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ECLIPSES, PARALLAX AND PRECESSION OF EQUINOXES

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ECLIPSES

Since remote antiquity occultations of heavenly bodies attracted the attention of man and inspired him to find the time of recurrence of such events. The occultations of the Moon with *yogatārās* (junction stars or the identifying stars) of asterisms on its path (lunar zodiac) and conjunctions of other planets too with stars, or any other planet, likewise generated great interest. The most important occultation of the Moon with the Sun resulting in solar eclipse and of Moon itself, by the Earth's shadow leading to the lunar eclipse, might have proved very fascinating events. The earliest attempts to know their recurrence timings were based on determination of empirical cycles by observing colours of eclipses and identifying them as brown colour shading or dark colour shading etc. These were the attempts to find repetition cycles of eclipses of the same kind (colour). In fact, the colour of an eclipse depends upon the elongation of nodes of lunar orbits w.r.t. Sun which determines how intense (brown dark or deep dark etc.) the eclipse will be as a result of relative gradations in extents of overlapping. In ancient civilizations like China, Babylon, etc. records of eclipses had been kept. Ptolemy had a record of eclipses from 747 B.C. to 150 A.D. These records on analysis gave metonic cycle. It is believed that the Chinese might have this cycle earlier. In India, in early Vedic times there were periods determined on the basis of such observational records over centuries. We find a period of 20000 days,[1] and it is clear that

2000 days = 56 lunar years + 3 months
= 675 lunations = 3 (18 years — 9 m)
= 3 Periods of lunar node (Rāhu).

Thus the above mentioned cycle is justified. Early Vedic Aryans distinguished 3 different colours of eclipses (solar and lunar separately), i.e. black, red and white and tried to recognize periods of eclipses with specific colours.[2] On dividing the above figure (2000 days) by the number of colours (3) one gets the period of occurrence of eclipses of the same colour. This is almost equal to the period of Rāhu or Ketu (the ascending and descending nodes of lunar orbit). The Jaina texts like *Sūryaprajñapti*[3] mention five colours of eclipses, which is an advancement over the observations of *Vedāṅga Jyotiṣa* tradition. There is mentioned a cycle of 6 months.[4] We know that

675 lunations = 3 Chaldean saros + 6 months.

Thus the period of 6 months is also one of the shortest cycles, easily conceivable in early studies of solar and lunar eclipses.

In the *Vedāṅga Jyotiṣa* we do not find algorithms for computing eclipses, but there is a reference to a phenomenon of colouration which is referred to as *Vyatipāta yoga*.[56] This was the phenomenon (of colouration of Sun and Moon) for predictions of which, there were developed some methods in Jaina traditional astronomy.[7] Reference to this phenomenon occurs in *Paitāmaha Siddhānta* also.[6] In Siddhāntic texts later, this phenomenon got interpreted as the occurrence of equality of declinations of the Sun and the Moon. It may be remarked that occurrence of *Vyatipāta yoga* later got connected with a general form equality of declinations (*Krānti sāmya*) irrespective of their signs (see appendix). The methods of computing timings of this phenomenon developed from Vedic times and we find detailed methods of computing the occurrence of this in Jaina texts too. *Jyotiṣa Karaṇḍaka*[7] gives detailed methods of computing its reoccurrence in five year *yuga* of Vedic and post-Vedic tradition. But there were no other *yogas* completing the list of 27 as found in later astronomical texts. Almost all the theoretical treatises after Āryabhaṭa have dealt with the problem of *krānti sāmya* (equality of declinations of Sun and Moon) in details.

The Jaina canonical text has reference to two types of Rāhus, one of them being responsible for waxing and waning of the phases of Moon[8] (1/15th part of the lunar disc every day) while the other one was considered responsible for eclipses. There are five categories of this Rāhu depending upon the direction of 1st and last contacts and nature of eclipses[9] (like being partial, total or annular). There are no records preserved in the literature on the basis of which they inferred the cycles but it was concluded that the Parva Rāhu (responsible for eclipses) covers[10] the Sun or Moon at least once in six months and excellently Moon in 42 months and Sun in 48 years as discussed earlier. Jainas had records of 5 colours and concluded these cycles on the basis of these colours. The mathematical analysis of these findings is discussed by Sharma et al.[11] using number of theoretical methods. In brief: for lunar eclipse 42 eclipse months $\simeq$ 41 lunations and for solar eclipse 45 years $\simeq$ 48 eclipse years.

In later texts, after Vedic and Jaina canonical literature, we see the development of computational algorithms to predict eclipses. The notion of Rāhu got developed and the lunar ascending node was named after the same.

In the *Pañcasiddhāntikā* of Varāhamihira, *Vasiṣṭha* and *Romaka Siddhāntas* give algorithms for computing eclipses using the longitudes of Sun and Moon. The rules for computing eclipses are as follows :—

Rule I

(1) (A) For the lunar eclipse, find the difference D in longitudes of the near node and the Moon. If this is less than 13° lunar eclipse will take place, when she is in opposition to the Sun.

(B) (a) Time* of the beginning of eclipse is given by :

$$T_1 = T - 3/20\sqrt{169 - D^2}$$

*Here the time is given in hours and minutes; if the time is expressed in *nāḍikās* the factors preceding the root should be 3/8.

(b) Time of the ending of eclipse is given by :
$T_2 = T + 3/20 \sqrt{169 - D^2}$
where T is the time of opposition.

Rule II

(2) (A) Let the latitude of the Moon at that time be β in minutes of arc. If $\triangle\lambda$ is the hourly change in elongation of the Moon with respect to Sun, then the time for the beginning of eclipse is given by :

$$T_3 = T - \sqrt{55^2 - \beta^2 / \triangle\lambda} \quad \text{(I)}$$

Time for the ending of eclipse is given by:

$$T_4 = T + \sqrt{55^2 - \beta^2 / \triangle\lambda} \quad \text{(II)}$$

(B) $T' = T \mp 21/5 \triangle\lambda \sqrt{25 - \beta^2}$ gives beginning and end of partial maximum or total eclipse. (III)

Here it is clear that when the difference in longitudes of the node and the Moon is $13°, +$ the latitude of the Moon is taken to be $55'$ which means that near the node the latitude is assumed to change $55'/13$ minutes per degree. The sum of radii of Sun and Moon is (from I) :

$$r_m + r_s = 55'$$

Their difference (from III) is

$$r_s - r_m = 21'$$

$$\therefore \quad r_s = 38' \qquad r_m = 17'$$

which are taken to be constants.

The middle of eclipse is at T, the duration is zero if $r_s + r_m = \beta$.

The results are based on mean motions of the Sun and the Moon as hourly variations in longitudes are neglected.

The first rule is more primitive as it ignores the latitude of the Moon. *Pauliśa-siddhānta* discusses the method for computing solar eclipses. Here the parallax in longitude is converted to the time correction for conjunction (see the discussion parallax Eq. (1) later on). *Romaka-siddhānta* gives an improved version for computing solar eclipse.[12] It is this text which uses *tribhona-lagna* (nona-gesimal) for the first time. This very text gives parallax correction in latitude (i.e. the *natisaṃskāra*) in addition to parallax correction in longitude.

The *Sūrya-siddhānta* in *Pañcasiddhāntikā* of Varāhamihira gives a better and more refined method for computing solar eclipse and uses far better constants. Like in *Romaka*, it uses true motions and consequently more correct parallaxes and angular diameters. The true motion of the Moon at the time of eclipse is used instead of true daily motion. Parallax-corrected conjunction time is computed using parallax-corrected longitudes. A separate chapter is devoted for graphical representation of eclipses. To mark the points of 1st and last contacts, the Moon's position is to be fixed with respect to east-west of the observer. For this purpose, two corrections *ākṣa valana* (depending upon latitude of the place of observation) and *āyanavalana* (depending upon the Moon's *ayana* w.r. to equinoxes) are to be applied to the east west point. But in computing *ākṣavalana* versed sine is used instead of sine of the hour angle which is a mistake as pointed out by Bhāskarācārya II. Vasiṣṭha does not give *āyana valana*, but the *Sūrya-siddhānta* gives fairly accurate formula. The *Romaka-siddhānta* is silent about directions of 1st and last contacts.

The *Āryabhaṭiya* and *Mahābhāskarīya* give same method of computing parallaxes, angular diameters and *valana*. These compute parallaxes separately for the Sun and Moon and then take the difference. In the case of the Moon the same is taken in her own orbit instead of in ecliptic. It is interesting to note that *Mahābhāskarīya* uses parallax in computing lunar eclipse also, but it does not result in any difference in final figures.

Algorithm for Computing Eclipses in Present Recension of Surya-siddhanta

Having surveyed the developments historically, let us discuss in brief the working algorithm for computing eclipses according to the present version of the *Sūrya-siddhānta,* then we would like to comment on the successes and failures of these methods in the light of the equations of centre being applied to the Sun, other constants being used and the theoretical formulations involved therein. Before starting the actual computations, one should first check the possibility of occurrence of eclipse. It may be pointed out that in Indian tradition, the ecliptic limit was taken to be 14° elongation of Rāhu at the moment of syzygies. The limit is same for lunar and solar eclipse; because it was computed using mean radii of Sun and Moon and the parallax was neglected.

Lunar Eclipse

At the time of ending moment of *purṇimā* (full moon) one should compute the true longitudes of Sun, Moon and ascending node (Rāhu). The apparent disc of the Sun in lunar orbit is calculated using their mean diameters. Also the cross section of earth's shadow in the lunar orbit is computed. From the diameters of the overlapping bodies, and latitude of the Moon, position of Rāhu, one can easily infer whether the eclipse will be complete or partial. The half of the time of eclipse $T_{\frac{1}{2}}$ *(sthityardha)* is given by

$$T_{\frac{1}{2}} = \frac{60}{V_m - V_s}\sqrt{\left(\frac{D_1 + D_2}{2}\right)^2 - \beta^2}\ ghaṭis$$

where V_m = daily velocity of Moon, V_s = daily velocity of Sun, β=latitude of moon and D_1, D_2 stand for angular diameters of overlapping bodies (Earth's shadow and

Moon in case of lunar eclipse). Thus the beginning (*sparśa* or 1st contact) and ending (*mokṣa* or last (4th) contact) are given by

$T_0 + T_{\frac{1}{2}}$ where T_0 is the time of opposition.

Similarly half of the time of full or maximum overlap *(vimardārdha)* will be given by

$$T'_{\frac{1}{2}} = \frac{60}{V_m - V_s} \sqrt{\left(\frac{D_1 \sim D_2}{2}\right)^2 - \beta^2} \text{ } ghaṭis.$$

and $T_0 \mp T_{\frac{1}{2}}$ will be the moments of beginning and ending of full overlap *(vimarda)*. (These are the timings for *sammilana* and *unmilana* in traditional terminology which indicate the positions when the two bodies touch internally). Thus one gets the 3rd and 4th contacts also.

In order to have better results, the positions of the Sun, Moon and Rāhu are computed at the instant of the middle of the eclipse and using these the required arguments are recomputed and again the *sthityārdha* and *vimardārdha* are computed. The procedure is recursive and is expected to improve the results. The *Sūrya-siddhānta* gives also the formulae for eclipsed fraction (maximum and instantaneous) which are easily provable on the basis of the geometry of the eclipse phenomenon. Also it gives the formula for remaining time of eclipse if the eclipsed fraction is given after middle of the eclipse which is just the reverse process.

After giving algorithms for computing eclipses, the *ākṣa*-and *āyana-valanas* are to be computed to know the directions of 1st and last contacts. The formulae are

$$\textit{ākṣā-valana} = \sin^{-1}\left(\frac{\sin Z \times \sin \theta}{\cos \delta}\right)$$

where $Z =$ zenith distance, $\theta =$ latitude of the place of observation and $\delta =$ declination.

If the planet is in the eastern hemisphere then *ākṣa-valana* is north and if the planet is in the western hemisphere then this is south.

$$\textit{āyana-valana} = \sin^{-1}\left(\frac{\sin \epsilon \times \cos \lambda}{\cos \delta}\right)$$

where $\lambda =$ longitude of the eclipsed body. If both the *valanas* have same sign, then

sphuṭa-valana = *ākṣa-valana* + *āyana-valana*.
If they have opposite sign, then
sphuṭa-valana = *ākṣa-valana* — *āyana-valana*.

The *sphuṭa-valana* divided by 70 gives the *valana* in *aṅgulas*. The *valanas* are computed for the 1st and last contacts. These give the points where the 1st and last contacts take place on the periphery of the disc of the eclipsed body with regard to east-west direction of the observer. One can also compute *valanas* for *sammīlana* and *unmīlana* too and decide also their directions.

Solar Eclipse

The *Sūrya-siddhānta* gives the formula for parallax in longitude and latitude. The algorithms of various texts for computing the same are discussed in the next section on parallax. Here we give the rules used in *Sūrya-siddhānta.*

(1) Compute *udayajyā* $= \dfrac{\sin A \times \sin \epsilon}{\cos \theta}$

where A = the *sāyanalagna* = longitude of ascendant at ending moment of amâvâsyā (computed using *udayāsus* or timings for rising of *rāśis*).

$\cos \theta$ = cosine of latitude = *lambajyā.*

(2) Compute the longitude of *daśamalagna* using *udayāsus.* Calculate the declination δ_D for this longitude.

(3) If δ_D and θ have same direction, subtract the two, otherwise add them. The result is the zenith distance Z_D of the *daśama lagna* (*madhya-lagna* in the terminology of *Sūrya-siddhānta* in chapter on solar eclipse).

$\sin (Z_D)$ is called *madhyajyā.*

(4) Compute *dṛkkṣepa* using the formula

$$\textit{dṛkkṣepa} = \sqrt{(\textit{madhyajyā})^2 - \left(\frac{\textit{madhyajyā} \times \textit{udayajyā}}{R}\right)^2}$$

where R is standard radius adopted for tables of sines etc. ($=3438'$ in *Sūrya-siddhānta*)

(5) *dṛggatijyā* $= \sqrt{R^2 - (\textit{dṛkkṣepa})^2}$ = *śaṅku.*

Approximately one can also take $\sin (Z_D)$ to be *dṛkkṣepa* and $\cos (Z_D)$ to be *dṛggati*. *Sūrya-siddhānta* gives this approximation too and defines

$$\textit{chheda} = \frac{R}{\textit{dṛggatijyā}}$$

$$\begin{aligned} \textit{viśleṣāṃśa}, V &= \textit{tribhona lagna} - \text{Sun's longitude, (0) } \phi\ S_L \\ &= A - 90 - S_L \end{aligned}$$

$$\textit{lambana} = \frac{\sin (V)}{\textit{chheda}} \text{ east or west in } \textit{ghaṭis}.$$

If the Sun is east of the *tribhona-lagna* then the *lambana* is east and if Sun is west of the *tribhona-lagna, lambana* is west.

Note that in the approximation here it has been assumed that zenith distances of *madhyalagna* and *tribhona-lagna* are equal (in fact these differ a little). This approximation does introduce some error in *lambana*.

(6) Compute also the *lambana* for the longitude of the Moon.

If $S_L > A—90°$, the Sun is east of *tribhona-lagna*. In this case subtract the difference of *lambanas* of Sun and Moon from the ending moment of amāvāsyā otherwise add the two. The result is the parallax corrected ending moment of amāvāsyā. Compute the longitudes of Sun and Moon for this moment and recalculate the *lambanas* and again the better *lambana* corrected ending moment of amāvāsyā. Go on correcting recursively till the results do not change.

Now compute the *nati saṃskāra* for correcting the latitude of the Moon using the formula :

$$\begin{aligned} nati &= \frac{(V_m - V_s) \times d\d{r}kk\d{s}epa}{15\,R} \\ &= 49\, d\d{r}kk\d{s}epa/R = 49\, d\d{r}kk\d{s}epa/3438 \\ &= d\d{r}kk\d{s}epa/70. \end{aligned}$$

Apply the *nati* correction to the latitude of the Moon. Using the parallax-corrected ending moment of amāvāsyā and *nati*-corrected latitude of Moon, compute the timing for 1st contact *(sparśa)* 2nd contact (*sammīlana*- time for touch internally, indicating full overlap) 3rd contact (*unmīlana*—start of getting out, indicating touch of the other edge internally) and the eclipsed fraction, *ākṣa-valana, āyana-valana* etc., using the same formulae as given in case of the lunar eclipse. The only difference is that here the eclipsed and eclipsing bodies are Sun and Moon, while these were the Moon and Earth's shadow in case of the lunar eclipse.

In the next chapter *(Parilekhādhikāra) Sūrya-siddhānta* gives the method of depicting the phenomena of contacts etc. diagrammatically using the *mānaikya-khaṇḍa* and *manāntara-khaṇḍa* $(D_1 \pm D_2)/2$ and the *valanas* (to indicate the directions of 1st and last contacts). Such a diagrammatical depiction of eclipses is found almost in every standard text of Hindu traditional astronomy. The details of the method employed are elaborately given by Mahavira Prasada Srivastava.[13]

The illustrative examples for computing lunar and solar eclipses are given by Mahavira Prasada Srivastava[14] and also by Burgess.[15]

It is worthwhile to discuss here how far successfully could *Sūrya-siddhānta* predict solar and lunar eclipses. It may be remarked that the methods as such are quite right but the data used sometimes lead to failure of predictions. The main difference lies in the equations of centre to be applied to the Moon. It may be remarked that the mean longitude of Moon in *Sūrya-siddhānta* is quite correct but the corrections like variation, annual variation, evection etc. (which result from expansion of gravitational perturbation function for the 3-body problem of Earth-Moon-Sun system in terms of Legendre polynomials of various orders) are lacking. There are thousands of terms for correcting longitude of the most perturbed heavenly body, the Moon. At least nearly fifty or eleven or most unavoidably 4 or 5 corrections are required to be applied to the longitude of Moon and to its velocity, to get satisfactory results. Even if only Muñjāla's correction (evection) is applied, there may result an error of the order of $\frac{1}{2}°$ in longitude of Moon[16] even at syzygies.

It may be remarked here that the *Sūrya-siddhānta* (*S.S.*) applies only one equation of centre (the *mandaphala*) in the longitudes of Sun and Moon. In fact the amplitudes for *mandaphalas* of Sun and Moon were evaluated using two specific eclipses. These were so selected as follows :

(1) One eclipse (solar or lunar) in which the Moon was 90° away from her apogee (or perigee) and Sun on its *mandocca* (lines of apses)

(2) Second eclipse in which the Sun was 90° away from its *mandocca* (or *mandanica*) and Moon was at her apogee.

Although we do not have records of these eclipses for which the data on *mandaphala* were fitted, it is evident that the eclipses might have been so selected that in one case the *mandaphala* of one of them is zero and maximum for the other and vice-versa in the second case. It is clear that the amplitudes of *mandaphalas* in these cases will be the figures used in *Sūrya-siddhānta*. The maximum *mandaphala* (1st equation of centre) for Sun is 2°-10' and for Moon its amplitude is 5°. The actual value in case of Sun being 1°55′ which along with the amplitude of annual variation 15′ amounts to the amplitude (=2°10′) given in *Sūrya-siddhānta*. This evidently indicates that the annual variation got added to the equation of centre of Sun with the sign changed which is also clear if the above-mentioned cases of fitting of data are analysed theoretically. It may be remarked that the *S.S.* equation of centre of Moon does not have annual variation so that at least the *tithi* is not affected by this exchange of the annual variation from Moon to the Sun (as the sign too got changed).

Now it is evident that only those eclipses which conform to the situations given above, (for which the data fitting was done) will be best predicted and the eclipses in which the Sun, Moon are not at their above mentioned nodal points, may not be predicted well or may be worst predicted if they are 45° away from these points on their orbits. The error in longitude of Moon is maximum near *aṣṭami* (the eighth *tithi*)[17] and it is minimum upto 1/2° near syzygies. There had been cases of failure of predictions in the past centuries and attempts were made by Gaṇeśa Daivajña, Keśava and others to rectify and improve the results. The timings may differ or even sometimes in marginal cases, the eclipse may not take place even if so predicted using data of *Sūrya-siddhānta* or sometimes it may take place even if not predicted on the basis of *Sūrya-siddhānta*. The difference in timings (between the one predicted on the basis of *Sūrya-siddhānta* and the observed one) are quite often noted in some cases even by the common masses[18] and for that reason now *pañcāṅga*-makers are using the most accurate data (although the formulae used in general are the same) for computing eclipses.

The modern methods of computing eclipses use right ascensions and declinations, while Indian traditional methods use longitudes and latitudes and parallax in the ending moments of syzygies (and *nati* in latitude of Moon). The instantaneous velocities are not used. The daily motions even if true, but without interpolations, on being used introduce errors. The locus of shadow cone and the geometry of overlap in the framework of 3-dimensional coordinate geometry is not utilised. The recursive processes do improve the results and the formulae as such are all right but the errors in the true longitudes and latitudes of Sun and Moon and in their velocities lead to appreciable errors.

In fact even Bhāskarācārya in his *Bījopanaya*[19] discussed most important corrections like hybrids of annual variation and variation but missed evection which was earlier found by Muñjāla in his *Laghumānasa*. In 19th A.D. Candrasekhara gave annual variation. If corrections due to Muñjāla, Bhāskarācārya and Candra Shekara are applied simultaneously, results improve remarkably.

In the last century of Vikrama Samvat and also in the last forty years of present century of Vikrama era many Indian astronomers like Ketakara[20] and others advanced the methodology of calculation of eclipses using longitudes and latitudes and prepared *sāranis* (tables) for lunar and solar eclipses (for whole of global sphere). These tables yield very much accurate results.

APPENDIX

If the Sun and Moon have equal declinations with same sign in different *ayanas*, the *yoga* was termed *vyatīpāta* and if the signs were opposite but still the magnitudes were equal in same *ayanas* then it was termed as *vaidhṛti* (See fig. 7.1—1(a)(b)). In later developments the *yogas* were given a much more general meaning and these

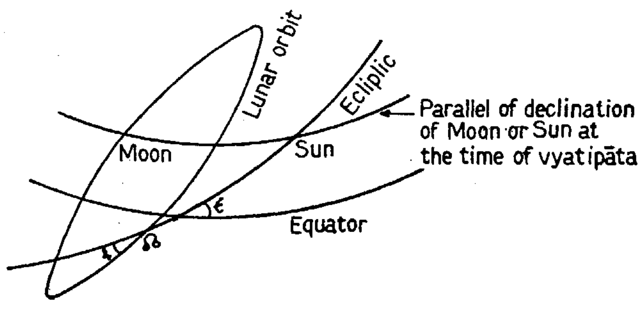


Fig. 7·1 (a)

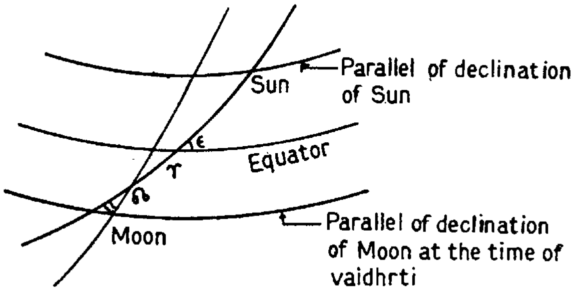


Fig. 7·1 (b)

were defined as sum of longitudes of Sun and Moon. *Yogas* were defined as a continuous function to know the time or day of *vyatipāta* and *Vaidhṛti yogas*. The idea of using this parameter is easily expected because if the latitude of the moon's orbit is neglected then for equality of declinations,

$$\sin S_L = \sin M_L$$

where S_L and M_L stand for longitudes of Sun and Moon respectively which shows, if $S_L = M_L$, $S_L = 180° - M_L$ or $S_L + M_L = 180°$.

Thus the sum of longitudes was treated as a parameter. In order to study the variation of this parameter there were defined 27 *yogas* in siddhāntic texts. This attempt may be visualised as one of the earliest attempts to compute the day (or time) of eclipse or to have an idea of occurrence of eclipse. Jaina texts mention *vyatipāta* and *vaidhṛti yogas*. *Jyotiṣkaraṇḍaka* gives a method of computing only *vyatipāta yogas* in a 5-year yuga. It may be noted that *vaidhṛti* was first defined in *Pauliśa-Siddhānta* (300 B.C.) But the list of 27 *yogas* was computed by Muñjāla (10th century A.D.). The method of computing *krānti sāmya* (timings of equality of declinations) is given in all texts (see "*Jyotirgaṇitam*" *Pātadhikāra*).

PARALLAX (LAMBANA)

Theoretically computed positions of planets (using *ahargaṇa* and equation of centre), are geocentric. Since the observer is in fact on the surface of the Earth, a correction on that account must be applied at the time of observations. The difference between the positions of a planet as seen from the centre and from surface of the earth is called *lambana-saṃskāra* (parallax-correction) or simply the *lambana*. In siddhāntic texts like *Sūrya-siddhānta* etc. it is discussed in the beginning of the chapter on solar eclipse, as this correction depends upon the position of observer and the zenith distance of the planet at the time of observation and thus must be applied in astronomical phenomenon like eclipse. Geometrically we have shown the geocentric position P_1 of the planet P as seen by an observer at the centre of the earth O. The observer

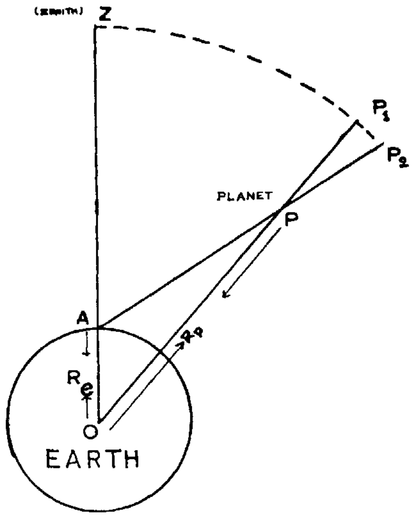


Fig. 7·2

is as the point A on the surface of the earth and his zenith being vertically upward point Z. The position of the planet as seen from A is P. The angle $<APO$ is the *lambana* in the zenith distance of the planet. This is given by

$$\sin p = \frac{R_e}{R_p} \sin z$$

where z = zenith distance
R_e = radius of earth
R_p = OP
= distance of planet
p = $<APO$ = *lambana*.

It may be remarked that the parallax was appearing in the data on lunar observations in early astronomical traditions of pre-siddhāntic period, because the observations were being performed at the time of moonrise and moonset. In these cases maximum value of parallax (horizontal parallax) appeared in their data. In *Purāṇas* and in Jaina literature in Prākṛta[12] there are statements in which it is mentioned that Moon generating its *maṇḍalas* travels higher than the Sun. The statement is usually misinterpreted as mentioning Moon being at larger distance from Earth than the Sun. In fact in such statements the "height" means the latitudinal or declinational height in the daily diurnal motion in *maṇḍalas* (i.e. in spiral-like paths). It is evident that Moon goes upto declinational height of 28°.5 and Sun only upto the declinational height of 23°.5 in Jambūdvīpa. In fact the statements give heights in units of *yojanas* which are just the heights like the ones above sea level. Thus the statements in Purāṇas and Jaina astronomical texts like *Sūrya-prajñapti* mentioning Moon travelling above the Sun, are justified. It is found that[22] 510 yojanas$=2\,\delta_{max}$

$=47°$ when δ_{max} is the maximum declination (or obliquity) of Sun and the Moon goes higher than Sun by 80 *yojanas* $=\left(\frac{80\times47}{51}\right)^{\circ}=7°.37$. Thus using the data given in Prākṛta texts of Jainas it is found that latitude of Moon arrived at is $7°.37$. The actual value of latitude of Moon including parallax is $6°.64$ (the actual value without parallax $=5°$). According to the Jaina literature the estimated parallax of the Moon is quite large due to experimental errors. In *Pauliśa-siddhānta* the latitude[23] of the Moon is given to be $4°\ 30'$, but one verse gives $4°40'$ and there is also a verse [24] giving $7°.83$. This very text gives parallax in longitude in terms of *ghaṭikās* to be added to or subtracted from the time of ending moments of *amāvāsyā* (new moon conjunction). The formula can be written in the following form[25]

parallax = 4 sin (hour angle of Sun) *ghaṭis*. (1)

In *Sūrya-siddhānta* we do not find much details in defining parallax geometrically but the later texts of the siddhāntic tradition have all relevant details. *Sūrya-siddhānta* starts discussing parallax in longitude and latitude stating that parallax in longitude *(harija)* of Sun is zero when it is in the position of *madhya-lagna*[26] (ascendant—90°) and the parallax correction in latitude (*nati* or *avanati*) is zero where the northern declination of the *madhya-lagna* equals the latitude of the place of observation. These facts can be easily visualised applying spherical trigonometrical formulae to solve the relevant spherical triangles. *Sūrya-siddhānta* and other texts in Indian traditional astronomy discuss the parallax corrections in longitude and latitude only.

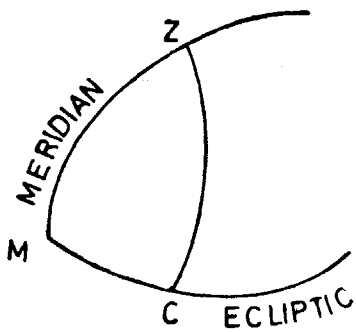


Fig. 7·3

In *Āryabhaṭīya* the parallax is computed as follows:[27]

Let Z be the zenith and M the point of intersection of the ecliptic and ZM the meridian of the place of observation. C is the point of shortest distance of the ecliptic from the zenith (i.e. ZC perpendicular from Z to the ecliptic (Fig. 7.3).
Define

madhyajyā = chord sine of $ZM = \sin(\widehat{ZM})$

udayajyā = chord sine of MZC = sine $(\widehat{MZC})$
where sign on the angular argument indicates that the trigonometric function is evaluated with standard radius (R)

$$\text{Since } \angle ZCM = \pi/2, \sin(\widehat{MC}) = \frac{\sin(\widehat{ZM}) \times \sin(\widehat{MZC})}{R}$$

$$= \frac{madhyajy\bar{a} \times udayajy\bar{a}}{R}$$

$$dṛkkṣepajyā = \sqrt{(madhyajyā)^2 - (\sin(\widehat{MC}))^2} \qquad (2)$$

$$dṛggatijyā = \sqrt{\sin^2(\widehat{ZP}) - (dṛkkṣepajyā)^2}$$

where ZP = zenith distance, sin $(\widehat{ZP})$ is called *dṛgjyā*.

$$\therefore (dṛggatijyā)^2 = (dṛgjyā) - (dṛkkṣepajyā)^2. \qquad (3)$$

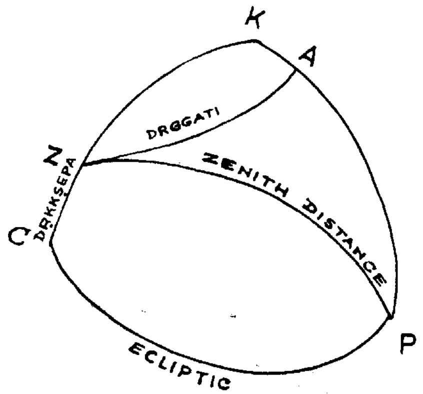


Fig. 7·4

This formula[28] can be proved as follows :
In Fig. 7.4, CP is the ecliptic, P being the planet. K is the pole of ecliptic, Z the zenith of the observer, ZA the perpendicular from Z on the secondary KP. Since ZC $\perp$ CP and $ZA \perp KP$, $\sin^2(\widehat{ZA}) = \sin^2(\widehat{ZP}) - \sin^2(\widehat{ZC})$

sin $(\widehat{ZC})$ is *dṛkkṣepajyā* and the chord sine of zenith distance ZP is *dṛgjyā*. Chord sine of ZA is *dṛggatijyā*.

$$\text{parallax in longitude} = \frac{R_e \times dṛggatijyā}{R}\ yojanas$$

$$= \frac{R_e \times dṛggatijyā}{\text{planet's true distance in } yojanas}\ \text{minutes}$$

$$\text{parallax in latitude} = \frac{R_e \times dṛkkṣepajyā}{R}\ yojanas$$

$$= \frac{R_e \times dṛkkṣepajyā}{\text{planet's true distance in } yojanas.}\ \text{minutes}$$

Bhāskarācārya I (629 A.D.)[29] in *Mahābhāskarīyam*, followed Aryabhaṭa's method. Brahmagupta[30] in his treatise *Brāhma-sphuṭa-siddhānta* criticized the approach by Āryabhaṭa. His objection is that *dṛgjyā* is the hypotenuse, *dṛkkṣepajyā* is the base, hence (2) is not valid, but we have shown that this is correct.[28] Brahmagupta's criticism is valid only if the arc between the central ecliptic point and the planet, stands for *dṛggati* as defined by him.

Brahmagupta's method of computing *lambana* is based on evaluating five R sines (chord sines)[31] as follows :

(1) If θ = the latitude of the place,
δc = the declination of the ecliptic point (c) on the meridian.
madhyajyā (as already defined) = R sin (zenith distance of the meridian ecliptic point)

$$= \sin \widehat{(ZM)} = \sin \widehat{(\theta + \delta c)}$$

(2) The R sine of the arc between ecliptic and equator on the horizon is *udayajyā*

$$= \frac{\sin \lambda \sin \epsilon}{\cos \theta}$$

where λ = longitude of the point of ecliptic in the east.
ϵ = obliquity of the ecliptic.

(3) *Dṛk-kṣepajyā* is the R-sine of the zenith distance of the central ecliptic point and is given by

$$dṛk\text{-}kṣepajyā = \sqrt{(madhyajyā)^2 - \left\{ \frac{udayajyā \times madhyajyā}{R} \right\}^2}$$

(4) *Dṛggatijyā* is the chord sine of altitude of the central ecliptic point.
$dṛggatijyā = \{R^2 - (dṛk\text{-}kṣepajyā)^2\}^{\frac{1}{2}}$ (4)
Note the difference from eq. (2).

(5) $dṛgjyā = \sin \widehat{(zenith\ distance)} = \sin (\widehat{z})$. It is given by

$$dṛgjyā = \left(R^2 - \left\{ \frac{dṛggatijyā \times \sin (\lambda - S_L)}{R} \right\}^2 \right)^{\frac{1}{2}}$$

where S_L = longitude of the Sun

$$lambana = \frac{dṛgjyā \times \text{Earth's semidiameter}}{\text{distance of the planet in } yojanas} \text{ (in minutes of arc).}$$

In eclipse-calculations the difference between *lambanas* of Sun and Moon is required. So sometimes this difference is called *lambana* (the parallax for computation of eclipses).

$$lambana\ P' = \left\{ \frac{(dṛgjyā \text{ of Moon})^2 - (dṛk\text{-}kṣepajyā \text{ of Moon})^2}{\text{Moon's true distance}} \right\}^{\frac{1}{2}} \times 18$$

$$= \left\{ \frac{(dṛgjyā \text{ of Sun})^2 - (dṛk\text{-}ksepajyā)^2}{\text{Sun's true distance}} \right\}^{\frac{1}{2}} \times 18 \text{ in minutes of arc}$$[32]

where the factor 18 is obtained from the value of the Earth's semidiameter.

This can be converted into *ghaṭis* using ratio proportion with difference between daily motions of the Sun and the Moon.

$$p \text{ (in } ghaṭis\text{)} = \frac{60}{d} \times P'$$

where d is the difference between daily motions of Moon and Sun in minutes of arc.

For solar eclipse, parallaxes in longitudes of Sun and Moon and the parallax correction in latitude of the Moon *(nati)* are required.

The *nati* is given by

$$nati = \frac{(dṛk\text{-}ksepajyā \text{ of Moon}) \times 18}{\text{Moon's true distance}} - \frac{(dṛk\text{-}ksepajyā \text{ of Sun}) \times 18}{\text{Sun's true distance}} \text{ in minutes of arc.}$$

Moon's true latitude $=$ Moon's latitude $\pm$*nati.*

Sūrya-siddhānta and Brahmagupta both compute the *lambana* and *nati* using the formulae:

$$lambana = \frac{\sin (M - S_L) \times dṛggatijyā}{(\sin 30°)^2} \quad ghaṭis$$

M = longitude of the meridian ecliptic point.

$$nati = \frac{dṛk\text{-}kṣepajyā\ (V_m - V_s)}{15\, R} \quad \text{(in units of those of velocities)}$$

where V_m and V_s stand for the daily motions of the Sun and the Moon.

Bhāskarācārya gave simpler algorithm for computing horizontal parallaxes of planets. According to this algorithm the daily velocity of planet divided by 15 gives the parallax.[33] This formula is quite evident because the parallax of any planet is the radius of the Earth in the planet's orbit. The radius of the Earth $=$ 800 *yojanas* and daily velocity of each planet according to *Sūrya-siddhānta* is equal to 11858.72 *yojanas.* We know that the ratio of the daily orbital motions $=$ ratio of the orbit's radii. Hence

$$\text{Parallax } p = \frac{\text{velocity of planet}}{15} \quad \text{(in units of those of velocity).}$$

Since day $=$ 60 *ghaṭis,* hence horizontal parallax is almost the angular distance travelled by planet in 4 *ghaṭis.* It may be remarked that in fact the distances (in *yojanas*), daily travelled by planets are not the same, hence the results were inaccurate. The following table shows the figures for comparison.[9]

Table 7.1. *Table showing Bhāskara* II's *horizontal parallax for each planet and modern values.*

Planets	Bhāskarācārya's horizontal parallax	Modern observations yield horizontal parallax Minimum	Maximum
Sun	236″.5	8″.7	9″.0
Moon	3162″.3	3186″	3720″
Mars	125″.7	3″.5	16″.9
Mercury	982″.1	6″.4	14″.4
Jupiter	20″.0	1″.4	2″.1
Venus	384″.5	5″.0	31″.4
Saturn	8″.0	0″.8	1″.0

Note that only the parallax of the Moon is fairly correct. This resulted in reasonable success in predictions of eclipses.

In later traditions for the computation of eclipses, *Makaranda-Sāranī*[34] is famous. This has the following algorithms for computing *lambana* and *nati*

(1) At the time of ending moment of *amāvāsyā* compute Sun's declination $= \delta_S$ and declination of *tribhona-lagna* (A = ascendant—90°) $= \delta_A$.

(2) Zenith distance of $A = Z_A = \delta_A \pm \theta$, (+ve if θ and δ_A are oppositely directed,—ve sign if these have same sign).

(3) If $\left(\frac{Z_A}{22}\right)^2 > 2$ subtract 2 from this.

(4) Compute $hara = \left\{\left(\frac{Z_A}{22}\right)^2 + \left[\left(\frac{Z_A}{22}\right)^2 - 2\right]\right\}^\circ + 19^\circ.$

(5) $lambana = \left[14 - \left(\frac{S_L - A}{10}\right)\right] \times \left(\frac{S_L - A}{10}\right)$ *ghaṭikas* to be applied in ending moment of amāvāsyā.

If *tribhona-lagna* $A > S_L$ then it is to be added to and if $A < S_L$ then it is to be subtracted from ending moment of amāvāsyā.

(6) $13 \times$ *lambana* = *lambana* in minutes of arc = (in minutes of arc)

(7) Compute $S_L - \Omega \pm l = a =$ *lambana* corrected latitude argument (*śarakendra*).

where Ω = longitude of Rāhu.

Using a as argument (*śara-kendra*) compute latitude of Moon, as per algorithm. given in the text (*Makaranda-sāraṇi*). Let it be denoted by P_m.

(8) $A \pm 6 \times P_m$ = *lambana-corrected tribhona-lagna* $= A'$ (say).
A'+angle of precession a *sāyanatribhona-lagna* $= A''$ (say).

(9) Compute the declination corresponding to the longitude A''. Let it be $\delta_{A''}$.

(10) $\theta \pm \delta_{A'}$ = zenith distance of *lambana*-corrected *tribhona-lagna* = $Z_{A'}$ (say).
(11) Compute $(18 - Z_{A'}/10)\, Z_{A'}/10$ in minutes of arc y (say).
(12) Compute $378 - y$ = Remainder (in minutes of arc) = r (say).
(13) *nati* = y/r.
It has same sign as that of $Z_{A'}$.
(14) Moon's latitude$\pm$*nati* = true latitude of Moon.

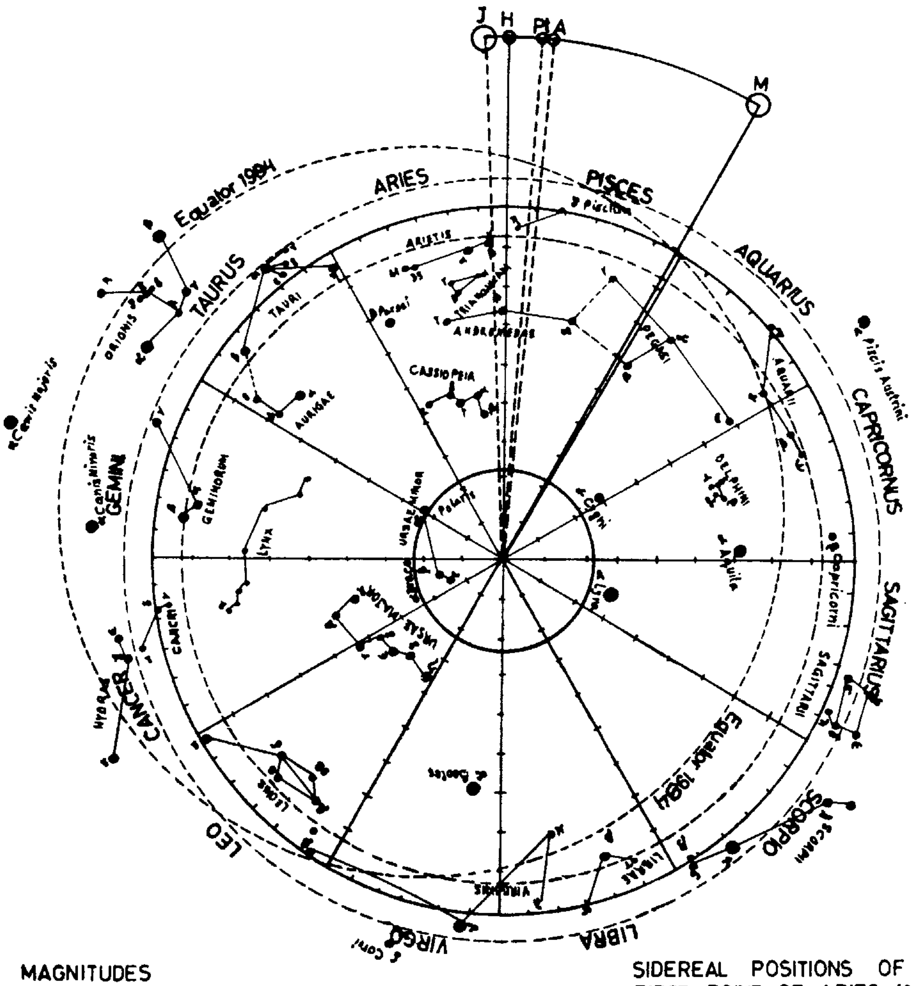
Later Kamalākara Bhaṭṭa who compiled his *Siddhānta-tattva-viveka*[35] in 1656 A.D. made an exhaustive analysis of the *lambana* and *nati* corrections. This is by far the most detailed analysis. He criticised Bhāskarācārya's approach as well as the treatment done by Munīśvara in *Siddhānta-sārvabhauma* and pointed out the approximations, used by them in their derivations. It may be remarked that Kamalākara's treatment is probably the most exhaustive of all the treatments available in astronomical literature in Sanskrit. He has categorised *lambana* corrections in various elements and gave sophisticated spherical trigonometric treatment in order to study the values in different geometrical positions for applications in solar eclipse computations.

It may be noted that in Indian astronomy, *lambana* is applied in observations of Moon, moonrise and moonset and in computing solar eclipses etc. but it was never applied in *tithis*, which have same ending moment all over the global sphere.[36] It was not applied in computing cusps of Moon but the same should have been applied.[37]

It may be pointed out that the advancements in developing formulae for computing *lambana* and *nati* by Indian astronomers upto Kamalākara Bhaṭṭa (before Newton) are very much appreciable, but these corrections were done in longitude and latitude only, in terms of parallax in zenith distance and no formulae for parallax corrections in right ascension and declination were developed because eclipses were calculated using ecliptic coordinates only and never the equatorial coordinates.

PRECESSION

From Vedic and Post-Vedic literature, it can be easily inferred that the phenomenon of precession was qualitatively known to Indian astronomers of those times although its velocity was not known. We find the lists of lunar asterisms *(nakṣatras)*, starting with Śraviṣṭhā (Dhaniṣṭhā—an asterism of lunar zodiac identifiable with the star alpha or beta delphini) at the time of *Vedāṅga-Jyotiṣa*[38] and the list found in Jaina literature[39] has asterism Abhijit (identified as Vega star α-Lyrae) as the 1st asterism (the positions of nodal points of ecliptic at the time of *Sūrya-prajñapti* are shown in the figure 8.1), which clearly indicates that the starting *nakṣatra* was changed from time to time, whenever an appreciable change got accumulated in the position of winter solstice among the stars. According to B.G. Tilak[40] there were also times when spring equinox was in the constellation Mṛgaśīrṣa (the "Orion") which corresponds to the position of winter solstice in the asterism Uttarābhādrapadā, 3 *nakṣatras* (about 40°) ahead of Dhaniṣṭhā *nakṣatra*. In fact there were two types of traditions developed in vedic and post-vedic period.[41] According to one tradition winter solstice marked the beginning of year and according to the second tradition the beginning of the year was with Sun's passage at spring equinox. At the time of the *Śatapatha Brāhmaṇa* the spring equinox was near the constellation Kṛttikās (pleiades).[42]



MAGNITUDES
First ●
Second ●
Third ●
Fourth ●
Fifth ●

* Period of occurrence of Winter solstice when sun's longitude used to be equal to that of beginning of Abhijit which is 630 muhūrtas (=276° 47′) from Aśvinī nakṣatra.

SIDEREAL POSITIONS OF THE FIRST POINT OF ARIES (♈) IN DIFFERENT TIMES.

M = Modern 1984 A.D.
A = Zero point of Aśvinī nakṣatra 285 A.D.
Pt. = Ptolemy 150 A.D.
H = Hipparchus 140 B.C.
J = Jaina period* 200 B.C.

Fig. 7·5
The Zodiac through ages.

Later this shifted to Bharaṇī asterism at the time of *Sūrya-prajñapti* and to the asterism Aśvinī (identified with the star alpha or beta Arieties) in the Siddhāntic period. These days the list of *nakṣatras* starts with Aśvinī. No change was done by the astronomers of the siddhāntic period. This is due to the reason that the *nirayaṇa* (sidereal i.e. without precession) zodiac got accepted in the tradition, although the Hindu year was tropical. (This fact will be discussed later). All the later treatises are using *nirayaṇa* longitudes and their respective *ayanāṃśas* as the difference between the analytically computed and observed position of the Sun. It may be noted that from early Vedic time upto the time of Brahmagupta including *Vedāṅga Jyotiṣa, Sūrya-prajñapti, Pañca-siddhāntikā, Āryabhaṭīya, Brāhmasphuṭa-siddhānta* etc., only the *sāyana* (Tropical) longitudes were used. In earlier literature we clearly find change in the order of *nakṣatras* in the zodiac list after an accumulation of precession equal to 1 *nakṣatra* (=13°20′) or more, but now the precession with respect to beginning of Aśvinī is about 23°38′ but 1st *nakṣatra* of the list has not been changed by any astronomer over about 2000 years. Here in figure 7.5 we have shown the position of spring or vernal equinox among the stars. Here the Jaina period has been fixed around 200 B.C. taking into consideration the list of nakṣatras in *Sūrya-prajñapti*.[43] For comparison the position of the vernal equinox at the time of Hipparchus (140 B.C.) too is shown in the diagram. As pointed out earlier in Aryan traditions, the beginning of the year was taken with Sun's position at spring equinox or winter solstice, but after the time of *Sūrya-prajñapti* the vernal equinox position of Aśvinī was almost exclusively taken as beginning of year and the winter solstice position got almost dispensed with in reckoning the beginning of the year.

Although Hipparchus (2nd century B.C.)[44] is accredited with the discovery and categoric statement of the shift due to precession in the positions of stars with respect to spring equinox from the time of Timocharis of Alexandria (280 B.C.) yet there is no doubt that even Vedic Hindus shifted the zero of their zodiac from time to time and thus, were knowing the phenomenon of precession. Ptolemy (150 A.D.) gave the velocity of precession to be about 36″ per annum which was improved by later astronomers. In the Indian tradition, although the 1st nakṣatra in the lists was changed from time to time, with the changes in the position of spring equinox, the actual magnitude and rate of precession in the Hindu tradition is not found before siddhāntic period. After the advent of siddhāntic (theoretical) astronomy, Viṣṇucandra in *Vasiṣṭha Siddhānta* mentions the *yuga* of *ayana* (precession)[45 46] which gives the rate of precession as 57″ P.A.[45 47] The phenomenon is referred to in *Romaka Siddhānta* also[46] and it is argued that the shift of solstices among stars is not possible without precession of equinoxes. These authors were criticised by Brahmagupta,[48] Bhaskara I[49] (600 A.D.) etc. Varāhamihira mentioned the *ayana calana* hypothesis[50] and suggested verifications through actual observations.[51] In fact because of small angle of precession or due to no belief in *ayana-calana*-hypothesis, the precession was ignored by Āryabhaṭa, Bhāskara I and Brahmagupta etc. It was Muñjāla (10th century A.D.) who started using *ayanāṃśa* which had accumulated to about 6° by that time. Probably at that time, there was confusion in *saṃkrāntis* (Solar ingresses or transits from one sidereal sign to the next) and he had to remove the confusion by fixing the value of *ayanāṃśa* through experimental observations. Viṣṇucandra and

Muñjāla believed in uniform regression of equinoctial point up to complete cycle, while present *Sūrya-siddhānta* uses the hypothesis of oscillatory motion (trepidation) of equinoxes. (This hypothesis will be discussed in details in the next section). The rate of precession according to Viṣṇucandra is about 56″.9 P.A. (number of cycles in a *kalpa*=189411). According to Muñjāla the rate is 59″.2 P.A. (number of cycles in a *kalpa*=199669). These are uniform regressional motions. According to *Sūrya-siddhānta* the cycle of oscillatory motion has a period of 7200 years, but it is strange that rate of precession is again uniform 54″ P.A. and it is not oscillatory which is against laws of oscillatory motion.

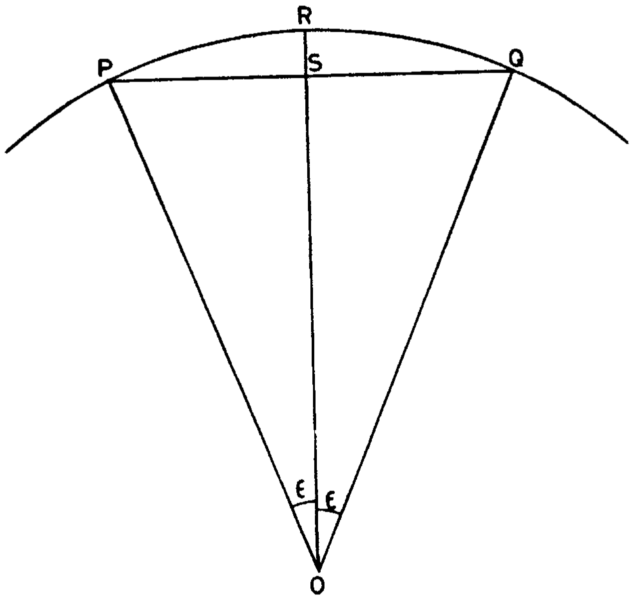


Fig. 7.6
The technique used for horizontal observation in *Sūrya-prajñapti* and *Śulba-Sūtras*.

The Length of Solar Year, Sidereal or Tropical

In *Vedāṅga-jyotiṣa* the length of solar year was taken to be 366 days and this was used in Vedāṅga calendar of 5-year *yuga* having two intercalations in 5 years. Later in *Sūrya-siddhānta* we find the solar year to be equal to 365 days 15 *ghaṭis* 31 *palas* and 30 *vipalas* (365^d—6^h—12^m—24^s). In fact, methods for determining solar year are given in *Sūrya-prajñapti* (and *Śulba-sūtras*). At that time there was not known any distinction between tropical and sidereal years. The Sun was observed while rising and setting every-day, throughout the year. The length of the year was computed by counting the number of days and a fraction thereof on the last day at any of the cardinal points preferably solstices, as the Sun appears to be stationary there. In diagram 7.6 is given the experimental technique described in *Sūrya-prajñapti*.

At winter solstice *Q* one starts observing the Sun while rising in the east with the help of a stick or a *nalikā* (tube).[52] A *davarikā* (thread) *PQ* was also used in these experiments. The arc *PRQ* is graduated in 124 divisions and 124 subdivisions in the fashion of Vedāṅga astronomical tradition (the figure 124 being the total number of syzygies in 5 year *yuga*). P is the summer solstice position of the Sun. At this point the fraction of the day is computed using ratio proportion and it was estimated that on the last day it went up to 144 ($=124+20$) subdivisions (i.e. 20 subdivisions in excess) and same situation occurred on the winter solstice day on Sun's back return journey. This way the length of the year can be shown to be $365+\frac{2\times 20}{124}=365\frac{1}{3}$ days.[53]

Generally it is believed that Indian astronomers did not determine the length of solar year experimentally and adopted the value as given in *Sūrya-siddhānta* from external sources but this decoding of experiment on determination of solar year in *Sūryaprajñapti* disproves this hypothesis. Such experiments were being performed in early vedic times too. The earliest report on such observations is found in *Aitrareya Brāhmaṇa*.[54] The position of Sun at any point on the horizon of any locality is specified by its declination, so it is evident that this method will yield tropical year and not the sidereal year. Thus it is clear that by the word "year" the Hindu astronomers of pre-siddhāntic period meant tropical *(sāyana)* year. In fact they were not aware of sidereal *(nirayaṇa)* year. This very traditional way of determining solar year is found in the *Siddhānta śiromaṇi* of Bhāskarācārya and other Siddhāntic texts but we do not find any method of determining really sidereal year in any Siddhāntic text of Indian Astronomy. In fact almost the same was the situation in other traditions of olden times. It was Piccard who actually determined the sidereal year length by taking record of right ascensions of Sun at specific times with an interval of 76 sidereal years (A.D. 1699-1745).[55]

From what has been discussed, it is evident that the Hindus in fact never intentionally used sidereal year. In all experimental methods (described in Siddhāntic texts) one gets only the tropical year and never sidereal year. This way one can infer that it was error in the determination of solar year (of the order of 24 minutes in excess to the actual tropical year)[56] which was responsible for creating confusion of taking tropical year for the sidereal year. There is no doubt that this value got into use in Hindu traditions and now no almanac maker is ready to accept tropical year for religious Hindu calendar. It is also possible that they intended to use year in sidereal sense but it happened to be tropical year within the available means they adopted to measure the "year" in general sense.

Hypothesis of Trepidation of Equinoxes

Inspite of the fact that the year length in Indian tradition was intended to be tropical, the sidereal year got enforced into use unintentionally. The later astronomers started believing in the hypothesis of trepidation of equinoxes, according to which the spring equinox oscillates (trepidates) upto $\pm 27°$ on both sides of the fixed zero of zodiac. It takes about 27000 years to complete this cycle. This way the zero of the zodiac was thought to get restored twice in each cycle and thus the same was not changed instead the return was awaited, after it reached 27° from the Aśvinī *nakṣatra*.

It may be pointed out that Āryabhaṭa II and Parāśara believed in $\pm 24°$ amplitude of trepidation.[57] The rate of precession adopted by Āryabhaṭa II was 46″.5 P.A.[58] It may be remarked that neither Muñjāla nor Bhāskarācārya nor others in Hindu astronomical tradition, have categorically mentioned the oscillation of equinoxes. Only in the present recension of *Sūrya-siddhānta,* we find a statement[59] which gives the method of computing *ayanāṃśa* using throughout a constant velocity of 54″ P.A. In this method, 27° get subtracted adhoc whenever it goes in excess to this amplitude. This has no mathematical or physical validity (as, unless velocity is gradually reduced to zero, its direction cannot change).[60] Moreover, even in *Sūrya-siddhānta,* the relevant *ślokas* evidently appear to be added later on; as these use a different style of enumerating figures by actual numbers and not as usual (in all other parts) by symbols like *netra* (eyes) = 2, *candra* (Moon) = 1, etc. Also in the body of the text, there is no use of *ayanāṃśa* as such in computing *lagna* (ascendant) etc. There is just a general statement asking the use of *ayanāṃsa* for computing gnomonic shadows, ascensional differences etc., in that very *śloka,* where the formula for computing the same has been given.[61]

Even ancient records show that *ayanāṃśa* was greater than 27°. For example, at the time of *Śatapatha Brāhmaṇa* when Kṛttikās are said to rise in the east.[62] the *ayanāṃśa* comes out to be 45°. Thus it is not worthwhile to wait for *ayanāṃśa* to accumulate to a value greater than 27° in future, and keep on believing the hypothesis of trepidation of equinoxes. The law of gravitation has discarded this hypothesis and explained the phenomenon of full cycle of precession on the basis of the torque exerted by Sun and Moon on the spheroidal earth.

Brenand[63] and Vijñanānanda Swāmī[64] tried to explain the oscillatory motion of the *ayana* by taking projections of ecliptic, solstitial colure etc., in one plane, but such an argument may hold for any value of maximum amplitude of the angle of precession.[65] In our opinion the oscillations described in *Sūrya-siddhānta* may not mean more than the declinational north south oscillations of stars on the eastern horizon due to precession of equinoxes over the period of 27000 years. Thus the hypothesis of trepidation of equinoxes has no physical validity. On the other hand, the theory of precession of equinoxes and its full cycle, is the final verdict of the law of gravitation.

The Zero of Hindu Zodiac and Various Schools of Ayanamśa

As we have pointed out earlier, the zero of the zodiac got fixed at Aśvinī after *Sūryaprajñapti* (200 B.C.) when theoretical astronomy started. The various treatises took their own computed position of the Sun to be exact and defined the *ayanāṃśa* as the difference between its observed and computed positions. Thus the velocity of *ayana* was erroneous. The sidereal solar year adopted in *Sūrya-siddhānta* is in excess to the actual value (determined by Piccard by taking observations over a range of 76 years)[66] by about $8\frac{1}{2}$ *palas* (=3.6 minutes). Due to this error the analytically computed celestial sidereal longitude of the Sun will be erroneous and as a consequence, the sidereal zero of the zodiac is invariably expected to be changed if one adopts the siddhāntic definition of *ayanāṃśa* and its velocity. The velocity of precession according to *Sūrya-siddhānta* is 54″ per annum while according to *Graha-lāghava* it is 60″ per annum and other texts have somewhat different velocities. These velocities include also the errors due to inaccuracies in the measurement of solar year. As already pointed out, the solar year as determined by the methods described above, is tropical *(sāyana)*. In fact there is a difference of about 20 minutes between the *sāyana* and *nirayaṇa* years, (*nirayaṇa* year being in excess). The methods adopted for determining the length of year were erroneous to the extent that error of the order of 20 minutes masked the difference between the two solar years and in fact, because of no notion of the difference between the two, the tropical year so determined, with that much error, got adopted as sidereal year and it happened to be in excess (by about 3.6 minutes) to the actual sidereal year length. As a result of adopting this year length and Sun's longitude as eaxct, zero of the zodiac went on shifting from the actual position and about 3½ degrees got accumulated in about 2000 years (since the time of switch over from *Vedāṅga* traditional calendar of 5 year *yugas*, to the siddhāntic tradition of calendar making). In the last decades of 19th century A.D. traditional *pāñcāṅga*-makers were trying to fix the zero of Hindu zodiac by adopting the modern value for precessional velocity or by back calculating the position of spring equinox in conformity with the data of *Sūrya-siddhānta* or other standard treatises of Indian tradition. Since the polar longitudes of stars etc. given in *Sūrya-siddhānta* can not yield a consistent *ayanāṃśa*, the problem of *ayanāṃśa* remained a vexed question. There were many rival publications by Govinda Sada Siva Apte and Dina Natha Shastri Chulet and many others.[67] These conflicts gave rise to two different prominent schools *Raivata pakṣa* and *Citrā pakṣa* among the followers of the modern school (called the *Dṛk-pakṣa*, the school believing in observations). The third one being the *Surya-siddhānta* school which has erroneous rate of precession. According to the *Raivata pakṣa* the *ayanāṃśa* is about 3½ degrees less than the value in the *Citrā* school. If the zero of zodiac is defined as the position of spring equinox at the time of switch over from the Vedāṅga to siddhāntic tradition of *pañcāṅga*-making, the *Raivata pakṣa* has sound justification, because if year length had 8½ *palas* error, the same must have been accumulated. According to the exponents of *Citrā* school the zero is 180° away from 1st magnitude star Spica or α-Virginis *(Citr)*. In fact the authors of various treatises over the past 1500 years too, did try to fix the zero of Hindu zodiac from time to time. Zeros of the zodiac are different in various treatises as tabulated below.[69]

Siddhānta-text	Year of zero *ayanāṃśa*
Present *Sūrya-siddhānta*, *Vasiṣṭha-siddhānta* etc.	421 *Śaka*
Muñjāla's *Laghumānasa*	449 ,,
Rājamṛgāṅka Karaṇaprakāśa, Karaṇa Kutūhala*	445 ,,
Grahalāghava	444 ,,
Bhāsvatī-karaṇa	450 ,,
Second *Āryabhaṭa Siddhānta*	527 ,,
Bhaṭatulya (by Damodara)	342 ,,

*It is interesting to note that one of the manuscripts of *Rājamṛgāṅka* has no *ayanāṃśa* formula. It is guessed that *ayanāṃśa* was added after *graha-lāghava*, which indicates that *ayanāṃśa* took long time to get into common use.[69]

It is clear that there were attempts to fix the zero of the zodiac to dates before Āryabhaṭa.

Note that changes in zero of zodiac were done upto the time of *Grahalāghava* (16th century A.D.). Thus there were quite different years of zero *ayanāṃśa* as shown in the table above. Indian Calendar Reform Committee[70] under the Chairmanship of late Prof. M. N. Saha, analyzed the problem using polar longitudes of stars given in the *Sūrya-siddhānta* and concluded that the zero was being fixed again and again by different Jyotiṣācāryas in their own times. There exists difference upto over 3° or so in different treatises of astronomy, but certainly the zero of Hindu zodiac is about 2° removed from that of Ptolemy's. From this analysis it is clear that it is not the zero of the Vedāṅga zodiac which we have in modern *Sūrya-siddhānta*. Thus not going by any old text, the followers of *Citrā pakṣa* fixed the zero at that very point to which the equinox had shifted upto 1800 śaka or so (as used by V. B. Ketakara)[71] and adopted the accurate value of precessional velocity so that no more error is accumulated in future years. A fact has to be stated that previously V. B. Ketakara too published *Jyotirgaṇitam* using *Raivata pakṣiya ayanaṃśa*, but on the advice of Sankara Bala Krishna Dixit, he destroyed all copies of the same and rewrote and published the *Citrā pakṣiya Jyotirgaṇitam*. He even deleted a *śloka*[72] (on the advice of S. B. Dixit) in which he had clarified that it was the *Raivata Pakṣa* which was justified but because of its unacceptability in the social practices he started *Citrā Pakṣa* which would go parallel to *Sūrya-siddhānta* for a century or so and thus get adopted by that time. The Calendar Reform Committee has advised to fix the *ayanāṃsa* at 23°15′ (a value for Jan. 0,1955 A.D.) and started taking most accurate expressions for precessional velocity and also the nutation. The Committee proposed a *Sāyana Śaka* year beginning with 21st March (22nd March in case of leap year) but it could not be accepted by traditional *pañcāṅga* makers. The zero of zodiac in this system is moving with same velocity as that of precession but it will always be at a constant distance of 23°15′ from the position of spring equinox. For defining lunar months it will be a good plan if the constant precession 23°15′ is used without further change with the precessional velocity.[73] This way the lunar months will be defined with respect to *sāyana* longitude of the Sun minus 23°15′ at the ending moment of respective *amāvāsyās*. This way, the change in the limits of lunar months with respect to tropical calendar (say Gregorian Calendar) accumulated up to 1955 A.D., will not increase any more in years to come. Consequently the lunar months will get pegged on to *sāyana* solar year and this change may not be noted by ordinary people to result in disorders in social practices. This way the luni-solar religious calendar will conform to seasons at least to the extent it was in 1955 A.D. But this technique cannot be accepted unless it gets the sanction of Hindu religious authorities. Since the names of lunar months are conforming to the asterisms occupied by the Moon at the ending moments of full-moon syzygies and many festivals are related with asterisms, the *pañcāṅga* makers find it difficult to adopt the constant precession system for deciding religious rites. This is the reason why the proposals of the Calendar Reform Committee did not receive recognition and in near future there is no hope to switch over to *sāyana* system or constant precession system of calendar-making for Hindu religious rites.