out of the 32-1 or 31 terms (*gaccha*) the 16th term is the middle term and its increase is 16 × 4 = 64. This is about the third sea.

No.	Number os terms	No.	Number of terms
1.	4	17.	68
2.	8	18.	72
3.	12	19.	76
4.	16	20.	80
5.	20	21.	84
6.	24	22.	88
7.	28	23.	92
8.	32	24.	96
9.	36	25.	100
10.	40	26.	104
11.	44	27.	108
12.	48	28.	112
13.	52	29.	116
14.	56	30.	120
15.	60	31.	124
16.	64		

From the above it is clear that in the third sea the middle sum appears at the 16th number as 64. Similarly, for the fourth island the middle sum 128 appears at the 32nd number, being twice its preceding.

Here the word *samīkaraṇa* appears for an equation, denoting relation between two or more sets. If n denotes the order of an island or sea, the number of moons in the first ring of the nth is $144 \times 2^{(n-2)}$, where 144 is the number of moons on the first ring of the Puṣkarārdha island. Then there are the concepts of the *ādidhana* and the *uttaradhana*. Thus they are depicted as follows :

$$S = (n/2)\,(2a + (n-1)d) = an + n(n-1)d/2$$

= *ādidhana* + *ūttaradhana*

= sum of the initials + sum of the successors ... (3.4)

The sum of the progression is called *sarvadhana*. There is one more technique adopted for convenience of calculations. It is by introducing the negative quantity *ṛṇa rāśi*.

On the first ring of the outer Puṣkarārdha Island there are 144 moons. There are 144 × 2 moons and the first ring of the third sea Puṣkaravara. The width of this sea is 32 lakh *yojana*, hence it has 32 rings the common difference of the progression being 4, the sum of the initials is 144 × 2 × 32 or 9216. This is the *ādidhana*.

The *uttaradhana*, n(n–1)d/2, is (32)(32–1)4/2 or 31 × 64. If 64 is added to this it becomes 32 × 64 and from it we may subtract 64 later on. When this *uttaradhana* is added to the sum of the initials or the *ādidhana* the sum is 11200. Thus for each successive island or sea the sum is four times that of its preceding island and the negative quantity is twice that of its preceding.

3. In order to find out the number of the astral bodies (deities) the number of the islands and seas are to be found out. According to the formula from the *Parikarma* there are as many islands and seas as is the number of logarithm of *rāju* to the base two as reduced by the number of logarithm of Jambu Island to the base two and then added by six. It may be noted here that the bisection points which lie on the Meru and the first five islands regarding the *rāju* have not been included here because this number of moons has already been stated. According to the author whatever measure is obtained of the astral bodies through the divisor $(256)^2$, if that is desirable then not only six but also numerate over six are required to be reduced from the logarithm of *rāju* to the base two apart from the reduction of the logarithm of diameter of the Jambū island to the base two. Only after this reduction the desired number of the numerator is obtained. Vīrasenācārya has adopted this plan in order to prove a correct figure for the statement from the *Parikarma* formula, and this procedure as he says should not be considered against the School and that one should not hold a prejudiced view about the value as well the plan.

4. In order to find out the number of moons situated on all the rings of all the islands and the seas upto the Svayambhūramaṇa Sea, the positive and the negative quantities are placed separately, and the total number of the rings obtained through the logarithm of *rāju* to the base two is placed in form of the number of terms. In order to obtain the positive quantity, three summations are necessary which is as follows : (i) first term 176 × 64, (ii) multiple common difference 4 (iii) number of terms. Here the number of terms is : (logarithm of *jagaśreṇī*) –3 – 6 – (logarithm of Jambu Island), which is established separately. The remaining number of terms is logarithm of *jagaśreṇī* to the base two. Herein and what follows the logarithms are taken to the base two unless stated otherwise.

5. All the number of terms are set up in mutual order of multiple of 4. Here the positive quantity is the number of terms given by logarithm of universe-line (*jagaśreṇī*). This is spread and to its every one 2 is given and then the spread distribution is mutually multiplied getting the *jagaśreṇī*. If the same logarithm of *jagaśreṇī* is spread and to its every one 2 × 2 is given and the distribution is again mutually multiplied, the square of *jagaśreṇī* is obtained. This is also called *jagapratara*.

6. Out of the three negative quantities, let us first find out the logarithm of the Jambū island which is 100000 *yojanas* in diameter of whose bisections or logarithm to the base two is 17 and one *yojana* is left. The 17 bisections are spread and to every one is given 2 × 2, after which mutual multiplication is performed getting 100000 × 100000. The remaining one *yojana* is equivalent to 768000 *aṅgulas* whose bisections or logarithm to base two is 19 when one *aṅgula* is left. When these bisections are spread and to every one is given 2 × 2 and the mutual multiplication effected, then the result is 768000 × 768000. Similarly when the logarithm of *aṅgula* to the base two are spread and to its every one 2 × 2 is given and then mutually multiplied as before, then the resulting product is *aṅgula* × *aṅgula* or (*aṅgula*) × (*aṅgula*) or *prataräṅgula*. This represents the set of points in a square made up of a finger-width. The point has already been defined to be the space occupied by an ultimate-particle (*paramāṇu*).

7. Now the 6 bisections which include those corresponding to first five islands and seas and one at Meru are spread and to its every one is given 2 × 2 and the terms are mutually multiplied, the product so obtained is 64 × 64. As the *jagaśreṇī* is equivalent to 7 *rājus* hence the bisections of 7 is approximately 3. These bisections are also spread and to its every one is given 2 × 2 and mutually multiplied resulting in 7 × 7.

8. In this way the sum total of the negative quantities is given by

$$1 \text{ lac} \times 1 \text{ lac} \times 768000 \times 768000 \times \textit{prataräṅgula} \times 64 \times 64 \times 7 \times 7.$$

This is negative hence as per logarithmic rule this amount goes to the denominator and the *Jagapratara* remains in the numerator. Thus the quotient is given by

$$\frac{\textit{jagapratara}}{1 \text{ lakh} \times 1 \text{ lakh} \times 768000 \times 768000 \times \textit{prataräṅgula} \times 64 \times 64 \times 7 \times 7} \quad \text{... (3.5)}$$

The above may be written as

$$\frac{\textit{jagapratara}}{\textit{prataräṅgula} \times 1 \text{ lakh} \times 1 \text{ lakh} \times \text{numerate} \times 64 \times 64 \times 7 \times 7} \quad \text{... (3.6)}$$

9. From the above the sum of the initials and the sum of the common-differences are calculated as follows : On multiplying the expression respectively by 288 and 64 we get

$$\textit{ādidhana} = \frac{288 \textit{ jagapratara}}{\textit{prataräṅgula} \times 1 \text{ lakh} \times 1 \text{ lakh} \times \text{numerate} \times 64 \times 64 \times 7 \times 7} \quad \text{... (3.7)}$$

$$\textit{pracayadhana} = \frac{64 \textit{ jagapratara}}{\textit{prataräṅgula} \times 1 \text{ lakh} \times 1 \text{ lakh} \times \text{numerate} \times 64 \times 64 \times 7 \times 7} \quad \text{... (3.8)}$$

The sum of the above two expressions, (3.7) and (3.8) is given by

ādidhana + *pracayadhana*

$$= \frac{352 \textit{ jagapratara}}{\textit{pratarāṅgula} \times 1 \text{ lakh} \times 1 \text{ lakh} \times \text{numerate} \times 64 \times 64 \times 7 \times 7} \quad \ldots (3.9)$$

From the expression (3.9) the negative quantity which is to be subtracted is a series whose first term is 64, common difference is 2, and the number of terms is the remainder obtained by subtracting logarithm of Jambu Island as in excess from the logarithm of *Jagaśreṇī* to the base two. Hence, the sum is given by

$$\frac{64 \textit{ jagaśreṇī}}{\textit{sūcyaṅgula} \times \textit{saṅkhyāta} \times 64 \times 7 \times 1 \text{ lakh}} \quad \ldots (3.10)$$

where the *saṅkhyāta* is the same as the numerate.

On subtracting the expression (3.10) from the expression (3.9) we have the following remainder as

$$\frac{352 \textit{ jagapratara} - 64 \textit{ jagaśreṇī } (\textit{sūcyaṅgula} \times \textit{saṅkhyāta} \times 64 \times 7 \times 100000)}{(\textit{pratarāṅgula} \times 1 \text{ lakh} \times 1 \text{ lakh } (\textit{saṅkhyāta} \times 16 \times 7 \times 7 \times 64 \times 64)/16} \quad \ldots (3.11)$$

The above expression reduces to

$$\frac{\textit{jagapratara}}{\textit{pratarāṅgula} \times 65536 \times 7}$$

$$\text{or } = \frac{4\,(65536)\,.\,(7)}{4\,(65536).(7)} \quad \ldots (3.12)$$

The above gives the total number of astral bodies. If it is multiplied by 7 we get the total number of astral deities residing there.

Conclusion

The above measure is just an approximation in which small terms have been considered negligible as compared with very large point-sets or instant-sets. The application of logarithms is worthy of attention here, involving smaller and smaller units of distances and lengths denoting sets of space-points as the universe-line (*jagaśreṇī*) and the linear-finger-width (*sūcyaṅgula*) which have been expressed in terms of the construction instant-set : *Palya*.

REFERENCES AND NOTES

1. The following texts describe the same topic :
   a) *The Tiloyapannattī of Yativṛṣabhācārya*, edited by H.L. Jain and A.N. Upadhye, Sholapur, vol. 2, 1951. Cf. *Tiloyapannatti with Commentary of Vishuddhamati Aryika*, edited by C.P. Patni Lucknow, vol. 3, 1988.
   b) *The Trilokasāra of Nemicandra Siddhāntacakravartī, with Sanskrit Commentary of Mādhavacandra Traividya and Hindi Commentary of Vishuddhamati Aryika*, edited by R.C. Mukhtar and C.P. Patni, Shri Mahaviraji, V.S. 2501.
   c) The *Dhavalā* commentary of the *Ṣaṭkhaṇḍāgama* of Puṣpadanta and Bhūtabali by Vīrasenācārya, edited by H.L. Jain, Amaraoti, vol. 4, 1942.
   d) The *Jambudivapannattī Saṅgaho of Paumanandi*, with an Introduction on the Mathematics of the *Tiloyapannattī* by L.C. Jain, edited by A.N. Upadhye and H.L. Jain, Sholapur, 1958. (Hindi). (a) ch. 7, pp. 419-441, latter, (b) ch. 4, pp. 295-312, (c) v. 4, pp. 150-157, (d) intr. app., pp. 99-102.
2. *Jambudivapannatti* v. 617, ch. 7, p. 419.
3. *Ibid.*, v. 618, ch. 7, p. 421.
4. *Ibid.*, v. 11, ch. 7, p. 246.
5. *Ibid.*, Cf. Muni Mahendra Kumar *The Mathematics of Tiloyapannattī* (Intr. Hindi), pp. 20-22; *Viśva Prahelikā* Bombay, 1969; pp. 255 et seq, R.C. Gupta, 'The First Unenumerable Number In Jaina Mathematics'. *Gaṇita Bhāratī*, vol. 14, nos. 1-4 (1992), pp. 11-24.
6. *Tiloyapannatti of Yati Vṛṣabhācārya* ch. 1, vv. 131 et seq.
7. *Ibid.*, ch. 1, vv. 93 et seq.
8. *The Trilokasāra of Nemicandra Siddhārtacakravartī*, ch. 1, vv. 76 et seq.
9. *The Dhanate Commentary*, 1(c), vol. 4, pp. 150-157.
10. *Tiloyapannattī of Yativṛṣabhācārya*, ch. 7, vv. 613 onwards, pp. 761-767 (latter).
11. *Ibid.*, pp. 427-41. This procedure has been adopted by the author, Vīrasenācārya for the treatment of the transfinite. *Cf.* L.C. Jain, 'On Certain Mathematical Topics of the *Dhavalā* Texts', *IJHS*, 2.2, 1976, pp. 85-111; Datta, B.B., 'Mathematics of Nemicandra', *Jaina Antiquary*, 1.2, 1935, pp. 25-44; 'The Jaina School of Mathematics', *BCMS*, 21, 1929, 115-145; R.C. Gupta, 'Circumference of the Jambudvīpa in Jaina Cosmography', *IJHS*, 10.1, 1975, 38-46; 'Jaina Cosmography and Perfect Numbers', *Arhat Vacana*, 4.2, 4.3, 1992, pp. 89-94. All the above papers may give a comprehensive view of the system of islands, seas, moons and other astral bodies in the Jaina School of Astronomy.