5 POST-VEDIC ASTRONOMY

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Vedic astronomy has a long span starting from remote antiquity upto the advent of siddhāntic texts. Vedic astronomical lore can be seen in the earliest strata of the *Ṛgveda*. We find an ancient report on conjunction of Jupiter with δ Cancri (Tiṣya or Puṣya Yogatārā) in the *Taittitrīya Saṃhitā*,[1] *Tāṇḍya Brāhmaṇa* etc. which can be shown to belong to a naked eye observation around 4650±80 B.C. on the basis of slow motion of the node of Jupiter's node over δ Cancri region.[2] There are ancient observations of Moon recorded in the *Ṛgveda,* which paved the way to five-year yuga system in calendar making. Although such ancient records are available in oral traditional literature of the Vedic times, yet there was no systematic text compiled earlier than 1400 B.C.[3] or so, when Ṛṣi Lagadha compiled *Vedāṅga Jyotiṣa* (*V. J.*). In fact there is a big gap of about twot housand years between *V. J.* and the siddhāntic tradition of Āryabhaṭa (last decade of 5th century A.D.). After *V. J.* we find Jaina canonical literature *Sūrya-candra prajñapti, Jyotiṣkaraṇḍaka* and Buddhist literature *Śārdūlakaraṇāvadāna* etc. *Pañcasiddhāntikā,* a compendium by Varāhamihira of five astronomical treatises, some ancient and some of comparatively recent times. In addition to this we find also the qualitative studies of kinematics of planets Mars, Mercury, Jupiter, Venus and Saturn in *Bṛhat-saṃhitā* and *Bhadra-bāhu Saṃhitā* etc.[4] These have also reports on cometary kinematics and include old analytic records on studies of meteors.

PLANETARY KINEMATICS

The planetary kinematics reported in *Bṛhat-saṃhitā, Bhadrabāhu Saṃhitā* and other saṃhitā texts are very primitive and qualitative and seem to be quite old. It is often commented that *V. J.* had only studies of kinematics of Sun and Moon before contacts with Greeks, but these reports on planetary kinematical studies indicate a good deal of attempt on studies of planetary velocities, retrogradations, their heliocentric rising and setting and conjunctions with stars etc.

Here we have tabulated (Table 5.1) the categorizations of retrograde motions of Mars. The study of motion was started whenever the planet became first visible after combustion. The number of asterisms between the point of first visibility and the point of retrogradation was reported and various names were assigned to Mars accordingly. For example, if it is retrograded in the 7th, 8th or 9th asterism after first visibility it was named "*Uṣṇa-Mukha*" according to *Bṛhat-saṃhitā* of Varāhamihira and Bhadrabāhu and "*Vakra-Mukha*" according to Vṛddha Vasiṣṭha. These names, though they have astrological prognostications, deserve critical analysis of the qualitative kinematical data.

Table 5.1

Categorization of Retrograde motions of Mars

Number of asterisms where it retrogrades (with respect to the asterism where it becomes first visible after combustion)	Varāhamihira & Bhadrabāhu	Vṛddha Vasiṣṭha
(1) In 7th, 8th & 9th asterisms	*Uṣṇa-Mukha*	*Vakra-Mukha*
(2) In 10th, 11th and 12th asterisms	*Soṣa-Mukha*	*Asru-Mukha*
(3) In 13th & 14th asterisms	*Vyāla-Mukha*	*Vyāla-Mukha*
(4) In 15th & 16th asterisms	*Lohita-Vakra*	*Raktānana*
(5) In 17th & 18th asterisms	*Loha-Mudgara*	*Musala*

Here as an example of study of motion of Mercury, we have tabulated three different categorizations of its velocities according to Parāśara Devala and Bhadrabāhu (Tables 5.2, 5.3 and 5.4) which seem to be gradual improvements in study of the motion of Mercury. The velocity was given names like *Ṛjvi, Ativakrā* etc. (see table 5.2) *Prākṛta, Vibhinnā,* etc. (see table 5.3) depending upon the number of days this planet took for its combustion. Its *Ṛjvi* (direct) velocity is for 30 days, *Vakrā* and *Vikalā* (stationary), velocity was noted to be for 6 days. The categorizations found in *Bhadrabāhu Saṃhitā* seem to be much improved (Table 5.4). All these qualitative data deserve critical mathematical analysis.

In the case of Jupiter too we find the study of its direct and retrograde motions. The motion was studied between its consecutive heliocentric risings. These studies yield sidereal time period of Jupiter$\simeq$12 years and on combining this period with Five Year Yuga, a 60-year cycle was designed. These consecutive 60 years are given different names and have much importance in Hindu religious calendar.

Table 5.2

Categorization of Mercury's velocities according to Devala

Type of velocity	No. of days of combustion or visibility
1. *Ṛjvi*	30 days
2. *Ativakrā*	24 days
3. *Vakrā*	12 days
4. *Vikalā*	6 days

Table 5.3

Categorization of Mercury's velocities according to Parāśara

Type of velocity	No. of days of combustion or visibility
1. *Prakṛtā*	40 days
2. *Vibhinnā*	30 days
3. *Saṃkṣiptā*	22 days
4. *Tīkṣṇā*	18 days
5. *Yogāntikā*	9 days
6. *Ghorā*	15 days
7. *Pāpā*	9 days

Table 5.4

Categorization of Mercury's velocities according to Bhadrabāhu

1. *Saumyā*	45 days
2. *Vimiśrā*	30 days
3. *Saṃkṣiptā*	24 days
4. *Tīkṣṇā*	10 days
5. *Ghorā*	6 days
6. *Pāpā*	3 days
7. *Durgā*	9 days

In the case of Venus the kinematical studies are very interesting. In Saṃhitās and in Jaina literature we find the study of motion based on estimates of its average velocities during heliacal combustion in different parts of the lunar zodiac.[5] During combustion Venus moves in lanes *(vīthis)* among stars. There are 9 lanes in all which are defined by the number of days it is heliacally invisible during inferior and superior conjunctions (Table 5.5). The lanes have definite *nakṣatras* starting with Aśvinī, Bharaṇī and Kṛttikā in Nāga[6] *vīthi* (not listed in table 5.5). The table also has zodiacal stretches listed for the case of inferior conjunction in units of *muhūrtas* of arc (1 *muhūrta*=the angular distance travelled by Moon in 48 minutes, and $819\frac{27}{67}$ *muhūrtas*=360°).

Table 5.5

Vīthi	Number of days for which Venus remains heliacally invisible		Zodiacal stretch
	Inferior conjunction	Superior conjunction	Muhūrtas of arc (Inferior conjunction)
1. *Vaiśvānara* (fire)	24	86	$84\frac{27}{67}$
2. *Mṛga* (deer)	22	84	75
3. *Aja* (goat)	20	86	120
4. *Jaradgava* (old bull)	17	75	105
5. *Go* (cow)	14	70	90
6. *Vṛṣa* (bull)	12	65	90
7. *Airāvata* (chief elephant)	10	60	75
8. *Gaja* (elephant)	8	85	105
9. *Nāga* (snake)	6	55	75

The names of *vīthis* indicate qualitative nature of the speed estimates. Still it can be shown that the perigee of Venus's orbit lies in *Vaiśvānara vīthi.*

In the case of Saturn the time period was estimated to be 30 years (approximately). All these studies on planetary kinematics, being qualitative, indicate their pre-siddhāntic chronology.

There are also studies on cometary statistics in Saṃhitā texts. It is undoubtedly true[7] that Indian astronomers believed in the periodicity of comets long before Edmund Halley claimed it for the comet observed in 17th century A.D. and known after him. Bhaṭṭotpala (A.D. 937) in his commentary on *Bṛhatsaṃhitā (Ketu-carādhyāya)*, gives a list of comets with their names after the names of Ṛṣis who studied their motions and recognized their reappearance in their lifetimes probably using previous records. Although *Bṛhatsaṃhitā* starts its chapter on comets with a general statement that the cometary motions cannot be computed, yet it lists definite loci of some comets in the lunar zodiac. Similarly other Saṃhitā texts and literature too give definite orbits of some comets and it is contemplated that these records have reports on old apparitions of Halley's comet. T. Kiang of China has decoded 29 apparitions of Halley's comet before 17th century A.D. in Chinese tradition (*Memoir of Royal Astronomical Society of England,* 1976), J. Brady of California has tried to decode still older records in European tradition, but the records before 240 B.C. are not much reliable. Indian records too can be decoded for such old reports on apparitions of Halley's comet,

JAINA ASTRONOMICAL TRADITION

The Jaina canonical text *Sūrya-prajñapti, Jyotiṣkaraṇḍaka,* etc. have records of pre-siddhāntic post-Vedic astronomical traditions. Although these records were compiled in the form of these texts quite late, there is no doubt that the observational records presented in *Sūrya-prajñapti* etc. belong to 2nd century B.C.[8] or even to an earlier period. We call the period between the *Vedāṅga Jyotiṣa* and siddhāntic astronomy to be the dark period, as these texts indicate no further advancement in Indian astronomical tradition partly, due to the reason that texts like *Sūrya-prajñapti* of this period are not well understood. These are undecoded due to the fact that the old tradition of ancient pre-siddhāntic astronomy was forgotten with the advent of siddhāntic astronomical schools.

Even Brahmagupta[9] and also Bhāskarācārya criticized the double counter Sun hypothesis of the Jainas. It may be remarked that now the paradox of two Suns is resolved and it has been shown that the relevant *gāthās* in *Sūrya-prajñapti* in fact belong to the daily astronomical observations of the Sun at the time of rising and setting.[10] These observations were meant for the experimental determination of the solar year. This post-Vedic tradition of Indian astronomy is very important and it has been decoded that the confusion regarding existence of two Suns resulted because the word *ardhamaṇḍala* for half of the diurnal path of the Sun got interpreted to mean the cutting of a *maṇḍala* (diurnal path) in two halves perpendicular to its surface. This is the traditional interpretation by Malayagiri and others (Fig. 5.1). Even Malayagiri[11]

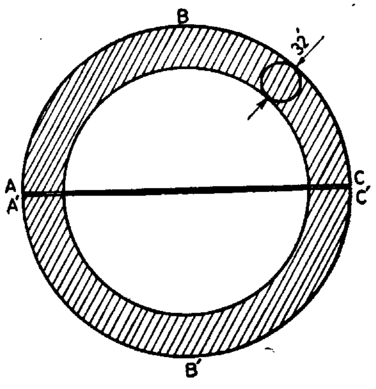

Fig. 5.1. The maṇḍala is the disc of sun, but this word now stands for the locus generated by the sun in diurnal motion; the word Ardha-maṇḍala (half of a maṇḍala) was taken to be half section as shown above.

accepts the inadequacy of his interpretation this way and begs pardon if that proves wrong in future. Now it has been shown on the basis of mathematical details and the linguistic approach that the word *ardhamaṇḍala* means a cross section of wheel like structure of diurnal path *(maṇḍala)* into two halves of half the width each, without distorting its circular structure as shown in Fig. 5.2. *Sūrya-prajñapti* states that the

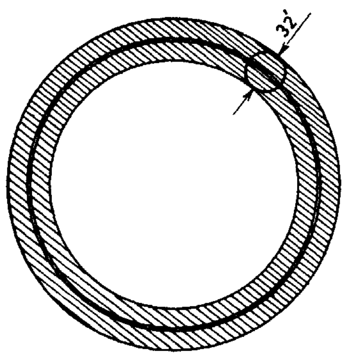

Fig. 5.2. In our interpretation the word *ardha-maṇḍala* a stands for the cross-section of this wheel like path. This *ardha-maṇḍala* is traversed by the sun in one day on an average or near equater. Radius of the disc being 16′.

Sun (on an average) traverses half of the *maṇḍala* (i.e. half of its own disc and consequently the wheel like locus generated in diurnal motion) every day, which is true taking into consideration the mean declinational *uttarāyaṇa* and *dakṣiṇāyana* (north south) motion of the Sun. There are shown plots of daily observations of the Sun in the morning and evening, which created confusion for two Suns. On the basis of these experiments, the solar year was determined to be of $365\frac{1}{8}$ days[12] which is found also reported in *Paitāmaha Siddhānta* in *Pañca siddhāntikā* of Varāhamihira. This length of solar year is not found used as there were not really a break-through in theoretical advancements in the dark period.

The Early Traditions of Siddhāntas

After post-vedic pre-siddhāntic developments, we find five schools as compiled by Varāhamihira in his *Pañcasiddhāntikā*. Out of the five siddhāntas the *Paitāmaha-siddhānta* is the earliest of all and belongs to the epoch of 11th Jan. 80 A.D.[12] Its elements have direct connection with *Vedāṅga-jyotiṣa* of Lagadha. The epoch of *Vasiṣṭha-siddhānta* is 3rd Dec. 499 A.D. although there existed another *Vasiṣṭha-siddhānta* at the time of Sphujidhvaja (around A.D. 70). The *Vasiṣṭha-siddhāntā* of *Pañcasiddhāntikā* belongs to a later version of the same under the name *Vasiṣṭha-samāsa-siddhānta*. This was an abridged version of the same as is evident from its name.

The *Romaka-siddhānta* was commented upon by Lāṭadeva. The epoch adopted in this text is the Sun-set at Yavanapura (Alexandria) on 21st March A.D. 505. Its elements like the use of sun-set epoch, tropical year, Hipparchus' Metonic cycle, computational methods etc. indicate without any doubt its Hellenistic origin.

The *Pauliśa-siddhānta* has methodology of Babylonian and Greek astronomy with strong influence of Indian traditional concepts. There seems to be no way to sort out the original *Pauliśa-siddhānta*. The *Sūrya-siddhānta* of *Pañca-siddhāntikā* was compiled by Lāṭadeva. He gave rules for computing *ahargaṇa* (number of days from an epoch) and methods of calculating solar eclipse etc. The system adopted here is the *ardharātrika* (mid-night system) of time reckoning and the epoch of this siddhānta is the midnight of Avanti 20/21st March 505 A.D. There is evidence of an earlier *Sūrya-siddhānta* which used somewhat different parameters for mean motions of the Moon, Rāhu (ascending node of the lunar orbit) and *candrocca* (lunar apogee)[13] etc. There is no doubt that the *Sūrya-siddhānta* was considered to be the most accurate. Its parameters were corrected and additions made in later centuries. The final version which we have today is much different from its version in *Pañca-siddhāntikā*. In fact, this belongs to the 9th or 10th century A.D. or even to a later date as is evident if one plots the errors in mean positions of planets against years over centuries.[14] A caution may be given in conclusions based on such methods as adopted by us and also in the ones adopted by David Pingree and others that it were the true positions and not mean ones which were observed and mean positions were fitted as *dhruvakas* (constant mean positions for the epoch) using the inequalities and the allied formulae adopted in those texts. Thus the mean positions computed by using modern accurate figures, even if these tally with the reports, may not prove the hypothesis concerning the epoch of the text. One should compute the true position ephemeride according to the texts in question, over centuries and also compute the ephemerides according to the most accurate data and inequalities of modern theories to get the true positions observable at those times. Wherever the two ephemerides give the best fit data observable, can be taken to be the epoch. Here allowance is to be made for observational errors or personal equations keeping in view the old observations to be the unaided eye records. Sometimes, a least square fit in data may also be necessary to eliminate the effects due to errors, and allowances for instrumental errors and the conditions under which those old observations were recorded are to be made.[15] Thus the epochs claimed on the basis of analytical computations using modern adopted figures only can not be taken with a grain of salt unless categorically mentioned in the text itself. Thus some of the claims, are to be rechecked and do not stand as a final verdict. Whatever the dates of these siddhāntas may be, these are earlier than 6th century A.D. or so (i.e. before Āryabhaṭa). Some of these may even go back to earlier dates.

Three Important Schools of Siddhāntic Astronomy

After the advent of siddhāntic astronomy (as evidenced from *Pañca-siddhāntikā*) three important siddhāntas (sometimes named as tantras) came into being over a period of about 500 years or so. These got into use in calendar making and in predicting astronomical phenomena throughout the country. These are *Āryabhaṭa-siddhānta* (due to Āryabhaṭa (A.D. 499)), *Brahma-siddhānta* (due to Brahmagupta), *Sūrya-siddhānta* (the latest version attributed to Asura Maya). These schools prevailed over centuries and are surviving even now in *Pañcāṅga* making (in preparation of religious Hindu calendars) in the country. There were teacher precept traditions in these schools which developed the formulations of the respective theoretical disciplines and went on advancing in theory keeping intact the basic constants and concepts of their respective schools. Many theoretical texts evolved in these traditions. These were called 'siddhāntas' (theoretical treatises) while the texts giving only the simple algorithms (without proofs) for computing planetary positions, eclipses, cusps of the Moon, heliocentric rising and setting and other astronomical phenomena were called *Karaṇas* (means of practical work out of ephemerides and calendars). These were based on one or the other of the three schools of astronomy. It may be remarked that the school of *Sūrya-siddhānta* became the most widespread and is thought to be the most accurate of the three as it was improved and additions were made upto 10th century A.D. or even later. Earlier it was a Sāyana school (with spring equinox as zero longitude), but when the tropical *saṃkrāntis* (transits of Sun in tropical signs) showed much deviation with respect to the fixed zero of ecliptic, it accommodated the *ayanāṃśa* (angle of precession of equinox) given by Muñjāla. This siddhānta is even now used in northern, eastern and western parts of the country and also got accepted in neighbouring countries like Nepal, Burma, Tibet, Bhutan, Ceylon and also in far off countries, where Hindus and Buddhists made their way and settled. *Brahma-siddhānta* is used in some parts of Rajasthan and Madhya Pradesh and survives even now in somewhat reformed version due to an astronomer Caṇḍu (about 500 years ago) who prepared good tables for computing calendars. *Āryabhaṭa-siddhānta,* the oldest of the three is still in use in southern parts of India (in *Vākya karaṇas* which are the *karaṇas* based on simple algorithmic sentences *(vākyas)* making use of constants of *Āryabhaṭīya* tradition). It may be remarked that in later centuries, there evolved some other schools too with some variations in formulations, *bīja saṃskāras* etc., like Bhāskarācārya's etc. These had their own teacher-disciple tradition *(guru-śiṣya-paramparā)* over centuries, and got into use in calendar making and in computing astronomical phenomena, heliocentric rising, eclipses etc., but in general only the three schools prevailed. Here we discuss the siddhāntic texts of these three schools one by one in brief.

Āryabhaṭa-siddhānta

Āryabhaṭa prepared his epoch-making treatise *Āryabhaṭīya* (A.D. 499). There was another Āryabhaṭa II who wrote *Ārya-siddhānta.* Sometimes the *Āryabhaṭīya* is referred to as the first *Ārya-siddhānta.* It has mainly two parts[16] *Daśagītikā* and *Āryāṣṭaśata.* The first one has 10 *ślokas* in *Gīti* meter and deals with system of depicting numbers by symbols used for brevity in the body of the text and for the constants like *bhaganas* (number of revolutions of planets in a *kalpa*) etc. The length of solar year adopted in this text is 365 days 15 *ghaṭīs* 31 *palas* 15 *vipalas.* The number of *yugas* in a *kalpa* (Brahma's day) is 72 and not 71 as in other treatises. There are no *sandhis* after every beginning and end of a *manvantara* in this treatise.

The part after *Daśagītikā* consists of *Gaṇita-pāda, Kālakriyā-pāda* and *Gola-pāda.* The first one deals with pure mathematical formulae, on squares, cubes, roots, circles, algebra and indeterminate equations. The *Kālakriyā-pāda* discusses the units of time, conjunction of planets, *vyatīpāta* (equality of declinations), anomalistic and synodic periods, Jovian era, intercalary months, lord of day, mean position and equation of centre etc. *Gola-pāda* discusses armillary sphere, position of ecliptic, position of living beings on the earth, increase and decrease in the size of the earth in a *kalpa,* rotation of earth, vertical circle, rising of signs, eclipses etc.

This text is very brief. It does not discuss many topics as done in later treatises. For example, it is lacking in methods of computing cusps of the Moon, conjunction of planets, *tithis, nakṣatras, yogas* and *karaṇas* and does not give latitudes and longitudes of stars. There are no algorithms for computing heliocentric rising and setting of planets and conjunctions etc.

It is interesting to note that this treatise has given the notion of daily rotation of earth. Later astronomers like Brahmagupta and others have criticized this notion. Brahmagupta argues, how the birds can come home after full day flying in the skies? In Fact, Āryabhaṭa was ahead of his times in giving this idea of rotation of the earth. The earlier and later astronomers lingered on to the old concept of *Pravaha vāyu* (air) thought to be responsible for rotation of heavens once in a day.

Brāhma-sphuṭa-siddhānta

It was written by Brahmagupta (A.D. 628). The text[17] deals with mean and true positions of planets, problems of direction, time and space, lunar and solar eclipses, heliocentric rising and setting of planets, cusps of Moon, shadow of Moon, conjunctions of planets and stars, criticisms of other tantras (siddhāntas), arithmetic problems on mean and true positions, indeterminate equations, algebraic equations, gnomonics, permutation and combination in meters, celestial sphere, instruments and some algorithms for fast computations in *Dhyānagrahopadeśādhyāya.*[18]

In fact, the section on constructing meters *(chandas)* using permutation and combination is the obscure part of the text. It is not decodable because of grammatical mistakes and in copying over centuries, and also partly due to the inconsistencies in mathematical formulations arising because of these mistakes. It may be possible to decode it if one tries all possibilities of correcting versions grammatically and at the same time checking the versions thus corrected for mathematical consistencies. In al-Bīrūnī's *India* there is discussion on Brahmagupta's work on meters. The subject was not clear even to al-Bīrūnī at that time.

It may be noted that Brahmagupta did not believe in the precession of the equinoxes and criticized Viṣṇucandra who advocated the theory of precessional motion. So the *Brāhma-sphuṭa-Siddhānta* has tropical longitudes. In fact, at that time the angle of precession was quite small.

Brahmagupta wrote another treatise *Khaṇḍa-khādyaka.* In its first part, he has given constants like those in the *Āryabhaṭīya* and in the second part he has given corrections to improve upon the results from part I (which are just those of Āryabhaṭa) to make them tally with observations. This had to be done by Brahmagupta because *Ārya-siddhānta* had much popularity at that time and the scholars could not dispense with the methods of this work at least in Brahmagupta's time. Brahmagupta was a great critic. He criticized Āryabhaṭa's works on various astronomical topics especially on the computation of parallax for solar eclipses etc. Brahmagupta's work got much popularity and appreciation by the time of

Bhāskarācārya and even earlier. Al-Bīrūnī and Bhāskara held Brahmagupta in high esteem. There were written two *karaṇagranthas* based on *Brāhma-sphuṭa-siddhānta* upto Bhāskarācārya (A.D. 1150). In the 15th century Caṇḍu (as astronomer) prepared tables on the basis of this siddhānta which are still used in Rajasthan and Madhya Pradesh by some traditional *Pañcāṅga*-makers.

Sūrya-siddhānta

The third important school is that of *Sūrya-siddhānta.* The author is Mayāsura. As already pointed out it is much different from the *Sūrya-siddhānta* of *Pañca-siddhāntikā.* It has 14 chapters and deals with mean and true positions (on the basis of epicyclic theory) and problems of space, time and direction, lunar and solar eclipses, diagrammatical representation of eclipse phenomenon, conjunctions of planets and stars, polar longitudes and latitudes of stars, cusps of Moon, heliocentric rising and setting, instruments, geography, celestial sphere etc. The year-length adopted is 365 days 15 *ghaṭīs* 31 *palas* and 30 *vipalas.* It allows only one equation of centre each for Sun and Moon and two equations of centre for other planets as in other treatises.

The text has reference to *ayanāṃśa* (angle of precession) while *Ārya-siddhānta* and *Brahma-siddhānta* did not use this at all. It may be remarked that in fact the relevant algorithms for computing *ayanāṃśa* were added to this text later. We have discussed this point in the section on *ayanāṃśa.* Muñjāla gave *ayanāṃśa* for the first time when it was about over $6\frac{1}{2}°$ or so. (Bhāskarācārya clarified this point that the same was given by Muñjāla). It was not at all noticed by scholars of the calibre of Brahmagupta and Āryabhaṭa, being small in their times. In fact before the introduction of *ayanāṃśa*, there might have been chaos in deciding the dates of *saṃkrāntis* (transits of Sun). Whatever be the situation at that time, the introduction of *ayanāṃśa* in *Sūrya-siddhānta* on the basis of Muñjāla's notion as expounded in *Laghumānasa,* proved to be a big shelter for the whole edifice of *nirayana* system of solar year reckoning and the astrology based on this system. Even though *ayanāṃśa* came into use, the year length was taken to be the same without any distinction between sidereal and tropical years over many centuries as the theory of trepidation of equinoxes got accepted in the algorithms for computing *ayanāṃśa* as given in *Sūrya-siddhānta.* This point is discussed in the section on *ayanāṃśa.*

This school got much popularity. Many *karaṇas* were written on the basis of this treatise, like *Grahalāghava. Makaranda-sāraṇī* etc., which were used for computing *pañcāṅgas* and astronomical phenomena over many centuries in all parts of India.

Besides these three siddhāntas, there were other treatises too, like *Mahābhāskarīya* of Bhāskara I (7th A.D.), Lalla's *Śiṣya-dhī-vṛddhida tantra*[19], Munjāla's *Laghumānasa* (A.D. 10th), *Siddhānta-śiromaṇi* of Bhāskarācārya II (A.D. 1150), *Siddhānta-sārvabhauma* of Munīśvara (early 17th century A.D.), *Siddhānta-tattva-viveka* of Kamalākara-Bhaṭṭa (A.D. 1656), *Siddhānta-darpaṇa* of Candrasekhara Samanta (19th century A.D.) of Orissa, etc.

Muñjāla's treatise is known for introducing *ayanāṃśa* in calendaric computations. This treatise *(Laghumānasa)* is also famous for giving an evection-like term in lunar theory which is a hybrid of evection and the first equation of centre. The amplitude of the evection term as given by him is quite correct. The texts like Munīśvara's *Siddhānta-sārvabhauma* and *Siddhānta-tattva-viveka* of Kamalākara too are based on *Sūrya-siddhānta*, but have their own advancements in the methods of computations and in developments of better formulae in various aspects of astronomical phenomena. The *Siddhānta-śiromaṇi* of Bhāskara II has unique features in the theory of indeterminate equations in advancing *kuṭṭaka-* (pulverizer) formulations, and also in 2nd degree indeterminate equations, especially in giving the advanced *cakravāla* -technique (1st given by Jayadeva). The theories of indeterminate equations were used by Bhāskarācārya in astronomical problems like in repetition of certain configurations of planets etc. Bhāskarācārya gave the detailed ideas about laws of gravitation of earth in his *Siddhānta-śiromaṇi*. He wrote a *karaṇagrantha* also under the name *Karaṇa-kutūhala*. He was also making constant efforts to improve the accuracy in the prediction of longitude of Moon and after having done daily observations of Moon over a long period, he wrote *Bījopanaya* (empirical corrections) giving sinusoidal empirical corrections to the longitude of Moon. The relevant formulae are just additive and subtractive constants varying with specific arguments. The arguments are these days realized on the basis of perturbation theory, but due to intermixing of many sinusoidal functions and lack of Fourier-like analysis his attempts did not result in clear cut identification of fortnightly variation, annual variation and evection functions. In fact, Bhāskara missed evection, as he observed Moon in specific positions where the same was zero, and he did not use the same on the mere authority of Muñjāla. This resulted in the failure of the analysis.[20] Candrasekhara Samanta of Orissa (19th A.D.) was an orthodox scholar of Indian astronomy. Samanta was not at all acquainted with the developments in the west. He devised some instruments and determined lunar inequalities independently. He gave annual variation for the longitude of Moon.[21]

In the seventeenth century A.D. Paṇḍita Jagannātha Samrāṭ under the patronage of Jai Singh Sawai wrote down *Samrāṭ Siddhānta* and translated the *Almagest* of Ptolemy in Sanskrit under the name *Siddhānta Samrāṭ*[22]. It has 13 chapters and 140 sections with 196 diagrams. Jai Singh Sawai with the help of Pt. Jagannātha could erect five observatories. It may be remarked that in the works of Jai Singh Sawai and Paṇḍita Jagannātha the lunar theory advanced more eccentric corrections and better constants for amplitudes were formulated. Jai Singh Sawai's tables *(zīj)* for computations of Sun, Moon, planets and *pañcāṅgas* deserve special attention. The tables like *Vedhopayogi Sāraṇis* (tables yielding observable positions for Moon etc.) are preserved in the library of his observatory in Jaipur. On analysis, the tables can furnish lot of information about developments in lunar theory and Sun's equations of centre, etc.

KARAṆAGRANTHAS AND SĀRAṆIS

Siddhāntic texts are theoretical treatises which give formulae with their proofs or explanations for computing ephemerides, *pañcāṅgas* and astronomical phenomena.

In these computations they use very big numbers as their epoch is usually the day of beginning of creation (according to the treatise) or Brahmā's day. The number of days elapsed since the date of creation is a very large figure (much larger than the Julian day number). The velocities of planets, nodes and apogees etc. too are given over a *Kalpa* (Brahmā's day, usually 432×10^7 years). Thus, it is clear that the figures involved in computations, using siddhāntic formulae, are very big and this renders the calculations very much cumbersome. The followers of the siddhāntic schools, although adhering to the basic constants of the treatise of their respective schools, wanted to simplify the computations and moreover, for actual operations, they needed ready-made formulae to fasten their calculations. This need gave rise to *karaṇa-granthas* which were intended to facilitate the calculations. Usually these have the following salient features :

(1) Unlike siddhāntic texts, these use *laghu ahargaṇa* (smaller number of days) from the epoch of compilation of the *karaṇa*. For the beginning date of the epoch, they provide the mean positions of planets and orbital elements and mean *tithis, nakṣatras* etc. (called *dhruvakas*) in tabulated forms. For computing the functions on any later (or earlier) day, the velocities of functions (planetary velocities in computation of their longitudes) are given. These are called *kṣepakas*. Usually these are given over a convenient cycle of 18 years or 19 years etc., or even over one year in which case the yearly *dhruvakas* are computed on the beginning day of every year. The *Grahalāghava* uses 14 years cycle with 444 śaka era epoch. For computing every day planetary ephemerides, *tithis, nakṣatras* etc. the daily speeds of the functions are used. For example, in computing daily *pañcāṅga* elements, the daily mean velocities of *tithis, nakṣatras* etc. are used and then corrections due to equations of centre are applied. The Makaranda tables use such techniques for computing *pañcāṅga* elements, making use of mean motions of Sun and Moon and the equations of centres of the *Sūrya-siddhānta* school.

(2) In order to provide convenient formulae in algorithmic form without going into the details of the proofs, etc. these use approximations in reducing formulae to simplest possible form without much loss of accuracy in approximations. For example, a bigger fraction may be reduced to simple form using continued fractions and terminating them at an appropriate stage. Sometimes, these are reduced to partial fractions to provide simple fractions with single denominators and fractions thereof, as additives or subtractives. Such techniques are used by Gaṇeśa Daivajña in his *Grahalāghava*. Sometimes even the sine functions are dispensed with under some approximations. *Grahalāghava* stands at the top in such treatments. Surprisingly enough its author did not use trigonometric functions as such, but gave the inequalities as additives or subtractives, after every (specific convenient) intervals for which simple ratio proportional interpolation is possible.

(3) Sometimes, *karaṇas* use empirical corrections in mean functions in order to have their results tallying with observations (no doubt the basic constants of the siddhāntic texts were retained as such). *Rājamṛgāṅka, Makaranda* and *Grahalāghava* applied empirical corrections in positions of planets but in none of the *karaṇas* any

empirical correction was applied to the longitude of the Sun. It is only the *karaṇas* of the 20th century like *Mārāṭhi grahagaṇitam* etc. by V. B. Ketakara which used corrected mean elements for Sun.

(4) Siddhāntic texts usually do not provide tables for computations (except some trigonometric functions etc. tabulated in some cases). On the contrary, *karaṇas* or *sāraṇis* try to provide necessary tables for quick computations. The *sāraṇis* have required elements tabulated against respective arguments. All the theoretical computations are already done in preparing the tables and for the user, only some simple arithmetic is left to get the final results.

Some Karaṇagranthas Based on Three Principal Siddhāntas

The Vedic and Vedāṅga astronomy made use of five-year *yuga* system. Even in the *Pañca-siddhāntikā,* we find *yugas* of small spans only. Moreover, the algorithms used in pre-siddhāntic astronomy were very simple. So no need was felt for preparing *karaṇa-granthas* in those times. It is only after the advent of siddhāntic astronomy by Āryabhaṭa that need was felt to prepare *karaṇa-granthas* in order to avoid big figures, arising from the use of *mahāyuga, kalpa* etc. in *ahargaṇa.* We do not have records of early *karaṇa-granthas.* But in the case of the *Brāhmasphuṭa-siddhānta,* we find two *karaṇas*[23] even before Bhāskarācārya. One of them used no empirical corrections in the results from *Brahma-siddhānta,* while second one, Rājamṛgāṅka by Bhoja, used *bīja* corrections in order to rectify the errors in mean positions accumulated over centuries. As mentioned earlier, astronomer Caṇḍu (16th century A.D.) prepared *sāraṇis* which are still being used by some *pañcāṅga* makers. Bhāskarācārya wrote a *karaṇa* text *Karaṇa-kutūhala* based on the constants of his treatise and wrote a separate booklet on sinusoidal empirical corrections in order to rectify the position of Moon. It may be remarked that the empirical correction (except in mean positions in some cases) were not approved for siddhāntas because in these theoretical texts only the corrections which had mathematical justification were allowed. So, in general, empirical corrections (specially sinusoidally varying *bīja* corrections) were not used in *siddhāntas.* The *karaṇas* used the corrections as temporary improvements for getting results tallying with observations. There are available *karaṇas* based on *Ārya-siddhānta* too, like *Vākya-karaṇa* by Sundara Rāja[24] of Southern India. There is another *karaṇa-grantha, Cāru-Candra Vākyāni* which is used for getting longitude of Moon. These *karaṇas* are based on *Āryabhaṭa-siddhānta* and give simple sentences which help in fast computations. The simple algorithms are put in simple sentences (*vākyas,* that is why the name *vākya-karaṇas*) which are very easy to remember. Sometimes, the sentences are very interesting having two meanings—astronomical and cultural.[25] It may be remarked that these texts have corrections to the position of Moon which are hybrids of lunar inequalities.[26] These *vākya-karaṇas* are used in preparing *pañcāṅga* in southern part of India.

There are many *karaṇa-granthas* based on the *Sūrya-siddhānta.* The *Graha-lāghava* of Gaṇeśa Daivajña (1522 A.D.) is famous among all these because of its simple algorithms and much simplified versions of complicated formulae. As already pointed out it has avoided the use of trigonometric functions as such and provided the elements at certain intervals of their arguments, which work quite well yielding satisfactory results. Gaṇeśa Daivajña himself has cautioned that in future his algorithms might give wrong results due to accumulation of errors with the lapse of time; in that case corrections should be applied after having verified the results with the help of instruments and thereby changing the arguments of the functions in accordance with observations. There are tables based on the *Sūrya-siddhānta* too. Makaranda's tables (A.D. 1478) are used for computing *pañcāṅga* elements and were popular for over 3 centuries or so. These tables too have applied *bija* corrections in planetary positions, except in the case of the Sun. Besides these there are many other *karaṇa-granthas* based on *Sūrya-siddhānta*. The earliest one is *Karaṇatilaka* of Vijayanandi.[27] That of Babilal Kochanna (A.D. 1298), *Bhaṭatulya karaṇa* (A.D. 1417), *Sūryatulya karaṇa* (A.D. 1523), *Grahakautuka-karaṇa* (A.D. 1496), *Bhāsvati-karaṇa* (A.D. 1520) and many more were written in later centuries.

This tradition of preparing *karaṇa-granthas* and tables was upheld by Kero Lakṣmaṇa Chhatre (19th century A.D.) and later astronomers of Indian tradition. Chhatre prepared tables for computing planetary positions using modern data. V. B. Ketakara prepared *Jyotirgaṇitam* (1898 A.D.), an epoch-making *karaṇa* type work with many tables provided for easy computations and used modern formulae based on gravitational perturbation theory. In the last half of 19th century a number of people rectified *pañcāṅga* elements by applying eccentric inequalities in case of Sun and Moon. Ketakara also wrote *Mārāṭhī grahagaṇitam* which provided many tables for calculations of *pañcāṅga* elements and planetary positions. Also there is *Ketakī grahagaṇitam* by V. B. Ketakara written in the style of *Graha-lāghava*.

In recent years too, there are *karaṇagranthas* produced by some scholars, e.g. *Karaṇa-kalpalatā* by K. L. Daftari (1976) and *Sarvānanda-lāghavam* by G. S. Apte. G. S. Apte wrote also another *karaṇa Sarvānanda karaṇa* again in the style of *Grahalāghava,* just improving the latter to get results tallying with observation. This text adopted *Sāyana* system of planetary longitudes. There are also tables prepared by various almanac makers like *Vṛhat-siddha-kheṭī* by Raja Ram Sharma. Besides these we find scattered materials like tables of *nakṣatras* of unequal spans as stated by Bhāskara. Also there are tables for computing *lagnas* (ascendants) etc. There is another text *Grahamālā* which can yield rough planetary positions and their retrogradations rising and setting etc. for any past or future years. These are ready reckoner tables based on bigger cycles of planets with respect to the Sun. Thus a prodigious amount of literature in various aspects of astronomy in Indian tradition was produced even in the present century.