THE VALUE OF π KNOWN TO *ŚULBASŪTRAKĀRAS*

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INTRODUCTION

The Greek letter π indicates the ratio, which is a constant, between the circumference of a circle and its diameter. In the Vedic literature a specific value of π is not found excepting in the *Baudhāyana Śulbasūtra* (*B.S.S.*I. 113)[1] which is mentioned with reference to the circumference of a pit required for fixing a sacrificial post (*yūpa*). It is an approximate value as in this case an accurate value of circumference is not necessary. From the study of the Vedic literature, it could be surmised that the Indians at that time might be knowing the value of π, although it is not possible to know its exact value that might have been assumed by them. From the study of *Śulbasūtras* (*Baudhāyana*, *Mānava*, *Āpastamba* and *Kātyāyana*) the value of π assumed at that time could be found out. The period of different *Śulbasūtras* are : *Baudhāyana*, 800 **B. C.** ; *Mānava*, 750 **B. C.** ; *Āpastamba*, 600 **B. C.** ; *Kātyāyana*, 200 **B. C.**

HISTORICAL BACKGROUND

The evidences to show that the relation between circumference and the diameter of the circle was known from Indus Civilization, ***Ṛgvedic*** period and then in ***Brāhmaṇa*** period are given below :

Indus Civilization

There are circumstantial evidences to show that people of Indus Civilization might be knowing different geometrical properties of a circle.

On plate XVIII, fig. 10a (Mackay 1948)[2], a flower with four petals circumscribed by a circle is shown (Fig. 1). It is to be noted that in order to draw this figure, the circumference of a circle is required to be divided into 12 equal parts. For this purpose knowledge of the construction, that with the known length of the radius of the circle, its circumference could be divided into six equal parts is presupposed. A flower with eight petals is also shown (Plate XVIII, fig. 15, Mackey 1948), which required division of of the circumference of a circle into 24 equal parts. This also confirms the above assumption.

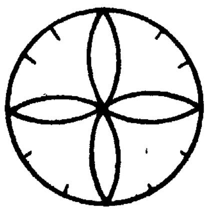

FIG. 1

Stone weights are extremely common in the excavation of the city of Mohenjo Daro. Spherical weights with flat top and base made of alabaster, lime stone, quartz, slate and jasper were found. These weights are very accurate. Could it be inferred that the people of Indus Civilization had the knowledge of the relation between volume, weight and density of the above materials ?

The bullock cart with wheels was known. These wheels might have been provided with metallic rims. For finding the length of the rim for fixing it on a wheel the relation between diameter of the wheel and its circumference might be known.

Vedic Period

Chariots are mentioned copiously in *Ṛgveda* (I. 102. 3 ; I. 53.9 ; I. 55.7 ; I. 141.8 ; II. 12.8 ; II, 18.4 ; IV. 46.2 ; etc.). Chariots might be one of the regular parts of the army (*ṚV.* I. 53.9 ; I. 102.3 ; II. 12.8 ; etc.). Chariots of different types are also mentioned in *Ṛgveda.*[3]

By the time of *Yajurveda* (16.27)[4] the number and variety of chariots that were manufactured were increased to such an extent that a separate guild of chariotmakers (*rathakāras*) was developed. A chariotmaker was grouped different from that of a carpenter. For fabricating metallic rim *(pavihi)* to be fitted around felly it is necessary to measure the perimeter of the felly very accurately. If the diameter of the rim is larger, even by a very small amount, it will be oversize for the felly. If the diameter is lesser than required then even after heating the metallic rim it will not be possible to fit it around the felly. As the variety in the size of wheels of different chariots was very large, the chariotmaker might be knowing the relationship between diameter of a wheel and its perimeter. This knowledge would enable him to fabricate a rim of the desired diameter. It may further be noted that the chariotmakers were of priest class who had helped much in advancing the frontiers of knowledge at that time. The names of chariotmakers that are mentioned in *Ṛgveda* are Bhṛgu, Ṛbhu, Tvastṛ and others[5].

Brāhmaṇa Period

The three *Śrauta kuṇḍas* namely, *Gārhapatya, Āhavanīya* and *Dakṣiṇa* are shown to be as old or may be older than *Ṛgveda*[6]. The sacred fires are mentioned in *Ṛgveda* (I. 15. 12 ; V. 11. 2)[7]. According to Datta (1932)[8]

"The first express description of the *Gārhapatya* as a circle of one square *vyāma (=puruṣa)* and of *Āhavanīya* being a square of same size appears in *Śatapatha Brāhmaṇa* (VII. 1. 1. 37 ; VII. 2. 2. 1)". In order to draw a square of area equal to that of a circle one must know the method of calculating the area of the circle (i.e. one must know the magnitude of π) apart from the knowledge of squaring a circle of equal area.

The above information gives circumstantial evidence that from the period of Indus Civilization to that of *Brāhmaṇas* some idea of the value of π may have been developed.

THE VALUE OF π AS DEDUCED FROM *Śulbasūtras*

(i) As given in introduction, in *B. S. S.* I.113 the approximate value of π equal to three was given, as more accurate value in that case was not necessary. The diameter of the *yūpa* is one *pāda*. The circumference of the pit in which the *yūpa* is to be fixed is given as three *pādas*. The value of π is, therefore, three.

(ii) An approximate method of obtaining a square of area equal to that of the given circle is given (*B. S. S.* 1·60[9] ; *M. S. S.* 10.3.2. 13 ; *A. S. S.* 3 6-8 ; *K. S. S.* 3.14). It is stated that the length of the side of a square of area equal to that of the given circle is 13 parts out of 15 parts in which the diameter of the circle is divided.

Let us assume that the diameter of the given circle is d. Then the length of the side of the square of equal area will be $\frac{13}{15}d$. The area of the circle $=\frac{\pi d^2}{4}$. The area of the square is $(\frac{13}{15}d)^2$. As the areas of the circle and that of the square are equal,

$$\frac{\pi d^2}{4}=\left(\frac{13}{15}\,d\right)^2 \qquad \therefore\ \pi = 3.004.$$

It is mentioned clearly that this is an approximate co-relationship which shows that the value of π was also known to be approximate.

(iii) *Baudhāyana Śulbasūtra* (1.58)[10] (also *M.S.S.* 10. 1. 1.8 ; 10.3.2.10 ; *A.S S.* 3.2-5 ; *K.S.S.* 3.13) had given a method of obtaining a circle of area equal to that of the square of the sides of given length. The construction is : *ABCD* is the given square with length of sides x. O is the centre of the square. Join OA and draw an arc OP. Add to the half length of the side of the square one-third the length of the half diagonal OA that remains outside the square. The radius of the circle of area equal to that of the given square is equal to $OE+\frac{1}{3}PE = OM$.

Let us assume that the length of the side of the given square is x. Then the radius (R) of the circle of area equal to that of the square is

$$R = \left[\frac{x}{2} + \frac{1}{3}\left(\frac{x}{\sqrt{2}} - \frac{x}{2}\right)\right].$$

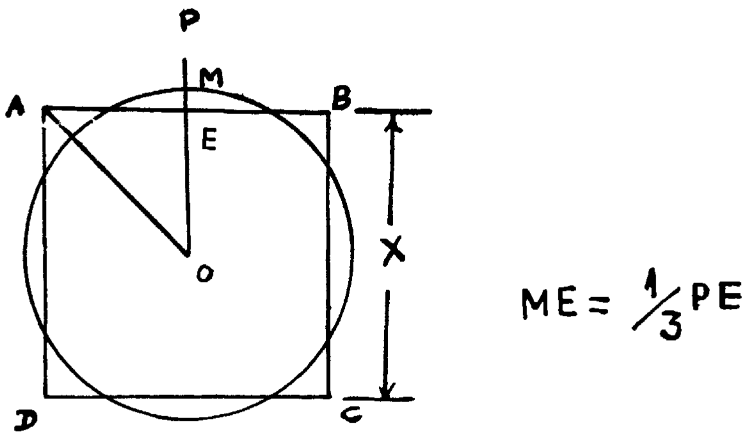

FIG. 2

Area of the square $= x^2 =$ area of the circle $= \pi R^2$

$$\therefore \quad x^2 = \pi\left[\frac{x}{2} + \frac{1}{3}\left(\frac{x}{\sqrt{2}} - \frac{x}{2}\right)\right]^2$$

$$= \pi\left[\frac{\sqrt{2}x + 2x}{6}\right]^2$$

$$= \frac{\pi}{36}\, x^2\,(6+4\sqrt{2})$$

$$= \frac{\pi}{36}\, x^2\left(6+4\times\frac{577^*}{408}\right) = \pi x^2\left(\frac{1189}{3672}\right)$$

$$\therefore \quad \pi = \frac{3672}{1189} = 3.088.$$

It is to be seen that if the value of $\pi = \frac{22}{7}$ was known to *Śulbasūtrakāras* then whether they would have arrived at the fraction $\frac{1}{3}$. This could be ascertained by reverse calculation. Let us assume that this fraction is unknown (Y) and value of π is known to be $\frac{22}{7}$.

$$x^2 = \pi\left[\frac{x}{2} + Y\left(\frac{\sqrt{2}x - x}{2}\right)\right]^2$$

$$= \frac{\pi}{4}\left[x + xY\left(\frac{577}{408} - 1\right)\right]^2$$

* According to *B. S. S.* (1.61)[11]

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3\times4} - \frac{1}{3\times4\times34} = \frac{577}{408}$$

$$\therefore \frac{4x^2}{\pi} = \left[x + \frac{169}{408} xY \right]^2$$

$$\frac{2}{\sqrt{\pi}} = 1 + \frac{169Y}{408}$$

$$\therefore Y = \frac{52.8}{169} \text{ (assuming } \pi = \tfrac{22}{7}\text{)} \quad \therefore Y \approx \tfrac{1}{3}.$$

Actually the nearest approximation would be given by $Y=4/13$ and this fraction would have been suggested by *Śulbasūtrakāras* if the value of π was known to them as 22/7.

(iv) According to *B.S.S.* (1.59)[12], the relation between the length of the side (*say* x) of a square with that of the diameter (d) of a circle cf equal area is given by the expression :—

$$x = d \left[\frac{7}{8} + \frac{1}{8 \times 29} - \frac{1}{8 \times 29 \times 6} + \frac{1}{8 \times 29 \times 6 \times 8} \right] = 0.8787\, d$$

Area of square $= (0.8787\, d)^2$

Area of circle $= \pi\, d^2/4$

As the areas of the square and the circle are equal,

$$\frac{\pi d^2}{4} = 0.772\, d^2 \quad \therefore \pi = 3.088$$

As this formula was developed by reversing the formula for circling a square of equal area (*B.S.S.* 1.58) (*see* ref. 10), the value of π obtained is same. If the *Śulbasūtrakāras* were knowing the accurate value of $\pi = 22/7$ then the fraction of the half diagonal to be taken to determine the radius of the circle of equal area would have been taken as 4/13 and not 1/3. If they would have taken the fraction 4/13, the above formula of squaring a circle would have been obtained as derived below :—

$$R = \frac{x}{2} + \frac{4}{13} \left(\frac{x}{\sqrt{2}} - \frac{x}{2} \right)$$

$$d = 2R = x + \frac{4}{13} \left(\sqrt{2}x - x \right)$$

$$d = x \left(\frac{9 + 4\sqrt{2}}{13} \right)$$

$$\frac{d}{x} = \frac{1}{13} \left(9 + 4 \times \frac{577}{408} \right)$$

$$= \frac{1495}{1326}$$

$$\frac{x}{d} = \frac{1326}{1495}.$$

The figure 1495 in the denominator is written as 1496. The error is negligibly small.

$$\frac{x}{d}=\frac{1326}{1496}=\frac{1326/187}{1496/187}=\frac{7+\frac{17}{187}}{8}=\frac{1}{8}\left(7+\frac{1}{11}\right)$$

$$\therefore\ \frac{x}{d}=\frac{7}{8}+\frac{1}{11\times 8}$$

$$\therefore\ x=d\left(\frac{7}{8}+\frac{1}{11\times 8}\right)$$

If the *Śulbasūtrakāras* were knowing the value of π to be equal to 22/7, they would have arrived at the relation between the length of the side of the square and diameter of a circle, $x=d\left(\frac{7}{8}+\frac{1}{11\times 8}\right)$. It, therefore, seems that the value of π known to *Sūtrakāras* might be 3.088.

(v) *Mānava Śulbasūtra* (i. 27) states that a square of two cubit square is equivalent to a circle of one cubit and three *aṅgulas* (Datta, 1932. p 149) (*see* ref. 8).

$$4=\pi\left(1\frac{1}{8}\right)^2$$

$$\therefore\ \pi=3.16049.$$

(vi) According to *Mānava Śulbasūtra* (Mazumdar, 1922)[13] the dimensions of *Gārhapatya*, *Āhavanīya* and *Dakṣiṇāgni* are as given below :—

Āhavanīya is a square of one *Aratni*, *Gārhapatya* is a circle of radius 14 *aṅgula* minus 1 *Yava* ($13\frac{7}{8}$ *aṅgula*), and *Dakṣiṇāgni* is semicircular of radius $19\frac{1}{2}$ *aṅgulas*.

Area of *Āhavanīya*=Area of *Gārhapatya*

$$\therefore\ \pi\left(13\frac{7}{8}\right)^2=57659\text{ sq. } aṅgulas$$

$$\therefore\ \pi=2.99.$$

Area of semicircular *Dakṣiṇāgni*=Area of *Gārhapatya*.

$$\therefore\ \frac{\pi(19.5)^2}{2}=576.$$

$$\therefore\ \pi=3.029.$$

(vii) *Mānava Śulbasūtra* (10. 1. 1. 8)[14] gives a construction to draw a semicircle of area equal to that of the given square. *ABCD* is a given square Draw a circle of area equal to that of the square *ABCD* with *O* as centre. Draw a square *EFGH* circumscribing the circle. Join *OF*. Draw a semicircle of radius OF. It has an area equal to that of the square $ABCD$.

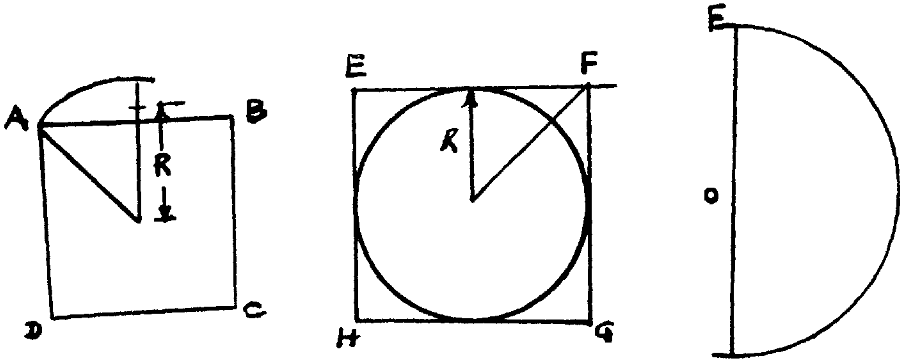

FIG. 3

Let us assume that the length of the side of a square = x. The radius of the circle of equal area = $\frac{\sqrt{2x}+2x}{6}$ = say R. The length of the side of the circumscribing square = $\sqrt{2}R$. The length of its one-half hypotenuse (i.e. OF) = $\sqrt{2}R$. A semi-circle of radius $\sqrt{2}R$ is drawn.

The area of the semicircle = area of the square of the square of side x.

$$\frac{\pi\,(\sqrt{(2)}\,R)^2}{2} = x^2 \;.\; R = \frac{\sqrt{(2x)}+2x}{6}$$

$$\therefore\; x^2 = \pi\left[\frac{\sqrt{(2x)}+2x}{6}\right]^2$$

$$\therefore\; \pi = 3{\cdot}088.$$

It is obvious that for developing the construction for drawing a semi-circle of area equal to that of the given square *Mānava Sūtrakāra* had assumed the value of $\pi = 3{\cdot}088$.

(viii) *M. S. S.* (10. 3. 2. 15)[16] had given a construction for circling a square of equal area. " He shall divide a square into nine parts the segments (of the circumscribed circle) into three parts each (by lengthening the lengths of the parts of the square), he shall remove the fifth part from the height (of such a lengthened line measured from its middle), together with dust (the circle with as radius the line from centre to the mark at the fifth part will be) as large as (the square)."

Let us assume that the length of the side of the given square is x. The circle circumscribing the square is say of radius R (= $x/\sqrt{2}$). Divide the square into nine small squares of equal area. Line BA was extended to cut the circle at C, $AC = Y$ (say). Draw OD perpendicular from the centre O of the circle to the side GE of the square and extend it to meet the circumference of the circle at F.

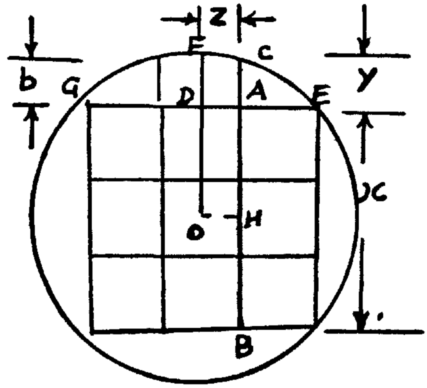

FIG. 4

$$AD = FC = Z; \quad DF = b.$$

$$AD = z = \frac{x}{2} - \frac{x}{3} = \frac{x}{6}$$

$$DF = b = R - \frac{x}{2} = \frac{x}{\sqrt{2}} - \frac{x}{2}$$

$$CA = y = b - R + \sqrt{R^2 - z^2}$$

$$= \frac{x}{\sqrt{2}} - \frac{x}{2} - \frac{x}{\sqrt{2}} + \sqrt{\frac{x^2}{2} - \frac{x^2}{36}}$$

$$= \sqrt{\frac{17}{36}x^2} - \frac{x}{2} = x\left(\frac{4\frac{1}{8}^*}{6} - \frac{1}{2}\right)$$

$$= \frac{3}{16}x.$$

The radius of the circle of area equal to that of the square

$$= \frac{x}{2} + \frac{3}{16}x - \frac{1}{5} \times \frac{33}{48}x = \frac{11}{20}x$$

The area of this circle $= \pi\,[\frac{11}{20}x]^2 =$ area of square of side $x = x^2$

$$\therefore \quad \pi = \left(\frac{20}{11}\right)^2 = 3.3.$$

* [$\sqrt{17} = \sqrt{16+1} = 4 + \frac{1}{8} = 4\frac{1}{8}$].

It is found that instead of decreasing the length *HC* by its 1/5th, if it is decreased by 1/6th * then a better approximation of value of π is obtained.

With this correction,

$$\text{the radius of the circle} = \frac{x}{2} + \frac{3}{16}x - \frac{1}{6} \times \frac{33}{48}x = \frac{165}{288}x.$$

$$\text{The area of the circle} = \pi\left(\frac{165}{288}x\right)^2 = \text{area of square} = x^2$$

$$\therefore \quad \pi = \left(\frac{288}{165}\right)^2 = 3.047.$$

Table 1 below gives the value of π as obtained by above given different constructions. It appears that the value of π known to *Śulbasūtrakāras* was 3.088 and not the modern value 3.142.

TABLE 1

	Reference	Value of π
i)	*B. S. S.* 1.113	3.00
ii)	*B. S. S,* 1.60	3.004
iii)	*B. S. S.* 1.58	3.088
iv)	*B. S. S.* 1.59	3.088
v)	*M. S. S.* 1 27 (Datta 1932)	3.16049
vi)	*M. S. S.* (Mazumdar 1922)	2.99 and 3.029
vii)	*M. S. S.* 10.1.1.8	3.088
viii)	*M. S. S.* 10.3.2.15	3.047.

CONCLUSION

The value of π known to Baudhāyana seems to be 3.088. The period of *Baudhāyana Śrautasūtra* is 800 **B. C.** The latest *Śulbasūtra* is that of *Kātyāyana*, B. C. **200**. This means that there was no improvement in the value of π as given by Baudhāyana upto **B. C**. 200. It is Āryabhaṭa (I) of 476 **A. D.** who had given the accurate value of $\pi = \frac{3927}{1250} = 3.1416$.

REFERENCES

1 *Tripādapariṇāhāni yūpoparūṇiti B.S.S.* (1.113).

2 Mackay, E. (1948), *"Early Indus Civilization"*. Second Edition, Luzac & Co. Ltd., London.

* The line *Utsedhāt pañcamam lumpet*......should be *Utsedhāt ṣaṣṭhamam lumpet*......

The manuscript of *Mānava S. S.* should be again examined and seen whether there is a mistake by the writer. Such mistakes are observed in *M. S. S.* For example in *sūtra* 10. 3. 3. 5, instead of the word *ekādaśam* the word *dwādaśam* should be mentioned. (See von Gelder 1963).

3 The chariot of Aśvinīkumāras was triangular having three seats and three wheels. (*R.V.* I. 34. 2 ; I. 118. 1–2)

A chariot that could carry seven or eight persons is described (*R.V.* IV. 2.4)

A chariot so large as to accomodate 67 persons is also mentioned (*R.V.* III. 6.9). Chariots with one horse (*R.V.* V. 56.6) or with pairs of horses from one to five are described (*R.V.* II. 18.4). Two types of wheels for chariots are mentioned. A wheel formed of hub, spokes, felly and rim (*R.V.*I. 32.17). Another is a solid wheel (*R.V.* II. 39.4).

4 *Namastakṣabhyo rathakārebhaśca namonamah. Yajurveda* (16.27).

5 *R.V.* IV. 16.20 ; X. 39.14, where Bhṛgu is mentioned as chariotmaker.

6 Oldenberg, "*Religion des Veda*". *S.B.E.* Vol. XXX, No. 2. P. 348 (from Datta B.B. 1932),

7 *Gārhapatyena santya ṛtunā yojdnanirasī.* *R.V.* I. 15.12. *R.V.* VI. 15.19 ; X. 85.27 mention of *Gārhapatya* fire is found.
The mention of three types of sacred fires is made in *R.V.* V. 11.2.

8 Datta, B.B. (1932), "*The science of the Śulba, A study in early Hindu Geometry*". University of Calcutta.

9 *Api vā pañcadaśabhāgānkṛtvā dwāwudharedeṣānityā caturaśrakaraṇī.* *B.S.S.* (1.60).

10 *Caturaśram maṇḍalam cikīrṣannkṣṇayārdham madhyāt prācimabhyapātaydyadatiśisyate tasya saha tṛtīyena maṇḍalam parilikhet.* *B.S.S.* (1.58).

11 *Pramāṇam tṛtiyena vardhayet tat ca caturthenātmac'atustriñśonena.* *B.S.S.* (1.61).

12 *Maṇḍalam caturaśram cikirṣan viṣkambhamastaubhāgān kṛtvā bhagamekonatriñśadhā vibhajyaṣtaviñśatlbhāgānudharet bhāgasya c'a ṣaṣthamaṣṭamabhāgonam.* *B.S.S.* (1.59).

13 Mazumdar, N. K. (1922), *Mānava Śulba Sūtram, Journal of the deptt. of Letters*, Vol. III, P. 327-342, University of Calcutta.

14 *Madhyātkoṭipramaṇena maṇḍalam parilekhayet* ।
Atiriktatribhāgena sarvam tu sahamaṇḍlam ।
Caturasre akṣṇayā rajjurmadhyatah sannipātayet ।
Parilekhya tadardhenārdhamaṇḍalameva tat ।
M.S.S. (*10.1.1.8*).

15 *Caturasram navadhā kuryāddhanuh kotyāstridhatridhā* ।
Utsedhāt pañcamam lumpet puriṣeṇeh tavātsamam ।
M.S.S. (*10.3.2.15*).