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CONSTANT-SET (DHRUVA-RĀŚI) TECHNIQUE IN JAINA SCHOOL OF ASTRONOMY

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In the Jaina School of Astronomy, the commentaries on the *Sūrya-prajñapti* and the *Candra-prajñapit* (*c*. 3rd century BC?) contain ancient Ardhamāgadhī Prakrit verses which make use of a term called the constant-set *(dhruva-rāśi)* for various types of astronomical calculations. The same term has been used in the Śaurasenī Prakrit verses of the *Tiloyapaṇṇattī* (*c*. 5th century AD) and the *Dhavalā* texts (*c*. 9th century AD). In the *Gommaṭasāra* as well (*c*. 11th century AD) a similar term, constant-divisor *(dhruva-hāra)*, appears for more or less the same purpose.

The mathematical technique of the constant-set appears to have an important bearing on the five-year *Yuga* system and appears to have possibly played a decisive role in the development of the enlarged *Yuga* system in India for astronomical calculations.

INTRODUCTION

At present, the commentaries on the *Sūrya-prajñapti* available are those due to Malayagiri[1], Amolaka-ṛṣi[2], and Ghāsīlāla[3]. Kohl[4] had contributed a text which was revised by A. Olz. Similarly, the commentaries of the Candra-prajñapti are available[5]. These texts are from the fifth sub-composition (*upāṅga*) of the Śvetāmbara Jaina School in the Ardhamāgadhī Prakrit. According to Jecobi and Schubring, they might have taken shape during the third and fourth centuries BC. According to Needham and Ling, they may go back to the later part of the 1st millennium. Certain descriptions are available in the contents rendered by Weber, *Ind. Stud.*, **10**, 254 ff. (1968), and Thibaut[6] *JASB*, **40**, comp. and also in his '*Astronomie*' in the Grundiss p. 20 ff., 29.

The *Tiloyapaṇṇattī*[7] was compiled by Yativṛṣabha (*c*. 5th century AD). This is in Śaurasenī Prakrit and belongs to the Digambara Jaina School. Similarly, in the same school, the *Dhavalā* commentary texts[8] were composed by Vīrasena (*c*. 9th century AD) of the *Ṣaṭkhaṇḍāgama* texts[9] composed by Puṣpadanta and Bhūtabali (*c*. 2nd century AD). The *Gommaṭasāra* texts[10] of this school form the summary texts of the *Ṣaṭkhaṇḍāgama* texts composed by Nemīcandra Siddhāntacakravartī (*c*. 10th century AD).

The term constant-set appears in the above texts, except the *Gommaṭasāra* in which the alternative term, constant-divisor (*dhruvahāra*) has been used. This appears to be an invariant of motion and is sometimes used as a parameter for generating a group of events which were periodic in character. The marked periodicity in the five-year *Yuga* system of India was thus tackled with the constant-set technique.

THE CONSTANT-SET TECHNIQUE IN THE TILOYAPAṆṆATTĪ

In the *Tiloyapaṇṇattī*, the technique of constant-set *(dhruvarāśi)* has been applied for finding out the distance between the orbits of the moon and the sun from the *Meru*[11].

The description of the three verses is as follows:

ekasaṭṭhī guṇidā pañcasayājoyaṇāṇi dasajuttā/
te aḍadāle vimissā dhuvarāsi ṇāma cāramahī//122//
ekaṭṭhisahassā aṭṭhāvaṇṇuttaraṁ sadaṁ taha ya/
igisaṭṭhīe bhajide dhuvarāśi pamāṇamuddiṭṭhaṁ//123//

$$\frac{31158}{61}$$

paṇṇarasehiṁ guṇidaṁ himakarabiṁbappamāṇamavaṇijjaṁ/
dhuvarāsīdo sesaṁ viccālaṁ sayalavīhīṇaṁ//124//

$$\frac{30318}{61}$$

Translation of the above verses is as follows:

Verse 122: On multiplication of five hundred and ten *yojanas* by sixty-one, and adding forty-eight to the product, the result (as divided by the denominator sixty-one) becomes the extension of orbital-ground, called the constant-set.

Note: $510\frac{48}{61}$ is equal to $\frac{31158}{61}$. This has been called the constant-set or orbital-field of the sun or the moon.

Verse 123: The quotient obtained on dividing thirty-one thousand and one hundred fifty-eight by sixty-one has been shown as the pole-set.

Verse 124: On multiplying the diameter of the moon by fifteen, the product is subtracted from the constant-set; the result is the measure of interval of all the remaining orbits.

Note: Diameter of the moon is $\frac{56}{51}$. Hence, $\frac{56}{51} \times 15 = \frac{840}{61}$. Now one can find the interval between the remaining orbits as equal to $\frac{31158}{61} - \frac{840}{61}$ or equal to $\frac{30318}{61}$.

Further Procedure

In the verse ahead, the following has been worked out. When (30318/61) is divided by 14, one gets the interval between every one of the orbits as $35\frac{214}{427}$ *yojanas*. Now to this amount is added the moon's diameter $\frac{56}{61}$ *yojanas*, getting the common-difference $36\frac{179}{427}$ *yojanas*. The first orbit is at a distance of 44820 *yojanas* from the *Meru*. The second orbit is at a distance of $44820 + 36\frac{179}{427}$ *yojanas* from the *Meru*. The third orbit is at a distance of $44820 + 2(36\frac{179}{427})$ *yojanas* from the *Meru*.

This is carried on till the last orbit.

Similarly, the method of the constant-set in the *Dhavalā* has been used for confirming the measure of the set of illusive-visioned bios *(mithyādṛṣṭi jīva rāśi)* through four analytical methods[12]. This is altogether a different procedure for the use of a technique of a constant set away from the convention of periodicity. However, the constant-divisor *(dhruvahāra)* concept in the *Gommaṭasāra*, is given in detail[13], where a geometric regression is produced with the constant-divisor as the common-ratio.

The Constant-set Treatment in the Sūryaprajñapti and the Candraprajñapti Commentaries

In various commentaries of the *Sūryaprajñapti* and the *Candraprajñapti*, the constant-set technique has been applied to calculate requisite sets which usually form an arithmetical or a geometrical sequence. First of all, the method is given to find out the measure of a constant-set for solving a particular problem of astronomy. Then the process of obtaining the subsequent progression of desired results is given.

There are as many as 12 examples of the use of different types of constant-sets in the commentaries of the above texts. The *muhūrta* is divided into 62 parts and further subdivided into 67 parts. Similarly, other units are divided and subdivided, the *Vedāṅga* system of time, however, being different for divisions and subdivisions.

For want of space we give only the following examples:

	Purpose	Constant-Set	Sequence-Form
1.	Finding the constellation with the sun at the end of any half-lunation	33+(2/62) + (34/62×67)	Geometric progression with constant-set as common ratio[14]

2.	Obtaining view-path in various orbits of the sun	$83+(23/60) + (42/(60\times61))$	Arithmetical progression with constant-set as the common difference[15]
3.	Finding out increase or decrease in length of sun's shadow in a particular solstice or in a *tithi* of half-lunation of a year in a *Yuga*	4 *padas* (decreasing below) and 2 *padas* (increasing above)	Arithmetical progression with negative and positive common-differences, the constant-set in individual case being the first or last term respectively[16].
4.	Finding out the orbital point of the moon at the end of bright-half 15th in a *Yuga*	32 as a constant-set in digits *(dhruvāṅka)*	Geometrical sequence with constant-set digits as common-ratio

An Example Thereof

First the constant set is calculated. In all there are 124 half-lunations *(parvas)*, i.e. 62 bright halves and 62 dark halves in the course of five year *Yuga*. During this course, there are five sun-constellation conjunctions. Hence, for one half-lunation one gets 5/124th part of a sun-constellation part period. The sun moves 1830 celestial parts in a *munūrta*, or moves through 1830 half-*maṇḍalas* or *ahorātras* in a *Yuga*. Hence, the quantity 5/124 is converted into *muhūrtas* multiplying it by 1830 and 30.

Thus, we get ($4575/62 \times 30$). This is converted into the conventional form of integral part, the 62th part and the 62×67th part. Thus, the constant-set is given by $33 + 2/62 + 34/32 \times 67$. Let us consider the problem for the first half-lunation. In this case, the constant-set is multiplied by one. The constant-set thus remains the same. Now the period to be subtracted corresponding to the first constellation, *Puṣya*, is determined. We know that 23/67th part of the *Puṣya* constellation is found to have elapsed. Hence there remains 44/67th part of the *Puṣya* constellation. On converting this quantity into *muhūrtas*, by multiplying it by 30, we get $44 \times 30/67 = 1320/67$ *muhūrtas* or when conventionally converted, this can be written as $19 + 43/62 + 33/62 \times 67$. This amount is subtracted from the former. Thus, we get $13 + 19/62 + 1/62 \times 67$ *muhūrtas*, which is the period which remains to be covered after the sun has passed the *Puṣya* constellation and occupies its position in the *Aslesa* for this much period. Just after this, the first half-lunation in the form of the coming dark 15th of the Sravana month comes to an end. This method may be applied to determine the *Yoga* of the sun and the corresponding constellation at the end of any half-lunation.

Digression on the Lunar Eclipse

Shamsastry, in his '*Draspa:* The Vedic Cycle of Eclipses' gets 18 lunar years and 9 months for one cycle of eclipse on the basis of three different colours in the recurrence of eclipses. In the Jaina School, the eclipse year is equal to 346.62 days and half of the eclipse year is equal to 173.31 days. Synodic month is equal to 29.53 days. Six synodic months are equal to 177.18 days. There is the possibility that 2 lunar and 5 solar or 3 lunar and 4 solar eclipses could occur in a year.

Parva Rāhu is known to cover the moon after 42 months. A lunar eclipse of one colour may recur after a period which would consist of days as an integral multiple of the days contained in half the eclipse year. The moon being full, the period contains a whole number of lunations.

$$\text{Number of lunations} = \frac{\text{length of half eclipse year} \times \text{integer}}{\text{length of lunation}}$$

$$\text{or } L = \frac{(173.3)\ Z}{(29.53)} = \frac{(346.62)\ Z}{(59.06)}$$

With the help of the parametric equation, it could be proved that the least value of Z consistent with the ecliptic conditions is 7. Hence, $L = (3.46.62/59.06) \times 7$, or $L = 41.0926$ days approximately

Now 41 lunations $= 41 \times 29.53 = 1210.73$ days,

and 42 months $= 42 \times 30 = 1260$ days.

Similarly, 42 eclipse months $= (346.62 \times 42)/12 = 1231.17$ days.

Thus, the lunar eclipse of the same colour appears after 41 lunations or 42 eclipse months.

That is how the rule given by Shamsastry (for finding sun's *parva-nakṣatra*) could be correlated with the constant-set technique in so far as the multiples in terms of positive integers Z for any recurrence phenomenon are concerned. In the example illustrated above, the multiples may range from 1 to 62. For further probe, the work of Lishk may be referred to[18].

Conclusion

The above probe into the technique of the constant-set appears in circulation round the fourth century BC, when the *Karaṇānuyoga* type of works in the Jaina School were compiled for set-theoretic studies of the mathematical *Karma* system theory[19]. The periodicity of natural phenomena and its calculations needed a group-theoretic study and the constant-set technique appears as one of the attempts towards it. The progressions and regressions also seem to find a place in their study. It also appears that this study might have played an important role later when the enlarged *Yuga* system was taking shape in India about the conjunction of planets through their mean motions as well as precise corrections through theory of epicycles evolved as a construction theoretic approach. It is still a secret why the counter bodies[20] were established in the Jaina School of Astronomy, moving in different regions, the description of the motion of the planets being lost and extinct in course of time. Mention may be made here about the work of Roger Billard[21], which is a work compendium of *Yuga* system through computerized deep and compact study worthy of attention, for the evolution of the constant-set technique creating various types of sequential structures.

References & Notes

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