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JEAN LE ROND D'ALEMBERT 1717-1783

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INTRODUCTION

The 17th century was described by A. N. Whitehead as the century whose genius nourished the modern mind with a wealth of ideas. More realistically than Pascal who suggested that human history would have meandered along a different course had Cleopatra's nose been a trifle shorter[1], Whitehead pointed out that the visage of Western civilization would have been altogether different if Galileo and Newton had not been born. Succeeding generations "have been living upon the accumulated capital of ideas provided for them by the genius of the seventeenth century"[2], he wrote. A crucial expression of that genius lay in the discovery that a quantitative and mathematical description of physical processes is perhaps the most fruitful way of interpreting the world.

The medieval philosopher Roger Bacon had already described mathematics as "the alphabet of all philosophy'[3], Others before and after him had expressed similar insights. But it was only in the course of the 17th century that the alphabet came to be used effectively. More than another century had to pass before Fourier's "mathematical poem"[4] could be composed. It is a long way from a, b, c to the masterpieces of literature. So it has been with the role of mathematics in physics also. The esoteric framework of current physics, using abstract spaces and group theoretic formalisms, may well be compared to surrealist poetry and other creative experiments in the literary world.

The centuries of Galileo and Newton recognized, initiated, and put to practical use many major mathematical techniques: analytical geometry, the calculus, and the theory of probability, to name only the most important of them all. But it was left to the 18th century to put on a firmer footing what is now called mathematical physics. The scientific firmament of the 18th century is also studded with stars of the first magnitude, as numerous and productive as of the century preceding, though their work was not as revolutionary in its break with the past.

Of the many who hold a just place in the initial history of mathematical physics during the 18th century, three names stand out above all others: Leonhard Euler, Daniel Bernoulli, and Jean le Rond D'Alembert. These three men sowed the seeds of what was to become one of the finest occupations of the human mind: the formulation and investigation of physical laws and processes in essentially mathematical terms. While the 17th century used mathematics as a valuable tool in the description of nature, mathematical physics practically revels in mathematical expressions and their consequences.

D'Alembert, Bernoulli, and Euler often, and inevitably, attacked similar problems and one another during their long years of productivity. They interacted with one another, both directly and indirectly, while contributing to the naturing of mathematical physics. Each had his own temper and charm. Historians of that delightful period have defended one against the other, sometimes exaggerating the importance of one, sometimes belittling the role of another.[5]

1983 is the 200 anniversary of D'Alembert's death. We will discuss in this paper the role of D'Alembert in the history of scientific ideas.

The Background

During the first decades of the 18th century Newtonian gravitation had not yet been universally accepted in Europe where the vortices of Descartes were still very much in vogue.[6] Similarly, the wave theory of light, because of its Continental origins, could not compete successfully in England with Newton's light corpuscles. Scientific research was carried out principally in various academies, of which the most important were the ones in London, Paris, Berlin, and St. Petersburg. In particular, Frederick II of Prussia and Catherine of Russia were both patrons of the arts and the sciences, took great interest in their respective academies, and encouraged scholars and scientists without regard to their national origin.

Paris was the intellectual capital of Europe. The salons of the great city attracted scholars, thinkers, and scientists who discussed everything from physics and philosophy to the theater and court gossip, exchanging ideas and quips in the process. The fine ladies who organized the parties were quite knowledgeable also, and they did not always subject themselves to the conventional codes of chastity.

Scientific academies challenged investigators to enter prize-winning competitions by submitting essays on proposed topics. Practising scientists wrote texts with historical introductions, and were generous in giving credit to themselves. Scientists in their correspondence were as bitter and sarcastic about others in the field as they often are in our own times. It was not yet the age of specialists, so that most scientific investigators wrote on more than one narrow field of research, often going even beyond scientific themes.

This was the setting in which D'Alembert, Euler, and Bernoulli worked. To students and teachers of physics these names simply recall principles, equations, or integrals. But once these were men in flesh and blood, arguing, fighting, and creating.

D'ALEMBERT'S EARLY LIFE

Claudine Alexandrine de Tencin had spent most of her youthful years as a nun, but she began to feel in her early thirties that perhaps life could offer her more exciting adventures. And so, by the time she was 32 she escaped from her convent, determined to make good for the years she had spent in spiritual spinsterhood. In less than a year she found refuge in the arms of an artillery officer. Their mutual affection continued for some years. And before they could agree on settling down as man and wife, the ex-nun discovered she was expecting the Chevalier Destouches' child. But when she was ready to announce this to the gentleman he had left town, perhaps unintentionally.

When the baby arrived on November 17, 1717—it was a boy—the lonely mother left it on the steps of the Church of St. Jean le Rond, perhaps out of fear of being forced back into a life of chastity by a punitive society for becoming an unwed mother. The screaming infant was picked up by an officer of the law, and entrusted to an orphanage. The boy was named after the Church as Jean le Rond. When the father returned to Paris, he heard about the child, took it from the orphanage, and gave it to a certain Madame Rousseau (no relative of the great philosopher). She in turn named the child E'Alemberg (with a 'g'). She and her husband turned out to be very affectionate foster parents.[7]

Jean is said to have displayed a high degree of intelligence from an early age. His father paid for his education. His teachers encouraged him to religious studies; but he pursued law, then medicine, and soon gave up both to turn his attention to mathematics which interested him most. He studied this subject mostly by himself, and in his readings spotted an error in a classic text on the calculus, which had been authored by a certain Charles Reynau.[8] What was impressive about this was that the book had been in circulation for over three decades, and the error had not caught anybody's attention. In fact, D'Alembert discovered the mistake in the second edition of the book.

D'Alembert brought this to the attention of the learned world in 1739 through a short paper which was presented to the Academy of Sciences by Clairault. He was not yet 22 years old. Clairault was very impressed, and spoke highly of the young man.[9] Thus did D'Alembert make his presence felt in the scientific community: by pointing out an error in another author's work. In retrospect, this was symbolic if not significant, for it revealed his critical mind and augured the polemical nature of a good deal of his later work. But this was soon followed by a number of other papers. These dealt with differential equations and theoretical mechanics. This was in the spirit of the age: an interest not only in mathematics, but also in its applications.

MECHANICS

In 1743 D'Alembert published his *Treatise on Dynamics*[10] whose aim, he declared, was to reduce "to the smallest possible number the laws of equilibrium and of motion of solid bodies". The three laws he stated in this work were the law of intertia, the law of parallelogram of motion, and the law of momentum conservation in collisions. In this treatise he formulated a method by which these laws could be applied to the solution of specific problems. The method consisted in regarding any motion as being made up of a natural intertial motion of which a part may have been destroyed as a result of external constraints. The motion that is thus destroyed is now treated in terms of a fictional agent or a constraining body. This approach, which Lagrange was to describe many years later as one that reduced dynamics back to statics, came to be known as D'Alembert's principle.

In the course of the 17th century the notion of force began to take on a quantitative aspect. Even though physics texts define and explore this concept neatly and with logical consistency, recalling the names of Galileo and Newton as the originators of it all, a great deal of confusion and controversy surrounded the idea of force during those early years of the modern scientific quest.[11] Galileo, for example, sometimes regarded force as being proportional to the velocity of a body, and sometimes as, to the square of the velocity. Descartes thought that the correct measure of force should be mv, whereas Leibniz argued that it must be mv^2. Newton wrote down in a notebook in 1664: "....it appears how & why amongst bodys moved some require a more potent or efficacious cause others a lesse to hinder or helpe their velocity. And ye power of this cause is usually called a force." And in the *Principia* he said: "The alteration of motion is ever proportional to ye force by wch it is altered." Indeed Newton used the word force in the sense in which we use the word impulse.[12] There was also discussion about whether force is to be taken as a cause or as an effect.

D'Alembert made a detailed analysis of the idea of force in the introduction to his treatise, noting at the outset that there was as yet no clearly defined meaning of the term. And he did not want to enter into any controversy on this important question.

He also remarked in this context that since all that one observed in any motion was displacement in space in a time interval, this ought to be the only fact from which the principles of mechanics should be deduced. He would talk of the observed motion, not of the *causes motrices*. He would avoid concepts like forces inherent to moving bodies which were obscure metaphysical entities. This brings to mind Heisenberg's matrix mechanics.

D'Almbert's treatise on dynamics, written at the age of 26, included other questions of a fundamental nature, such as whether the laws of mechanics were necessary or contingent. And in this work he also suggested that time may be regarded as a fourth dimension.

The following year, i.e. in 1744, D'Alembert published another treatise, this one on hydrostatics and hydrodynamics.[13] Euler and Bernoulli were, let us recall, the acknowledged masters in the field of mathematical physics. There were many reasons why D'Alembert's interest was drawn to the subject, not the least of which was the fact that he had already made a study of solid body mechanics in his first treatise. However, when D'Alembert not only wrote another treatise on fluid mechanics but also claimed that his approach was better than that of Bernoulli, the did not exactly win the latter's affection.[14]

In 1746 the Berlin Academy proposed as the theme for its prize essay the cause of winds. D'Alembert submitted his Reflections on the General Cause of Winds,[15] not knowing that Bernoulli was also a contestant. D'Alembert's essay won the prize, while Bernoulli's was not even published.

The physical content of D'Alembert's essay was not considerable. But the work reveals the attitude of the mathematical physicist in approximating nature to the extent of making a problem mathematically solvable. He noted here that "the majority of physico-mathematical questions are so complicated that it is first necessary to consider them from a general and abstract point of view, and then raise them step by step from the simple to the more complex cases". It was also in this work that he sowed the seeds of the calculus of finite differences. The germinal ideas that were to lead to partial differential equations may also be found here.

Participation in this contest brought D'Alembert into the acquaintance of Frederick the Great of Prussia to whom he dedicated the work in a Latin verse whose English translation reads:[16]

> Swifter than the winds, while of the winds I write
> The foes of conquering Frederick speed their flight;
> While laurel o'er the hero's temple bends,
> To the tired world the olive branch he sends.

The same year D'Alembert also presented a paper on the famous three body problem, which bas attracted some of the ablest mathematical minds in history, including Euler, Clairault, Lagrange, and Laplace.[17] Five years later he published a more complete work on lunar theory which, in the opinion of Joseph Bertrand, was by itself "sufficient to make him immortal".[18]

Another of D'Alembert's major work in analytical mechanics was an essay on the precession of the equinoxes.[19] After nineteen years of patient observations

(1728-47) J. Bradley had confirmed that the nutation of the earth's orbit was caused by the action of the moon. Less than two years after this announcement was made, D'Alembert gave a mathematical exposition of the phenomenon, deriving it as a consequence of Newtonian mechanics. He used his principle in attacking this problem. This is generally regarded as a major triumph of celestial mechanics.

The Wave Equation

The importance of the wave equation in mathematical physics can hardly be overstated. To D'Alembert is due the credit of introducing it first into theoretical physics. The equation was derived in 1747 in an attempt to reformulate the problem of the vibrating string.[20] A good deal of work in the field had alredy been done decades before, especially by Euler and Bernoulli. But it was D'Alembert's memoir which contained for the first time the partial differential equation which has come to play such an important role in the development of mathematical physics.

D'Alembert's approach to the problem of the vibrating string provoked protests from the competing experts, notably Euler and Bernoulli. D'Alembert, using only the assumption that the vertical restoring force in a stretched string is proportional to the curvature and to the tension of the string, arrived at what he believed to be a more general solution of the string problem than what had been obtained thus far. Bernoulli complained that this was accomplished only because its author had stripped the problem of all physically meaningful conditions. Such a general treatment, he felt, was unacceptable. Yet, this was precisely the point. The method of mathematical physics was to describe situations with the utmost generality first, and then only to consider particular cases, as D'Alembert had stressed in his Essay on Winds.

Why did D'Alembert take up this problem at this time? There is one possible explanation for this. D'Alembert is known to have entered Parisian social life in 1746 through the salon of Madame Geoffirin. Here topics of conversation ranged over a wide variety of subjects: intellectual, cultural, artistic, and scientific. One such topic could very well have been Jean Philippe Rameau's opera, based on Votaire's *La Princesse de Navarre*, which had been acclaimed as a great success. Now, Rameau had incurred the displeasure of music critics some twenty years earlier by his treatises on music.[21] But now, because of the success of the opera, he had come into the good books of the Music Academy. Rameau's musical theories must also have been discussed in the salons.[22] D'Alembert's interest in the matter could have been aroused in these discussions. Indeed D'Alembert himself wrote a popular exposition of Rameau's theories.[23]

Hydrodynamics

D'Alembert had been interested in fluid mechanics even as early as 1740. We have already referred to his first treatise on this subject. Thus, when the Berlin Academy announced in 1750 that the topic for its 1752 prize essay would be the theory of fluid resistance D'Alembert jumped at the opportunity. But his eassy did not win a prize. Nor did anybody else's. The judges declared that none of the contestants deserved the prize for that year. D'Alembert was not only disappointed, but quite angry because the Academy stated that the participants should compare their theory with experimental results, a requirement that, in his view, had not been spelled out in the original statement of the problem. Instead of resubmitting the work with the suggested amendments, he published it on his own.

This work, entitled *An Eassay on a New Theory of Fluid Resistance*,[24] embodies a number of germinal ideas of fluid mechanics. The concept of the velocity field first appears here. The field concept, in its quantitative sense, first arose in fluid dynamics. D'Alembert's work also contained special cases of the equation of continuity, and the condition for potential flow.

In this treatise D'Alembert referred to the contributions of Bernoulli and Euler, and offered a justification for his own work by pointing out to what seemed to him to be some shortcomings in Bernoulli's work. He expressed himself very tactfully by saying: "My intention here is far from underrating the work of these great men; but sciences such as these are by nature such as to be always perfected more and more, aided by the light that savants who have preceded us have shed on these obscure questions; we are sometimes fortunate enough to advance a little farther than they on the route which they themselves have drawn for us; and if we dare challenge them it is with the arms which they have furnished us."

MATHEMATICS

The 18th century was so much enchanted by the successes of the techniques of the calculus that sometimes it tended to exploit the resources of that discipline with very little concern for the reliability of its foundations. Indeed the situation was not much different from what obtained soon after the formulation of quantum mechanics in the 1920's. True, there were a few philosophically inclined physicists who grumbled at the introduction of what seemed to them to be questionable concepts. But the vast majority of the investigators built on what appeared to be a fruitful framweork. Thus, although some challenged notions like infinitesimals and limits, their objections were largely ignored. Moreover, they were generally negative critics in that while they took great delight in pointing to the conceptual difficulties of the new mathemiatics, and strongly recommended that one abandon the pursuit of the calculus, they offered no alternative answers.

Euler, in his prolific outpour on practically all branches of mathematics and physics, was elated by the fine and fancy formulae which his magical pen produced in profusion, and was not always bridled by considerations of rigor and legitimacy. As A. Zygmund has noted, "Though Newton and Leibnitz avoided divergent series, formal and unhibited use of the latter was bringing such mathematical rewards that the temptation was too much to withstand, and divergent series became, mostly through the work of Euler, an accepted tool of investigation".[25]

D'Alembert, having himself experienced the potentialities of the calculus, was not exactly for abandoning the subject. But he was very aware of the inherent difficulties involved in the underlying notions. If he made the famous remark, "Go ahead, and you will acquire faith"[26] it was not only because he knew the practical value of playing the game, but also because he was no less conscious of the conceptual difficulties in them.

As far as infinite series were concerned D'Alembert was quite explicit as to his views on them "As for myself," he wrote, "I must confess that all reasonings and calculations based on series which are not convergent or which may be supposed not to be so, are very suspect."[27]

Understandably he was also very particular about the notion of the limit in the calculus. "The theory of the limit," he said, "is at the foundation of the real metaphysics of the differential calculus. We are not dealing here, as is commonly said, with infinitesimally small quantities, rather only with the limits of finite quantities..."[28] D'Alembert's definition of limits was considered to be good enough by Cauchy during the next century when he adopted it in his classic work, "Cours D'Analyse."[29]

D'Alembert felt that more effort should be spent in developing the principles and the foundations of the calculus, than on mere details of calculations. Referring to texts written with little consideration to rigor, he noted, "careful attention is all the more necessary because those who have explained the rules of the infinitesimal calculus have either neglected its true principles or have presented them in an incorrect manner."[30]

At the same time he did not approve of the obscurantist rigor which, instead of clarifying, could confuse the reader. He referred, not without a touch of humor, to "a great modern mathematician famous during his lifetime in Germany as a philosopher, who began his elements of geometry with the theorem that a part is smaller than the whole, and demonstrates this with a reasoning so obscure that its effect is to make the reader doubt the correctness of the proposition."[31]

D'Alembert's concern for rigor and legitimacy of reasoning, when it extended to geometry, almost prophesied the rise of non-Euclidean geometry. Granting, as his contemporaries believed, that the basic axioms of Euclidean geometry were true, he spoke of the different geometrical propositions (axioms) which their "inventors grasped at a glance, but whose proof is necessary for mathematical rigor."[32] The theory of parallels intrigued him considerably, and he declared very significantly: "I dare predict, and I am not afraid of being contradicted by those who may have reflected over this matter, that the proposition that we are presenting here, and in general the theory of parallels, is one of the most difficult points in the pirnciples of geometry; I may add that this theory will be much advanced by this proof."[33]

PROBABILITY THEORY

D'Alembert's work on probability is universally regarded as the weakest portion of his creative work. Both contemporary and later writers have attacked and ridiculed D'Alembert on this score.[34] There have been exaggerations and even misrepresentations of D'Alembert's views in some of these discussions.

Ironically, while D'Alembert saw the value of pursuing the calculus while still recognizing its dubious foundations, he was far less enthusiastic about the theory of probability. He expressed his objections to this field of mathematics for the first time in an article published in 1754 in his famous Encyclopedie,[35] exactly a century after Pascal's famous letter to Fermat on the Chevalier de Mere's problem which initiated the mathematical theory of probability.[36] Joseph Bertrand noted that in spite of the works of Pascal, Huygens, and Jacques Bernoulli, D'Alembert refused to regard the calculus of probability as a legitimate branch of mathematics.[37] There is an implication of some sort of irrational obstinacy on the part of D'Alembert. Yet it is equally true that in spite of the works of Bohr, Born, and Heisenberg, Einstein refused to regard the probabilistic interpretation of the psi-function as a legitimate description of the microcosm. The point to remember is that D'Alembert, like Einstein, was obsessed with the logical foundations of the theory, as indeed he was with those of the calculus also.

It was not even true that he refused to accept the theory. Indeed he stated, "I wish only that the calculus of probability be clarified and modified."[38] And if he entertained certain doubts on it, it was only because he felt that "the theory of probability, such as it is presented in the books that deal with it, is on many questions neither as enlightening nor as complete as one might imagine."[39]

That the basic premises of probability theory did require further clarification is amply vindicated by the subsequent history of the subject. Indeed D'Alembert was perhaps the first mathematician of any repute to examine the whole subject from an epistemological point of view. Although in some of his contentions and consequent calculations he did err, there is some truth in Condorcet's declaration: "..If this calculus (of probability) stands one day on firmer footing we shall be indebted to D'Alembert for it."[40]

PHILOSOPHY OF SCIENCE

D'Alembert's intellectual activities were not confined to attacking technical problems in mathematics. His field of interest and the themes of his writings embraced a whole panorama of cultural creativity, from music and the arts to poetry and philosophy. Inspired by Francis Bacon whom he regarded as "the greatest, the most universal, and the most eloquent of philosophers," he sought constantly to correlate theory with observation and experiment. In many of his scientific writings D'Alembert referred to and insisted upon the importance of correlating theory with experience.

Thus, for example, while speaking of hydrostatics he noted, "Experience alone has been able to teach us the details of the laws of hydrostatics, which the subtlest of theories could never have enabled us to suspect. ."[41] Similarly, in his work on the precession of the equinoxes he wrote,". . of the different assumptions that we may make to explain this effect, only those which by their nature furnish us with unmistakable ways of assuring us of their correctness are worthy of our attention."[42]

Instead of saying as bluntly as Laplace was to say a century later that he did not need the hypothesis of a deity, D'Alembert suggested in his Treatise on Dynamics that "the nature of the Supreme Being is too hidden for us to know directly what is or is not in conformity with His wisdom, and we can only perceive the effects of that wisdom in our observations of the laws of nature when mathematical reasoning reveals to us the simplicity of these laws, and experiments shows us their applications and range."[43] We must remember that it was unusual for eminent scientists of the period not to make references to the Almighty.

In D'Alembert's view the ultimate aim of science was the development of satisfactory theories to explain facts of observation. However, he also recognized clearly the possibility that entirely different hypotheses could equally well serve in the adequate explanation of a given phenomenon. As Hermann Weyl pointed out "in this recognition of the 'ambiguity of truth'. Hobbes and D'Alembert preceded the modern positivists."[44]

D'Alembert was completely against the blind application of mathematical techniques in the description of the physical world without any regard to its nature, a practice in which some theorists have often indulged. While fully emphasizing the importance of mathematics in physics he also noted, "It must be admitted, however, that mathematicians sometimes abuse this application of algebra to physics. In the absence of proper experiments to serve as a basis for their calculations they allow themselves the most convenient hypotheses to suit their needs, but which are often quite distant from what is found in nature."[45]

As to the aims and potentialities of physics D'Alembert foresaw the dreams of the current struggles at unified field theories when he wrote: "The universe, if we may be permitted to say, would only be one fact and one great truth for whoever knew how to embrace it from a single point of view."[46]

At the same time D'Alembert was very particular in recognizing the limits and domains of the physico-mathematical methodology. It has always been a favorite sport among some scientists and philosophers to prove through scientific results and reasonings religious beliefs and theological doctrines. Whether this is done to please the public or out of genuine conviction, such attempts usually lead to confusions and contradictions. These arise because one loses sight of distinctions between faith and reason. As a result of which, said D'Alembert, "on the one hand some great geniuses have been sorely mistaken, and on the other hand, the defenders of religion have sometimes imagined too easily that it is being attacked."[47]

D'Alembert himself publicly declared that there was ample proof for the existence of God. However, in some of his private correspondence he tended towards an atheistic materialism.

It is not uncommon to see, even in our times, the tendency of intellectuals with only a marginal acquaintance with or training in the sciences, trying to use a scientific framework in propagating ideas that have intrinsically nothing to do with science. In the 18th century, for example, Rameau attempted a pseudo-scientific explanation for everything from geometry and the duality of the sexes to the Trinity. D'Alembert, not always moderate in his words of criticism, deplored this 'furious mania', and rebuked those who indulged in such practices whose purpose, he said, was simply to impress the ignorant.

Concluding Remarks

We have surveyed very briefly some of the positive contributions of D'Alembert to physics and mathematics. D'Alembert was one of the keenest intellects of the 18th century. Works on a variety of themes: cultural history, the philosophy of the Enlightenment, the history of music, the history of ideas, the mainsprings of the French Revolution, all have to include his name. Above all else, he was a physicist and mathematician.

D'Alembert is also one of the few eminent scientists of any period whose reputation has suffered significantly in the writings of historians of science. Indeed the general trend in histories of science has been to give the impression that D'Alembert was always the erring, confused savant who, by his incessant polemics and obscurantism, brought little light to the problems discussed, while Euler was invariably the clear headed thinker par excellence.[48] D'Alembert was certainly not flawless, but his flaws have received more than decent exaggerations in the writings of some historians. While the commendable work of Pappas[49] removed many of the misconceptions as to the docile character of D'Alembert, and the scholarly biography by Grimsley[50] analyzes his philosophical contributions in detail, there seems to be no adequate account in English of D'Alembert, the scientist. On the other hand his rivals Euler and Bernoulli have received well deserved recognition from enthusiastic scholars.

D'Alembert was not meticulous in the details of his calculations where he often let simple errors slip in. Much has been made of this unfortunate trait of his. But it is well to remember that errors in arithmetic and algebra are not as serious as errors in analysis or in the elucidation of principles. Considering the range and volume of his writings, it is understandable that he did not pay the most scrupulous attention to everything that he sent for publication.

Anyone who undertakes to head a project such as the Encyclopédie should be given some margin for possible misstatements, and even factual blunders, especially when he says quite frankly at the outset "..we declare without hesitation, in the name of our colleagues and our own, that we will always be disposed to confess our inadequacies and to profit by the enlightenment communicated to us."

Whatever the view of some of his rival contemporaries, and even of later scholars, many creative mathematicians and physicists, from Lagrange and Laplace to Louis de Broglie and Hermann Weyl have spoken of D'Alembert's scientific achievements only with the utmost respect. It is in such a spirit that we remember him on the 200th anniversary of his death.

Notes and References

[1]Pascal, B., "Le nez de Cléopatre: s'il eut été plus court, toute la face de la terre aurait changé." Pensées, ii, 162.
[2]Whitehead, A. N. "*Science and the Modern World*," N.Y. (1925) p. 68.
[3]Bacon, Roger "Opus majus," opus tertiam, U Penn Press (1928) p. 29.
[4]Lord Kelvin's description of Fourier Series. E. T. Bell, *Men of Mathematics*, London (1957 ed.) p. 217.
[5]Euler's student and admirer N. Füss on the one hand, and D'Alembert's protégé Condorcet on the other were the two major representatives of these views in the 18th century, as revealed in the Eloges of their heroes.
[6]See, in this context, the recent publication of H. Guerlac, *Newton on the Continent*, Ithaca (1983).
[7]D'Alembert's mother was called Claudine Alexendrine de Tencin, and she held a salon at her home. It was in this context that she met the cavalier officer Destouches-Canon and bore the mathematician.
[8]*L'Analyse Démontrée* by Father Charles Reynau discussed the mathematics of Descartes, Leibniz, and the Bernoullis. It was in the 1736 reprint of this work that D'Alembert detected the error.
[9]Bertrand, Joseph, *D'Alembert*, Paris (1889) p. 2.
[10]A modern edition of this and other treatises by D'Alembert has been issued by Editions Culture et Civilization, Bruxelles.
[11]André Fontaine, another 18th century mathematician claimed that he had known an equivalent of D'Alembert's principle years before the publication of D'Alembert's work. This caused controversies. *Collected Works*, A. Fontain ed. Paris (1764).
[12]See, in this context, V. V. Raman, "The Second Law of Motion and Newton's Equations," *The Physics Teacher*, March 1972, p. 136.
[13]"Traité de l'equiliber et du mouvement des fluides." Ref. 10.
[14]In various letters to Euler, Bernoulli expressed his anger and contempt for D'Alembert because of the publication of this work.
[15]The full title of the subject was: "Déterminer l'ordre et la loi que le vent devroit suivre si la terre étoit environnée de tous cotés par l'océan, de sorte qu'on put en tous temps trouver la direction et la vitesse du vent pour chaque endroit." This was an early attempt at theoretical weather forecasting.
[16]Haec ego de ventis dum ventorum ocyor alis
Palantes agit Austriacos Fredericus, et orbi
Insignis lauro ramum praetendit olivae.
[17]"Méthode générale pour determiner les orbites et les mouvements de toutes les planétes, eu égard a leur action mutuaille." This was, in fact, a generalization of the three body problem. D'Alembert presented this memoir, as a surprise, on the very day in which Clairault presented his, on the same subject, to the Académie, an act that won him Clairault's immense displeasure.
[18]D'Alembert's work on lunar theory, though completed in 1751, was published only in 1754. The

works of Clairault, Euler, and of D'Alembert on lunar theory, a subject that had been considered by many before, were revolutionary in that their approach to the problem was analytical rather than geometrical.

[19]"Recherches sur la précession des equinoxes et sur la nutation de l'axe de la terre," was published in 1749.

[20]"Recherches sur la courbe que forme une corde tendue mise en vibration," *Hist. de l'Acad. de Berlin*, 3, 1747, pp. 214-219, and 220-49.

[21]"Traité de l'harmonie reduite à ses principes naturels." A thesis put forward by Rameau was that in any musical composition there is always a fundamental base from which may be derived all the higher chords in it, and that every chord in it follows from the harmonic series of partial tones.

[22]Marmontel, J. F., *Oeuvres Complètes*, Paris (1819), p. 300.

[23]"Eléméns de musique théorique et pratique suivant les principes de M. Rameau."

[24]"Essai d'une nouvelle théorie de la resistance des fluides" is a classic in the history of science.

[25]*Encylopedia Britannica*, (1966) 20, p. 366.

[26]"Allez en avant, la foi vous viendra," quoted in Ref. 9, p. 57.

[27]"Pour moi, j'avoue que tous raisonnements et les calculs fondès sur des séries qui ne sont pas convergentes ou qu'on peut supposer ne pas l'être, me paraîtront toujours très suspects." *Op. Math.*, Vol. 5 (1768) p. 183.

[28]"La thèorie de la limite est la base de la vraie mathèmatique du cal cul differentiel. Il ne n'agit point, comme on le dit ordinairement, des quantitès infiniment petites; il s'agit uniquement des limites des quantitès finies." Article on 'Limite' in the *Encyclopedie.*

[29]Knopp, K., *Theory and Application of Infinite Series*, N. Y. (1949), p. 459.

[30]"Ce soin est d'autant plus nécessaire, que la plupart de ceux qui ont expliqué les règles du calcul de l'infini, ou en ont nègligè les vrais principes, ou les ont presentès d'une maniére trés fausse." *Elem. de phil. géom.* p. 275.

[31]". . . .un grand mathématicien moderne, cèlébre de son vivant en Allemagne comme philosophe, commence ses éléments de géometrie par ce thèoréme, que la partie est plus petite que le tout, et le prouve par un raisonnement si obscur, qu'il ne tendrait au lecteur d'en douter". *ibid.* p. 276.

[32]". . . .que les inventeurs ont apercues comme un coup d'oeil, mais dont la dèmonstration est nècessaire en rigueur mathématique." *ibid.* p. 269.

[33]"J'ose avancer, et je ne craint point d'être contredit par ceux qui y réflechiront, que la proposition que nous présentons à démontrer ici, et en générale, la théorie des paralléles, est un des points les plus difficiles dans les élémens de géometrie; et j'ajoute que cette théorie serait bien avancée par cette démonstration." *ibid.* p. 279.

[34]For a systematic criticism of D'Alembert's errors in Probability Theory see, *History of the Theory of Probability*, N. Y. (1965 ed.) Ch. on D'Alembert.

[35]Originally inspired by Chamber's Cyclopedia, this work finally came under the editorship of Denis Diderot and D'Alembert. The first of its 28 volumes appeared in 1751 with D'Alembert's famous "Discours Préliminaire" (Preliminary Discourse), a fine essay that embodies the scientific spirit (praising Bacon, Newton and Locke), profound insights, and liberal thought, all in excellent French.

[36]As is well-known it was while trying to solve a problem in a game of chance that the basic principles of probability theory were first stated. However, the problem itself had been discussed earlier in the works of Cardan, Tartaglia, etc.

[37]*op. cit.* 9, p. 49.

[38]"Je désire seulement qu'il (le calcul des probabilités) soit éclairci et modifié." D'Alembert, Opuscules, IV, p. 297.

[39]"La théorie des probabilités, telle qu'elle est presentée dans les livres qui en traitent, n'est sur bien des points, ni aussi lumineuse, ni aussi complète qu'on pourtait le croire." *ibid.* p. 451.

[40]Éloges, *ibid.* Vol. I, p. xx.

[41]"L'expérience seule a pu nous construire en détail les lois de l'hydrostatique, que la théorie la plus subtile n'aurait jamais pu nous faire soupçonner, ..." Elèments de philosophie, p. 331.

[42]". . . .parmi les diffèrentes suppositions que nous pouvons imaginer pour expliquer un effet, celles qui par leur nature nous fournissent les moyens infaillibles de nous assurer si elles sont vraies, sont les seules dignes de nos examens." *Prec. des equin. Op.*, p. 337.

[43]"La nature de l'Etre suprême nous est trop cachée pour que nous puissions connaître directement ce qui est ou n'est pas conforme aux vues de sa sagesse, nous pouvons seulement entrevoir les effets de cette sagesse dans l'observations des lois de la nature, lorsque le raisonnement mathématique fait voir la simplicite de ces lois, et que l'expèrience nous en aura montrè les applications et l'ètendue." *Traite de Dyn.*, p. 404.

[44]Weyl, Hermann, *Philosophy of Mathematics and Natural Science*, Princeton (1949) p. 153.

[45]"Il faut avouer pourtant que les géometres abusent quelquefois de cette application de l'algébre à la physique. Au defaut d'expériences propres à servir de base à leur calcul, il se permettent des hypothéses les plus commodes à la vérité qu'il leur est possible; mais souvent trés eloignées de ce qui est rèllelement dans la nature." *Discours*, prel. p. 29.

[46]*ibid.* p. 30.

[47]". . . .d'un coté quelques grands génies sont tombés dans l'erreur, de l'autre des defenseurs de la religion ont quelquefois supposé trop légèrement qu'on lui portait attenite." *ibid.* p. 33.

[48]In recent years the writings of the eminent scholar C. Truesdell have denigrated D'Alembert in the most erudite manner possible. See, in particular his estimates of D'Alembert in the collected works of Euler, edited by him. eg. Vol XII (seg. Ser). His assessment of D'Alembert's work: "One searches for the little solid matter, as a sparrow pecks out a few nutritious seeds from a dungheap—a task not altogether savory."

[49]Pappas, J. N., *Voltaire and D'Alembert*, Bloomington (1962).

[50]Grimsley, Ronald, *Jean d'Alembert*, Oxford, 1963.