ON THE SPIRO-ELLIPTIC MOTION OF THE SUN IMPLICIT IN THE *TILOYAPAṆṆATTĪ*

L. C. JAIN*
Professor of Mathematics,
Govt. P. G. College, Sehore.

The possible forms of the implied geocentric motion of the sun as described in the *Tiloyapaṇṇatti* of Yativṛṣabha (473-609 A. D. ?) are investigated. It is found that the geometry of the path of the sun is in the forms of opening-cum-closing as well as closed spiro-elliptic curves. They are comparable with the spiral of Archimedes (287 ? --212 B. C.) as well as the ellipse of Pappus (third Century A. D). The dynamical laws of the implicit motion are derivable from the equations of the paths. They are in addition to those given by Newton (1642-1727) and Einstein (1879-1955).

1. INTRODUCTION

As a sequel to quin-centenary of Nicolaus Copernicus (1473-1543 A. D.), celebrated in India, and abroad several research papers have appeared particularly in the *Indian Journal of History of Science* (1974—May and November) on varied aspects of the Hindu, European and modern astronomy. The motivation of J. V. Narlikar in his article on "Copernicus and modern Astronomy" has been to explain the significance of the work of Copernicus in the light of the Greek Astronomy, emphasizing the impact of its dynamical aspect in view of relativity which puts the pictures of Ptolemy (127-151 A. D.) and Copernicus as equivalent[1]. In accordance with his view, the present investigation is meant to expose the kinemetical and dynamical aspects of the ancient Indian astronomy of the dark period of history of Indian mathematics from texts on *Karaṇānuyoga.*

Astronomy being a small part of Jaina cosmology, a set of 619 verses of the seventh chapter of the *Tiloyapaṇṇatti* (a Prakrita Text of *Karaṇānuyoga* group[2]) describes the complete astronomical universe, excluding the details of the motion of planets of which the record is stated to have perished in course of time[3].

The author, in his earlier papers[4], has elaborated the following features of the Jaina Astronomical system implied in the *Tiloyapaṇṇatti* and the *Trilokasāra* :

* *Present Address :* Principal, Government College, Jaora, Dist. Ratlam, 457226.

(i) The implicit orbit of the astral bodies described in form of bounded circles is doubled by topological deformation and counter bodies are established at diametrically opposite ends, in Jambūdvīpa.

(ii) The angular division of the celestial sphere or orbit into 109800 ***gagana khaṇḍa*** (celestial parts) is a dual structure, the workable division consisting of 54900 celestial parts equivalent to 360°. The unit of angular measure is a celestial part and the unit of time is usually a ***muhūrta*** or a set of forty-eight minutes.

(iii) The *Meru* (*Mandara* mountain) is regarded as a celestial axis from where are measured the linear distances of various heavenly bodies in units known as *yojanas*. The measure of '*Yojana*' is controversial[4a]. The sun and the moon move with continuously increasing distance from the Meru, implying winding and unwinding spirals.

In the Jaina School of Mathematics, topological deformations have been resorted to for evaluating areas and volumes of surfaces and solids[5]. In case of motion of astral bodies, the dual structure has to be brought back to the original shape for the purpose of developing the classical formalism. Some of the results have already been calculated and this paper is devoted to probe into the implicit and relativistic elliptical form of motion of the bodies, tacitly envisaged by the Jaina School.

Archimedes[6] gave the name helix to a spiral perhaps (?) already studied by his friend Conon. Its polar form is $r=a+b\theta$, derivable from a more general form $r=a+b\theta^n$ Fermat proposed the spiral curve $r^2=\theta$ in 1636 A. D. Jacques Barnoulli (1692 A. D.) studied the logarithmic or equiangular spiral given by $r=ae^{b\theta}$ or $r=a^{\theta}$. It was called spira mirabilis and was the first non-algebraic plane curve rectified by him.

2. Data Regarding Motion of the Sun

The Jambū island[7] is one lac *yojanas* in diameter, at the centre of of which a conical (celestial) axis, known as *Meru*, stands one lac *yojanas* in height. The diameter of the lower base of *Meru* is $10090\frac{10}{11}$ *yojanas*, and that of the upper base is 1000 *yojanas*. The plane of the orbit cuts the axis at a point which may be regarded as a focus, round which the implied spiral motion of the sun could be described in terms of r and θ. For an observer located below, on the plane of *Citrā*, the path being the intersection of spiro-cylindrical base and cone, the projected picture will be different in cylindrical coordinates. Basing our study on the original image of the sun, one degree is found equivalent to 152.5 celestial parts. The angular velocity of image is 1830 celestial parts or 12° per *muhūrta*[8]. The angular velocity of the constellations is 1835 celestial parts or $12\frac{1150}{4575}$ degrees per *muhūrta*.

Thus relative to the *Nakṣatras* the sun has a motion at the rate of 5 celestial parts every *muhūrta*, completing the zodiacal path once in 366 days or a solar year approximately. Stationed at the first orbit, the distance of the sun from the *Meru* (axis) is 49820 *Yojanas*[9] and its linear velocity is $5251\frac{29}{60}$ *yojanas* per *muhūrta*[10]. In the last (184th) orbit, its distance from the *Meru* (axis) is 50330 *yojanas* and the linear velocity is $5304\frac{14}{60}$ *yojanas* per *muhūrta* approximately[11]. This is the orbit from where its journey back towards the *Meru* starts, no details being available regarding the exact time and distance, when it completes 366 revolutions The width of the set of orbits is $510\frac{48}{61}$ *yojanas*, where $\frac{48}{61}$ *yojanas* is the diameter of the disc of the sun[12]. The height of the sun above the plane of *citrā* is 800 *yojanas* and that of the constellations is 884 *yojanas*[13]. The beginning of the Jaina *yuga* of five years is reckoned at the same point of *Abhijit* constellation on the same day of the solar or the lunar year, with the commencement of *dakṣiṇāyana* (from summer solstice)[14].

THE GEOMETRY OF THE IMPLIED PATH OF THE SUN

The following figure No. 1. shows the implied spiral motion of a sun as implicity described in the *Tiloyapaṇṇatti*. *O* is the point of intersection

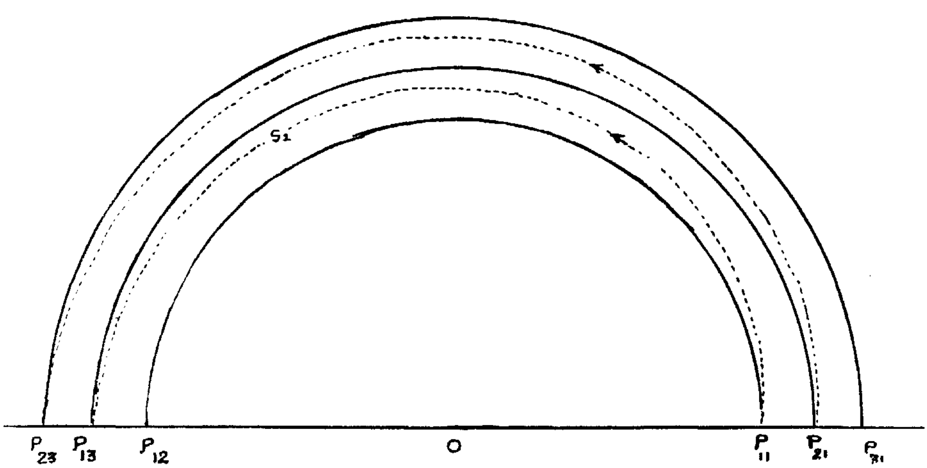

FIG. 1.

of the plane of the orbits and the *Meru* axis. P_{11} is the starting point distant 49820 *yojanas* from *O*. P_{12} is the opposite end of the diameter of the circle with radius OP_{12}. The path of the sun is the spiral $P_{11}\ S_1\ P_{11}$

described in 30 *muhūrtas*, displacing the sun to P_{13} where the distance $P_{13}P_{12}=\frac{170}{61}$ *yojanas*. The next point of the movement will be P_{21}, the orbit ending at P_{23} with the same details.

The next figure 2, is the possible topological deformation of the path described above with radii and displacements halved. The points P_{11}, P_{13}, P_{21}, P_{23} etc. are now depicted as P'_{11}, P'_{13}, P'_{13}, P'_{23} and so on.

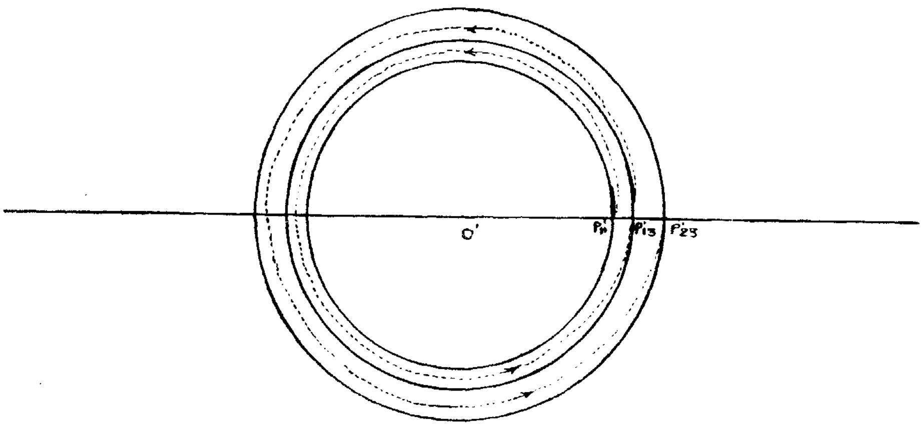

FIG. 2.

The above figures and detailed data suggest the following polar equations of the path of the sun during the solar year of 366 days :

$$r=\frac{a+b.2\pi x}{1+c\cos\frac{\pi x}{183}}, \qquad ..\ (3.1)$$

for all real x, when $0 \leqslant x \leqslant 366$.

Here a, b, c, are constants which are determinable from the boundary conditions detailed in the *Tiloyapaṇṇattī*

$$x=0,\ r=\frac{49820}{2}\ ;\ x=183\ ,\ r=\frac{50330}{2}\ ;\ \text{and}\ x=366,$$

$r=\frac{49820}{2}+E$, where E is the amount of extra displacement of the sun from original starting point just after a lapse of 366 days. E is the observational datum, whose introduction is essential for developing the equation of an opening-cum-closing spiral with the given description.

The above gives the following results for the values of a b, and c :

$$a=\frac{2507440600}{100150+E},\ b=\frac{100660\ E}{732\ \pi\ (100150+E)},$$

$$c=\frac{510-E}{100150+E}. \qquad ...(3.2)$$

When $E=0$, $b=0$, c the eccentricity reduces to be 0.005, and the path becomes an ellipse. Restoring E and neglecting c, the path becomes an Archimedean spiral, however, as it is a periodic motion the term containing c is significant.

Now considering the motion of the sun relative to the constellations, the relative motion of the sun may be put in polar form as follows :

$$r=\frac{f+g\,\theta}{1+h\cos\theta}, \qquad \ldots(3.3)$$

where θ is in radian measure and $0 \leqslant \theta \leqslant 2\pi$.

Here f, g, h, are constants, determinable from data in the *Tiloyapaṇṇattī* as under :

$$\text{For } \theta=0,\ r=\frac{49820}{2};\ \theta=\pi,$$

$$r=\frac{50330}{2};\ \theta=2\pi,\ r=\frac{49820}{2}+E.$$

On calculation, one finds that

$$f=\frac{2507440600}{100150+E}, \quad g=\frac{100660\,E}{2\pi\,(100150+E)},$$

and

$$h=\frac{510-E}{100150+E}. \qquad \ldots(3.4)$$

Thus the motion may be completely determined with the aid of the above polar equations, once E is known on observation.

4. Dynamical Laws of the Tacit and Closed-Spiral Motion, Envisaged in the Jaina School

From the geometry of the path of a body, one can derive the laws of motion under which it moves. In the last article, if $E \neq 0$, the dynamical law under which the sun moves relative to the constellations can be determined from the equation :

$$r=\frac{f+g\,\theta}{1+h\cos\theta},$$

$$Or \quad r=\frac{f}{1+h\cos\theta}+\frac{g\,\theta}{1+h\cos\theta} \qquad \ldots(4.1)$$

The first term on the right hand side of the (4.1) gives the elliptic motion for which the force is that under the law of inverse square of distance, i. e. $P \propto \frac{1}{r^2}$. Now there seems to be an additional force-contribution due to the second term on the right hand side of (4.1). Denoting the equation as follows

$$R = \frac{g\,\theta}{1+h\cos\theta}, \text{ and putting } R = \frac{1}{u},$$

one has
$$u = \frac{1+h\cos\theta}{g\,\theta}. \qquad \ldots(4.2)$$

Thus

$$\frac{du}{d\theta} = -\frac{u}{\theta} - \frac{h}{g} \cdot \frac{\sin\theta}{\theta}, \qquad \ldots(4.3)$$

where the second term on the right hand side could be neglected due to small value of h.

Hence
$$\frac{d^2u}{d\theta^2} = \frac{2u}{\theta^2}, \qquad \ldots(4.4)$$

where u could be assumed proportional to inverse of θ from (4.2) for small value of h, and therefore,

$$\frac{d^2u}{d\theta^2} = 2ku^3, \qquad \ldots(4.5)$$

where k is a constant.

The above result (5), for a central force alone, gives

$$P = h_1^2\, u^2 \left[\frac{d^2u}{d\theta^2} + u\right],$$

$$Or \;\; P = h_1^2\, u^3 \left[1 + 2u^2k\right] \qquad \ldots(4.6)$$

in which h_1 is taken to be $r^2 \frac{d\theta}{dt}$ for negligible eccentricity here.

The equation 4.6 shows that the additional force is that of inverse cube of the distance to a second approximation as also found by Einstein and which could explain the motion of perihelion of planet mercury[15]. However, the above equation (4.6) proposes an additional force $P \propto \frac{1}{r^5}$ apart from $P \propto \frac{1}{r^2}$ and $P \propto \frac{1}{r^3}$, and might be helpful someway or other, the transverse force not being considered in this approximation.

5. Concluding Remarks

The set of the sun's heliacal risings day to day as described in the *Tiloyapaṇṇattī* does not give explicit details of the motion of the sun in spiro-elliptic form which necessarily implied due to continuous motion of the sun about the *Meru* with its gradually increasing radial distance every instant. Yativṛṣabha, the author, was not in possession of this geometry but for the circumscribing circles of the kinematical orbits. They, however, implied a unified kinematical system with a diurnal and an annual motion of the sun. The derivation of the dynamical laws from the tacit path shows the historical importance of the laws of nature hidden in so simple a geometry which was envisaged by the ancient Indian cosmographers.

REFERENCES AND NOTES

1. Vide. *I. J. H. S.* Vol. 9, No. 2, 151-157.

2. From the present records, the controversial date of Yativṛṣabha puts him contemporary to Āryabhaṭa I (b. 476 A. D.) Although there is no evidence of exchange of knowledge of astronomy and mathematics between the two great authors, Jha has not denied the possibility of the gaining of this knowledge of the Jaina School by Āryabhaṭa I. (Cf. Jha, P., Āryabhaṭa I : His School, *Journal of the Bihar Research Society* vol. IV, Parts I and IV, Jan.-Dec., 1969, 102-114.) Yativṛṣabha was also possibly the author of *Cūrṇi-svarūpa* and *Karaṇa-svarūpa*, having gained special proficiency in *Kaṣāyaprābhṛta* from Nāgahasti and Āryamaṅkṣu. (Cf. *Tiloyapaṇṇatti*, pt. II, Sholapur, 1951, introduction, p. 3.)

3. *Parihisu te carante tāṇam kaṇayācallassa viccālam*
aṇṇam pi puvvabhaṇidam kālavasādo paṇaṭṭhamuvaesam (458)
"Those (planets) move along these circumference. Their distances from the *Meru* mountain and all that mentioned earlier, (in form of) teaching thereof, have become extinct in course of time" *T. P.* & 458.

4. Jain, L. C. *Tiloyapaṇṇatti ka Gaṇita*, Sholapur, 1958, (Abbr. *T. P. G.*) Jain, L. C., Kinematics of the Sun and the Moon in *Tiloyapaṇṇatti, Tulsī Prajña, J. V. B.*, Jan.-Mar., 1975, 60-67.

4(a) Cf. *T. P. G.*, pp. 18-20.

5. Cf. *T. P. G.*, op. cit., pp. 24-38, 85. Cf. also Singh, A. N.. *The Jaina Antiquary*, Vol. xvi, no. ii, Dec.-1950, Arrah.

6. Cf. Smith, D. E. *History of Mathematics*, vol. ii, Dover, 1958, p. 329; Bell, E. T., *Development of Mathematics*, New York, 1945, p. 80; Cajori, F., *A History of Mathematics*, New York, 1953, pp. 36, 50 and 224.

7 *Tanmadhye merunābhirvṛtto yojana śatasahasraviṣkambho jambūdvīpaḥ* (3.9)
Joytiṣkaḥ suryācandramasau grahanaksatraprakīrṇakatārakaṣca (4.12)
Meru pradakṣiṇā nityagatayo nṛloke (4.13)
"In the central portion of these (oceans and islands) is Jambūdvīpa, which is circular and one lac *yojanas* in diameter. (B.9) Mount Meru is like a navel at the centre of the constellations and the scattered stars." (4.12)
"In the human region, they are characterized by incessant motion around Meru." (4.13)
Cf. *Tattvārthavārtikam*, Benaras, 1915, ch. 3, v. 9; ch. 4, vv. 12, and 13 and commentary. Cf. also *T. P. G.*, op. cit., pp. 66-64.

8. *Sattarasaṭṭhaṭṭhiṇi hu cande sūre bisaṭṭhiahiyamca*
sattaṭṭhi vidya bhagaṇā carai muhutteṇa bhāgāṇam (507)
"The moon traverses seventeen hundred sixty-eight celestial parts in a *muhūrta*. Relative to this the sun moves sixty-two celestial parts more, and the constellation-class moves sixty-seven celestial parts more" *T. P.* 7.507.

9. *Saṭṭhijudam tisayāṇam mandararundam ca jambudīvassa*
vāse sodhiya dalide sūrādim paha suraddi viccālam (221)

"The distance between the first path of the sun and the *Meru* is obtained by halving the remainder which is obtained by subtracting three hundred and sixty *yojanas* as well as the diameter of the *Meru* from the diameter of the Jambūdvīpa *T. P.* 7.221

10. *Pañcasahassāṇi duve sayāṇi igivaṇṇa joyaṇā adhiyā*
uṇatisakalā paḍhamappahammi diṇayara muhutta gadimāṇam (270)
"Along the first orbit the measure of the (linear) velocity of the sun per *muhūrta* is five thousand two hundred fifty-one as well as twenty-nine parts out of sixty." *T. P.* 7.270.

11. *Pañcasahassā tisayā pañca cciya joyaṇāṇi adirego*
coddasakalāo bāhira pahammi diṇavai muhuttagatimāṇam (271)
"Along the outmost orbit, the measure of the (linear) velocity of the sun per *muhūrta* is five thousand three hundred five as well as fourteen parts (out of sixty)." *T. P. 7.271.*

12. *Diṇavai pahantarāṇim sohiyadhuvarāsiyammi bhajidūṇam*
ravibiṃbeṇa āṇasu ravimagge viuṇabāṇaudi (243)
Diṇavaipahasūcicae tiya sidi judasadeṇa saṃgṇide
hodi hucārakkhetam bimbūṇam tajjudam sayalam (243)
"When the remainder, obtained by subtracting the (set of) intervals of the paths from the eternal set, is divided by the sun's image (diameter), the (set) of all the paths of the sun is obtained as twice of ninety-two." *T. P.* 7.242.
"Whatever is obtained on multiplying the increase in the width of the sun's path by one hundred and eighty-three, becomes the orbital region (of the sun) without its diameter). When the (diameter) is added to this (amount), the (measure of the) whole (orbital region) is obtained." 7.243.

13. *Cittovarimatalādo uvarim gantūṇa joyaṇaṭṭhasae*
diṇayaraṇayaratalāim ṇiccam ceṭṭhanti gayaṇammi (65)
Aṭṭhasaya joyaṇāṇim causidijudāṇi uvari cittādo
gantūṇa gayaṇamagge huvanti ṇakkhattaṇayarāṇim (104)
"The city-plane of the sun is ever situated in the sky eight hundred *yojanas* vertically above the upper plane of the (earth) *Citrā*." *T. P. 7.65.*
"The cities of the constellations are eight hundred eighty-four *yojanas* above the *Citrā* (earth), along the celestial paths." *T. P. 7.104.*

14. *Dumaṇissa ekkaayaṇe divasā tesidiadhiyaekka sayam*
dakkhiṇaayaṇam ādī uttara ayaṇam ca avasāṇam (525)
Āsāḍhapuṇṇimie jugaṇipatti du sāvaṇe kiṇhe
abhijimmi candayoge pāḍiva divasammi pārambho (530)
"In a single *ayana* (interval between two solstices) of the sun there are one hundred and eighty-three days. Out of these two *ayanas*, beginning is with the southern *ayana* and the ending is with the northern *ayana*. *T. P. 7.525.*
"The five year *yuga* (cycle) ends on the *āṣāḍha pūrṇimā*. That *yuga* begins with the conjunction of the moon with the *Abhijit* constellation on the *śrāvaṇa kṛṣṇa pratipadā*." *T. P.* 7.530.
For details of Jaina Calendar, cf. Das, S. R., The Jaina Calendar, *The Jaina Antiquary*, vol. iii, no. ii, sep. 1937, 31-36, Arrah.

15. Cf. Weber, J., *General Relativity and Gravitational Waves*, New York, 1961, p. 67. Cf. also, Einstein, A., "The Foundation of the General Theory of Relativity", *The Principle of Relativity*, pp. 109-164, Dover, unabridged republication of 1923 translation.