REPORT

SEMINAR ON ASTRONOMY AND MATHEMATICS IN ANCIENT AND MEDIEVAL INDIA

A seminar on astronomy and mathematics in ancient and medieval India was held on May 19, 20, and 21 1987 at the Ramakrishna Mission Institute of Culture, Calcutta, under the joint auspices of the Asiatic Society and the Indian Council of Philosophical Research.

The Seminar was inaugurated by Swami Lokeswarananda who noted its novel character holding a dialogue between traditional and modern scholars. Prof. A.K. Saha, Chairman of the Organizing Committee explained that the seminar had been structured into three broad areas concerned with (a) some major themes in astronomy in ancient and medieval India, (b) similar themes in mathematics, and (c) a special dialogue between philosophers and traditional scholars and university trained scientists. Dialogues, he maintained, had always constituted an important method for extending the boundaries of knowledge. In this century one of the most famous dialogues of all times took place between one group led by Albert Einstein and another, the Copenhagen School, led by Niels Bohr about the real nature of quantum mechanics. The dialogue appeared in the form of a series of papers from the two groups which are now classics in the literature on physics. Galileo adopted the dialogue technique when he wrote his 'Dialogue concerning two chief world systems' which ushered in the age of modern science. Further back, in ancient Greece, Plato preferred the dialogue method in expounding the thoughts of Socrates, although much of the materials presented there concerned Plato's own thoughts, his concept of utopia, theory of ideas, cosmogony and so on. The importance of the dialogue method in India, Prof. Saha pointed out, was fully recognized as the most effective method for the exchange of knowledge between the teacher and the taught. The *Saṃhitās, Brāhmaṇas,* and *Āraṇyakas* provide numerous instances of dialogues, but by far the most impressive are those of the *Upaniṣads.* The *Milinda Panha,* recording a dialogue between Minander and Nagasena, is an instance of Buddhist interest in this form of exchange of knowledge.

Of the two major subjects of the seminar, astronomy, Prof. Saha observed, was perhaps the first science to be studied in a systematic manner. The study was somewhat handicapped by its association with the requirements of religion and rituals. The case of mathematics was different. it, no doubt, emerged through the world of experience, but very soon it developed its own intellectual structure and achieved great beauty and profundity only through the consistency of its inner logic.

Earlier, Dr. Kalyan Kumar Ganguly, in his welcome address, traced the genesis of the seminar. Dr. Debi Prasad Chattopadhyay, representing the Indian Council of Philosophical Research, explained the importance of a seminar of this type as a joint venture between his organization and the Asiatic Society.

The first four sessions were devoted to astronomy in the following order : Session : 1 Historiography, Session : 2 *Saṃhitās*, Epics and *Purāṇas* ; Session 3 & 4 : Concepts, Theories, and Procedures. In Session 1 on Historiography, Chaired by Prof. K.V. Sarma, five papers were read. Amalendu Bandyopadhyay described the instruments used by Samanta Chandrasekhar, an eminent astronomer of the traditional Siddhānta School and showed how he was able to detect the three important irregularities of the moon, namely, evection, variation and annual equation. Śatānanda's fame rested on his *Bhāsvatī* (A.D. 1099) ; in a little over hundred *ślokas* he gave all the important rules necessary for astronomical computations and rendered the compendious *Sūrya-siddhānta* practically redundant. Ramatosh Sarkar showed that ancient sanskrit astronomical texts not unoften contain statements which, upon analysis, lead to erroneous situations. Such statements are found, for instance, in the *Vedāṅga Jyotiṣa* and the *Arthaśāstra*. Traditional scholars could be helpful in providing correct interpretations. Ruma Bandyopadhyay, in her interpretation of Maya to whom the astronomy of *Sūrya-Siddhānta* had been revealed, traded on uncertain and controversial grounds. A.K. Chakravarty dealt with a number of Aśokan inscriptions containing auspicious dates according to a calender which appears to be inferior to the contemporary Greek calendar. Astronomical datings found in some Śaka and Kuṣāṇa inscriptions are already based on the Metonic cycle, suggesting Greek influence.

Professor Bibhuti Bhusan Bhattacharyya presided over Session 2 devoted to the astronomy of the *Saṃhitās*, Epics and *Purāṇas*, in which six papers were presented. In their paper on "some astronomical facts recorded in the *Ṛgveda*", Krishna De and S.S. De interpreted Asvinis, the twin physician gods, as the two stars in the constellation of Hyades in Taurus. Moreover, the twin gods represent two important great circles, the ecliptic and the equator. S.S. De and B. Basu compared the astronomical references in the *Vedas* and the *Qurān* and noticed a striking similarity between them. Padmakar Vishnu Vartak discussed an astronomical statement found in the *Mahābhārata* at the beginning of the Kurukṣetra war and interpreted it as an evidence in favour of the planets Uranus, Neptune and Pluto being known at the time of the great epic. The interpretation is conjectural and farfetched. The dating of the *Mahābhārata* as 5562 B.C. is again an unsubstantiated conjecture abandoned long time ago by the scholarly world on historical grounds. Jagatpati Sarkar discussed the case of the Sun as an astronomical deity. In Vedic times, Sūrya, Savitr, Mitra and Ādityas constituted a Sun-god group, from which the sun eventually emerged as the astronomical deity par excellence. V.L. Gandhi presented an atlas of the sky based on ancient Indian astronomy as found in Vedic and Purāṇic works.

Astronomical concepts, theories and procedures were the main theme of Sessions 3 and 4. They were chaired by Prof. Jagadish Chandra Bhattacharyya. Out of ten papers communicated six were presented at the seminar. Uma Dey discussed some observational aspects of astronomy while K.V. Sarma and S. Hariharan specially dealt with the contributions of Keralan astronomers. K.V. Sarma quoted from Parameśvara of Vaṭaśreṇī (A.D. 1360 - 1455) and Nīlakaṇṭha Somayāji (A.D. 1444 - 1545) to prove that great emphasis was placed on observation to arrive at correct planetary computations (*Pratyakṣasiddhāḥ spaṣṭāḥ syuḥ grahāḥ śāstreṣu itiritam, Dṛgganitam,* 1.1.2). In his *Jyotirmīmāṃsā,* Nīlakaṇṭha maintained that astronomical theories should be constantly checked by observations and astronomical constants corrected from time to time to enable computational results to agree with observation. S. Hariharan referred to the contributions of Mādhava of Saṅgamagrāma who flourished in the latter half of the 14th and the first quarter of the 15th century and exerted considerable influence upon later astronomers, specially in the field of spherics, trigonometrical functions, and infinite series expansions. Mādhava gave exact method of finding the declination of a heavenly body not moving along the ecliptic. L.V.S. Mani discussed ancient and modern methods of eclipse calculations and the equation of time. S.N. Sen discussed the traditional interpretation of *manda* and *śighra* corrections and the procedure for half such corrections in obtaining true planetary positions. He further gave modern geometrical rationale for a better understanding of the traditional interpretations.

Sessions 5, 6 and 7 were concerned with problems of mathematics and general problems in astromomy and mathematics. Session 5 was chaired by Prof. M.C. Chaki and Prof. Parimal Kanti Ghosh presided over the remaining two. In his opening paper, Bibhuti Bhusan Bhattacharyya discussed some conceptional discrepancies or obscurities regarding the formulae of Indian mathematics as adopted in European mathematics and astronomy. For example, Western mathematicians of the 17th century were confused in finding the proper significance of the series they found in Indian arithmetical treatises translated into Latin from Persian rendering of the original Sanskrit text.. A number of other instances were furnished. In Jaina mathematics, Navjyoti Singh dealt with unnamable finite numbers. The Jainas classified their numbers into three groups : *Saṃkhyāta, Asaṃkhyāta,* and *Ananta. Saṃkhyātas* are finite ordinals and *Anantas* transfinites. *Asaṃkhyātas* are difficult to interpret. According to Dhaval's commentary *Asaṃkhyāta* numbers are finite but cannot be represented by finite ordinals. Given any number naming scheme, infinite number of finites will always be left out, and such numbers will not lend themselves to mathematization. Navjyoti Singh showed that the Jainas solved this problem and developed the mathematics of the bizarre *Asaṃkhyāta* numbers.

Algebra, algebraic equations of various types, the indeterminate equations of first and second degree formed the subject matter of a number of papers presented by Parameshwar Jha, M.B. Pant, B. Chaki and S. Chaki, R.S. Lal, and N.K. Chakraborty. Dr. Jha traced the orogin of algebra in India to the *Saṃhitās* and *Brāhmaṇas,* particularly to the *Sulbasūtras.* The Jaina canonical texts and the *Bhākśālī* manuscript contain germs of algebraic equations. But the science of algebra proper, with its symbolism, analytical procedures, *etc.,* developed with the creative work of Āryabhaṭa and later astronomers.

B. Chaki, S. Chaki and Chakraborty more or less covered the same ground, while Pant concentrated exclusively on the *Kuttaka* (pulverization) method of solving first degree indeterminate equation of the type $ax + b = cy$, the second degree equation of the type $ax^2 + b = y^2$, and Bhāskara II's cyclic quadrilaterals.

In geometry, Euclid's fifth postulate baffled attempts to develop a proof, including the celebrated Gauss. M.C. Chaki showed that some attempts were also made in India to prove the postulate albeit without success. R.C. Gupta traced the history of developing an exact formula for the volume of a sphere in India. The correct formula was given by Archimedes in the 3rd Century B.C. ; in India the correct formula appeared in the works of Bhāskara II, although approximate formula had been in use before him. A.P. Singh gave a general account of the development of trigonometry in ancient India. A refreshing departure from all these was Yukio Ohashi's paper on Varāhamihira's orthographic projection. Ideas of these projections are contained in the first eleven verses of the 14th chapter of the *Pañcasiddhāntikā.* The explanations given by Thibaut and Dvivedi and more recently by Neugebauer and Pingree are inadequate. Ohashi showed that all the rules gave either correct value or reasonable approximation and were based on the theory of orthographic projection.

In Session 7 J.C. Bhattacharyya discussed the relationship between mathematics and astronomy. R.K. Kochhar observed that in the Indian milieu knowledge was treated as revelation, an attitude which encouraged the study of mathematics and metaphysics but not physical enquiry. According to Bandana Chakraborty's findings the social objectives of astronomical studies in ancient India were (a) to prescribe suitable time for performing sacrificial rites, and (b) to calculate the appropriate seasons for agricultural activities. M.S. Khan gave an account of the teaching of astronomy and mathematics in medieval educational institutions. M. Damodhar derived an approximate value of the velocity of light from an enigmatic statement 'Sūrya ratham travels 2202 Yojanas in nimeṣa-ardha' on the basis of doubtful assumption, while Mira Roy showed in her paper that many heavenly phenomena were invoked in alchemical preparations, in metallurgy, processing of gems, *etc.*

For Session 8, chaired by Prof. A.K. Saha, five broad issues were formulated, *viz.,* (a) the nature of mathematical knowledge, its difference from other branches of knowledge such as Āyurveda, linguistics and astronomy, (b) nature of mathematical objects, (c) the method of validation of mathematical knowledge and its essential difference from that obtaining in other *Śāstras*, (d) relation of mathematics with other branches of knowledge, and (e) relationship between mathematics and astronomy and the special features of the latter that necessitate such relationship.

In his opening remarks, the chairman summarized in the first place his reactions to the papers and discussions of the previous seven sessions. He noticed an understandable pride in the achievements of Indian astronomy and mathematics, but was disappointed not to find any discussion on the limitations of thinking, equally important for the understanding of the historical development of thought. In his own view, astronomy and mathematics in India were never allowed to be delinked from the demands of utility in practical life such as land surveying, trade, interests, fixation of timings of religious rites and festivals. All this preoccupation was true of mathematical development throughout the ancient world in general, but in Greece and later on in Europe it began to give way to more abstract and fundamental thinking. This did not happen in India. He further thought that this obsession with practical matters was partly responsible for lack of interest among ancient Indian mathematicians in the role of proof in mathematics, and in evolving a deductive method, which characterized Greek mathematical endeavour. On the nature of mathematics, the main theme of the dialogue, Prof. Saha chose to express the view of the formalists inspired by Hilbert. According to this view, mathematics is the highest abstract product and construction of the human mind, which stands on the basis created by its inner logical consistency, rigid rigour and freedom from contradictions. Furthermore, this construction does not need any validation from the universe outside the mind. Sometimes the mathematical constructs may be very strange, as for example, the imaginary quantity $\sqrt{-1}$, but when its mathematical operations were developed, it became useful to the hard headed physicists in introducing brevity and compactness in the analysis of many physical phenomena. He believed that there were other views on mathematics which the learned participants would discuss in the dialogue.

V. Shekhawat in his key paper dealt with the issue no. 2. By mathematical objects he meant arithmetical objects like natural numbers, algebraic objects like sets and variables, and geometrical objects like points and lines. He attempted to explain these mathematical objects in the light of the Indian experience as recorded in the *Vaiśeṣika sūtra, Sāṃkhya sūtra, Nyāya,* the *Sūrya-siddhānta, Gaṇitasāra saṃgraha, Līlāvatī* and *Bījagaṇita*. As to the concept of number, this is implicit in the enumeration of *guṇas* associated with *dravyas* in the Vaiśeṣika Sūtra and the *Sāṃkhya* characteristics or *lakṣaṇas*. It is possible that the science of enumeration originated with the Sāṃkhya theory. Although Vaiśeṣika and Sāṃkhya Sūtras both agreed on the reality of number, the important difference is, while the Vaiśeṣikas regard number as property of *Kāla*, the Sāṃkhya exponents treat number as an evolute of *prakṛti* or manifestation of *buddhi.* In other words, the concept of number developed intuitively. This is also true of the concept of arithmetical operations (*Parikrama*). The concept of negative and positive numbers was not possibly intuitive as it can develop, and probably did, in the context of exchange of money. Sekhawat further suggested that *Sāmānya* in the Vaiśeṣika Sūtra is close to the concept of a set. A set in order to be conceived as a set must be a collection of more than one objects. It would be impossible to conceive of *Sāmānya* if there were not more than one *Viśeṣas.* He then analyzed the relationship between set and number. In the discussions that followed P.K. Ghosh, M. Chakravorty, Bibhuti Bhusan Bhattacharyya and M.D. Srinivas participated.

Parimal Kanti Ghosh, in his keynote paper maintained that the concept of mathematical knowledge (issue No.1) could not be fully defined. It is generally admitted that mathematics is to a large extent subjective, an expression of the human kind. Its hard core is axiomatic, and it proceeds by intuition and logic to various constructions. Another characteristic of mathematics as a product of the creative mind is that it avoids dogmas. The evolution of mathematics makes it abundantly clear that it proceeds from concrete to abstract and then from abstract to concrete. In this way it is seen how integers led to fractions, the irrationality of the diagonal of a square, the concept of a line as a continuum filled by points, from the finite to the infinite. He also talked of number class relationship, the Jaina conception of number, and Cantor's theory of transfinite. He concluded that mathematics had not found its final philosophy and when it would find it, it would be dead.

In his keynote address on issue no. 3, Bibhuti Bhusan Bhattacharyya discussed the validation of any sort of knowledge from the point of view of the Nyāya-Vaiśeṣikas. This is determined by the validity of the instrumental cause of the perception of an object which fulfils the requisition of the perceiver. A thirsty person receives a fluid cool and transparent as water and drinks it. If the object perceived as water quenches his thirst the fulfilment of the requisition of the perceiver validates the instrumental cause of such perception. If mathematical knowledge, being the resultant of the mathematical formula keeping its instrumental cause, yields the requisite number of things enquired, then the formula concerned is valid and in due course the resultant knowledge also becomes valid. He further held that Śāstric formulas, when applied correctly, should yield correct results provided the terms used are properly understood. He cited a verse from the *Arthaśāstra,* which gives an idea of cost accountancy in Kauṭilya's time, provided the technical terms used, *e.g., prakṣepa, paṇyaniṣpatti, śulka, vṛddhi, abakraya* and *anyavyaya* are correctly understood.

M.D. Srinivas discussed the question of validation of mathematical knowledge from the consideration of proof. Historians of mathematics generally hold that Indian mathematical tradition, although rich in significant results and processes, failed to present proof of them. Srinivas showed that this view was totally incorrect. Instead of using proof as a method of validation, characteristic of Greek mathematics, the Indian tradition used the method of *Upapatti,* which loosely translated meant justification, derivation, demonstration, validation, and proof. Indian mathematical literature, particularly the commentaries are rich in *Upapattis.* Several examples were given from the texts of *Līlāvatī* and *Bījagaṇita,* and commented upon by Gaṇeśa Daivajña and Kṛṣṇa Daivajña. As the notion of *Upapatti* in Indian tradition is different from that of proof in the Greek tradition, the real purpose of the former being the same as that of the latter escaped the attention of historians of mathematics.

The issue no. 4 raised the question of relationship between different branches of mathematics. D.K. Sinha while leading the discussions on this question recognized that any area of knowledge, that is, discipline, including mathematics, was essentially an aggregate of parts which conditioned the very structure of the discipline. This idea of a discipline consisting of parts is also called 'elementalism'. The same thing is described by another jargon, 'holism', in which parts are linked to the whole. Then in connection with any discipline he introduced the idea of essence and object and the relation between the two. The nature of the two and their relationship, which is specific for mathematics, can be understood only through intuitive thinking. Intuitive thinking, he explained, can be simply stated as a capability to place 'objects' and see their relative importance. In the evolution of mathematics, this intuitive thinking has always played an active part but it has also significantly interacted with logical thinking. In fact, there exists a complementarity between logical thinking and intuition, a back and forth mobility between the plethora of experiences and a spate of abstractions. In his view, each branch of mathematics developed on a particular intuition which endowed it with primitive notions and truths. With the passage of time it acquired a formalized language of its own and built up an entire edifice from one theme, as for example, the theory of sets.

(S.N. Sen)
Convener of the Seminar