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ARTICLE

Mahādevī-sāriṇī: A unique table providing true longitudes of planets

B. S. Shubha[1] · B. S. Shylaja[2]

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Abstract
Finding the true longitudes of planets had always been a challenge for early astronomers. As keen observers, they noted the small drift of the planets from the calculated positions and rectified them from time to time. Such efforts are traceable in many texts. One such table is the *Mahādevī-sāriṇī* of the fourteenth century. The paper discusses the methodology of reading the true positions directly from the table using the mean positions and compares it with some later manuals of the seventeenth century. An example of the application of the conjunction of December 21, 2020, reveals that these tables for true longitudes are relevant even today. The efforts to improve accuracies achievable from naked-eye observations can be traced.

Keywords Indian astronomy of fourteenth century · *Mahādevī-sāriṇī* Planetary longitudes · Planetary tables · *Brahmatulya Udāharaṇam* · Conjunction of December 21, 2020

1 Introduction

The names of Āryabhaṭa, Brahmagupta, and Bhāskarācārya are well known for their contributions to astronomy and mathematics. The legacy of the work done in these fields continued in several areas across the Indian continent for the next 1000 years and beyond. Unfortunately, most of these works remain unknown to the academicians for various reasons. In the last century, many palm-leaf manuscripts have been edited and published, bringing to light the contributions of many astronomers. One such work is by Mahādeva of the early fourteenth century.

One of the important tasks of the astronomers was to prepare the almanacs giving the positions of the sun, the moon, and the planets, which were utilized for marking the dates of important events. Based on the *siddhāntic* texts like the *Sūryasiddhānta* (*SS*, hereafter), handy manuals were prepared every year to render the computations easier. These guides were referred to as *sāriṇī* or *koṣṭaka*. Some of the widely used tables include *Makaranda-sāriṇi*, (Uma et al., 2022, p. 104) and *Karaṇakutūhala-sāriṇī* (Shailaja et al., 2022, p. 73). Amongst these one such *sāriṇī* is *Mahādevī-sāriṇī* also known as *Gṛahasiddhi* by the author Mahādeva (Dikshit, 1981, vol. II, p. 123). The preparation of the tables was initiated by Cakreśvara and then continued by Mahādeva. This is indicated in a preliminary study of *Mahādevī-dīpika*, a manual for these tables (Shubha et al., 2022, p. 147). From the value of *palabha* (equinoctial shadow) of 4.5, we know that he lived at latitude 24.8N and a reference to the name Sirohi (a southern town in the state of Rajasthan) confirms it. *Mahādevi-sāriṇī* tables were brought to light by Neugebauer and Pingree in the year 1967 (pp. 69–92) providing an explanation for all the rows and columns in great detail. They had arrived at the arithmetic progression of the first row. This is clearly the increase in the mean value for the sun every 14 days at the mean *gati* of 59′ 8″ per day. Thus, the whole year is divided into 27 units leaving the last one as a fraction.

2 The mean and true values of longitudes

The general procedure for computations of the true longitudes as mentioned *SS* in the chapter entitled *spaṣṭādhikāra* (Balachandra Rao & Uma, 2008; Burgess, 1860, pp. 47–59; Dikshit, 1981; Uma et al., 2022) can be briefly summarised

B. S. Shubha
shubhashivaniyer@gmail.com

B. S. Shylaja
shylaja.jnp@gmail.com

1 Purnapramati Centre for Integrated Learning, Girinagar, Bengaluru 560085, India

2 Jawaharlal Nehru Planetarium, High Grounds, Bengaluru 560001, India

as (i) calculation of the total number of revolutions since the epoch for the given day, (ii) finding the mean positions of planets assuming a uniform motion, (iii) applying the correction for the elliptical orbit (*manda* or first equation), and (iv) applying the correction for the relative motion of the sun and the planets (applicable only for the five planets referred to as *tārāgraha*; star-like planets) (*śīghra* or second equation). It should be noted that the sequence cannot be integrated into a single series or progression. Each step will have to be carried out based on the result of the previous step. All the authors have stressed the need of applying corrections from time to time. For example, Nīlakaṇṭha stated (Ramasubramanian & Sriram, 2011, pp. xxxvi–vii):

> One has to accept that [each of] the five *siddhāntas* had been authoritative at one time [though they might not be so now]. Therefore, one has to look for a system which tallies with observation. The said tallying has to be verified by contemporary experimenters at the time of eclipses etc. ...the correlation of the computed Moon etc, with actual observation at a particular place, the revision of computation on the basis of such correlation, logical inference there from being transmitted as tradition, it being again correlated [with observation and again revised] and transmitted further down to others– this is how tradition is continued without interruption, and hence its [continued] authoritativeness.

It is also evident in handy manuals prepared at the beginning of the year. One such correction was discussed in *Brahmatulya Udāharaṇam*, called *Rāmabīja* (Shubha, 2020, pp. 50–56) and attributed to an astronomer Rāmacandra.

Generally, all texts have the simple table for obtaining the sine function in steps of 10° which are needed for the two corrections mentioned above.

The calculations of these steps are unavoidable. *Sāriṇīs*, for example, *Makaranda-sāriṇī*, provides only the corrections of the first and second equations; the astronomer is expected to carry out the rest of the calculations on his own (Uma et al., 2022). It is rare to find the true values tabulated.

The blue line represents M_a, the *mandocca*. M is the mean position, taken as 0, in this example. Red lines correspond to the first equation corrections (M with suffixes). Black lines correspond to the second equation corrections. The calculated values are included in Table 1. Fig. 1 explains the meaning of the first correction, which is decided by the angle between the mean positions of Mars and the Sun, in this example. M is the mean position which is taken as 0°, the apogee is marked as M_a. After the first correction is decided by the angle MOM_a, the position is M_1, which is used to get the second correction to get the position as S_1. This point is used for the next step, MOM_1 decides the value of MOS_2. In the next step, MOS_2 fixes M_3 and then S_3, and so on. This process of iteration is termed *asakṛt* as defined in *Brahmatulya Udāharaṇam*. We calculated following this procedure and the result is provided in the last column of Table 1.

For *Kuja*, (or *Bhauma,* Mars) the step involves half corrections in the first iteration as shown below. The date of calculation is *Vikrama samvat* 1676 (*Ś Śaka* 1541) *Jyeṣṭha kṛṣṇa caturdaśī,* Sunday corresponding to May 15, 1612 CE.

Ahargaṇa value = 156762
Madhyama Ravi value = $1^R|4°|14'|35''$
Madhyama Bhauma value = $9^R|27°|40'|39''$
Manda kendra (M) =
Mandocca − Mean value (*Mandocca)* = $3^R|8°|30'$
Manda kendra of *Bhauma* = $6^R|9°|41'|25''$
Dorjyā = $20°|37'|46''$

$$Mandaphala = (a \div r)\, sinM \tag{1}$$

where a = periphery of *Kuja* = 72° and r = 360°

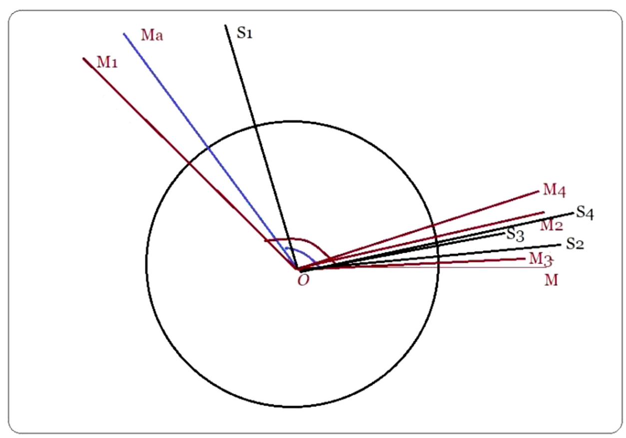

Fig. 1 The steps of iteration to arrive at the final position S_4 with O as the observer on Earth. The order of iteration is indicated by the numbers, M for mean positions and S for second equation corrected ones

Table 1 *Jyākhaṇḍas* (Rsine tables; R = 120)

Aṅkas	1	2	3	4	5	6	7	8	9
Differences	21	20	19	17	15	12	9	5	2
Jyākhaṇḍa	21	41	60	77	92	104	113	118	120

Source: *Karaṇakutūhala* of Bhāskarācārya II by S. Balachandra Rao and S.K. Uma

Table 2 The value of true longitudes from *Brahmatulya Udāharaṇam* and *Makaranda-sāriṇī* for *Ś Śaka* 1541 *Jyeṣṭa kṛṣṇa caturdaśī*, Sunday corresponding to May 15, 1612 CE. compared with the value from *Mahādevī-sāriṇī*

	Mahādevī-sāriṇī	***Brahmatulya Udāharaṇam***	***Makaranda-sāriṇī***	**Our calculation using BU procedure**
Mean Value	9^R |27°= 297°	296° |42' |49"	296° |21' |13"	296° |42' |49"
	Interpolated From tables #47 and 48	I: 10^R |14°|49' |54"	I: 10^R |16° | 32'| 32"	I: 4^R |23°|30'|14"
		II: 11^R | 4° |10' |44"	II: 11^R | 4° | 49'|34"	II: 11^R|11°|35'|23"
		III: 11^R | 4° |24' |44"		III: 11^R| 6°|51'|22"
		IV: 11^R | 4° |24' |47"		IV: 11^R| 6°| 2'|25"
		V: 11^R | 4° |24' |47"		
		VI: 11^R | 4° |24' |47"		V: 11^R | 4° | 6' |22"
True Value	**330° | 6' |27"**	**334° |24' |47"**	**334° |49' |34"**	**334° |6' |22"**

Mandaphala value (–) = 1°|55′|40″
Half of the above value (–) = 0|57′|50″
Mandaphala corrected value of *Bhauma* = 9^R|26°|43′|49″

Now *śīghraphala*

Bhauma śīghra kendra = 3^R|7°|30′|46″
Bhuja of the above value = 2^R|22°|29′|14″
Koṭi of the above value = 0^R|7°|30′|46″
Jyā of the above value = 15°|46′|36″.
Parākhya = 81°

Śīghrakarṇa

$$= \sqrt{(parākhya^2 \pm 2 \times parākhya \times koṭijyā + 14400)} \tag{2}$$

$$Śīghraphala = \frac{(parākhya \times bhujajyā)}{Śīghrakarṇa} \tag{3}$$

Śīghraphala = 36°|12′|11″
Half of the a11bove value = 18°|6′|5″

Adding this value to the *manda* corrected *Bhauma* value
From (3) above, *manda* corrected value of *Bhauma is* 9^R|26°|43′|49
First *Śīghraphala* corrected value of *Bhauma* = 10^R|14°|49′|54″
(I iteration value in Table 2)

Now the second iteration
Mandakendra of *Bhauma* = 5^R|8°|48′|41″
Dorjyā = 20°|37′|46″
Mandaphala value (–) = 4°|2′|34″
Half of the above value = 2°|1′|17″
Mandaphala corrected value of *Bhauma* = 9^R|29°|42′|56″
Bhauma śīghrakendra = 3^R|4°|31′|39″
Bhuja of the above value = 2^R|22°|29′|14″
Koṭi of the above value = 0^R|4°|31′|39″

Parākhya = 81°
Using Eqs. (2) and (3)
Śīghraphala = 35°|23′|55″
Adding this value to the *manda* corrected *Bhauma* value we get
Manda spaṣṭa value = 11^R|4°|10′|44″ (II iteration value in Table 2)
and the procedure continues. Thus, we have,
I iterated value of *mandaspaṣṭa Bhauma* is 10^R |14°|49′|54″
II iterated value of *mandaspaṣta Bhauma* is 11^R|4°|10′|44″
III iterated value is 11^R|4°|24′|44″
IV iterated value is 11^R|4°|24′|47″
V iterated value is 11^R|4°|24′|47″. These are listed in Table 2.

Note: The above procedure demonstrates that the corrections were repeated successively until the values converged.

Final value of *Spaṣṭa Bhauma* is **11^R|4°|24′|47″.**

For obtaining the true longitude of the planet the following are the steps followed in *Sūryasiddhānta* (Shukla, 1974), and this procedure is the same as followed in *BU* as shown above:

1. Half *mandaphala* is used for the correction.
2. Half of *śīghraphala* is added to the longitude of the planet.
3. The entire *mandaphala* calculated with step 2 is added to the iterated mean value.
4. The entire *śīghraphala* is added.

Fig. 2 The first page of the *sāriṇī* showing the tabulations for the mean values 0° and 6° of *Bhauma* (Mars), referred to as M1 in the text

We recalculated the corrections for all the steps and these are also included in Table 2. As shown above, *Brahmatulya Udāharaṇam* (Shubha, 2020, pp. 50–56) achieves the final result after 6 iterations. *Makaranda- sāriṇī* provides 2 steps of iterations (Uma et al., 2022, p. 104).

The entire procedure is eliminated by using *Mahādevī-sāriṇī*. The mean value, *Madhyama Bhauma* listed above, of 297° takes us to tables 47 (296°) and 48 (302°). We have to interpolate for the intermediate value of 297° to get 330°|6′|27″ as the true value. This is in the second column of Table 2.

Let us understand the rationale.

In Table 2, the true longitudes for the examples provided from two different texts *Brahmatulya-udāharaṇam* and *Makaranda-sāriṇī* for *Ś Śaka* 1534 *Jyeṣṭha kṛṣṇa caturdaśī*, Sunday corresponding to May 15, 1612 CE, are compared with the value read from the table of *Mahādevī-sāriṇī*. Our calculations following the method of *Karaṇakutūhala* also are included in the last column.

3 The tables of *Mahādevī-sāriṇī*

Mahādevī-sāriṇī can be considered a unique manual since it provides the true values of the longitudes of the planets directly readable from the mean positions arranged in increasing order. The *sāriṇī* was written during the epochal year 1238 *Śālivāhana Śaka* corresponding to 1316 CE. This would be the epoch of the tables too. However, the methodology renders it applicable to any year, since the mean values with all the necessary corrections incorporated are applied. The idea of fixing the mean value of sun at 0, for tabulating the true values appears very innovative. Since the value may not be 0 on the equinox day, (unlike the true value) a corresponding correction can be added later on.

The tables for all the planets are arranged at fixed intervals. The initial positions of the mean longitude of the Sun and the planet are set to 0°. The first column entry is at the end of 14 days. Subsequent columns are set at intervals of 14 days so that each table covers the entire year. The second column provides the true position at the end of 28 days, the third at the end of 42 days, and so on.

The next table has the initial position (called *cālaka*) of the Sun at 0° and that of the planet has been set at 6°. The columns are again set at intervals of 14 days. The subsequent tables are for mean positions 12°, 18°, and so on. Thus, there are 60 tables for each planet [360/6, with λ in steps of 6°].

The columns of this *sāriṇī* also provide the instantaneous speeds of the planets in the successive rows so that the interpolation is made easy. In the previous section, we have explained the derivation of the true longitudes. It is compared with similar procedures from the other texts. Here we are discussing the planetary tables of the five planets. A sample of the table is shown in Fig. 2, from manuscript number 429 of 1895–98 in folio 27 of the archival collections of Bhandarkar Oriental Research Institute, Pune. The manuscript does not start with any introductory lines or any text explaining the contents.

The hard work of the author, which becomes immediately apparent looking at the total number of tables, leaves us spellbound. It is a mammoth task. For every planet there are 60 tables and every table has 14 columns—every column has four different quantities. All the entries have been meticulously (hand) calculated by the author.

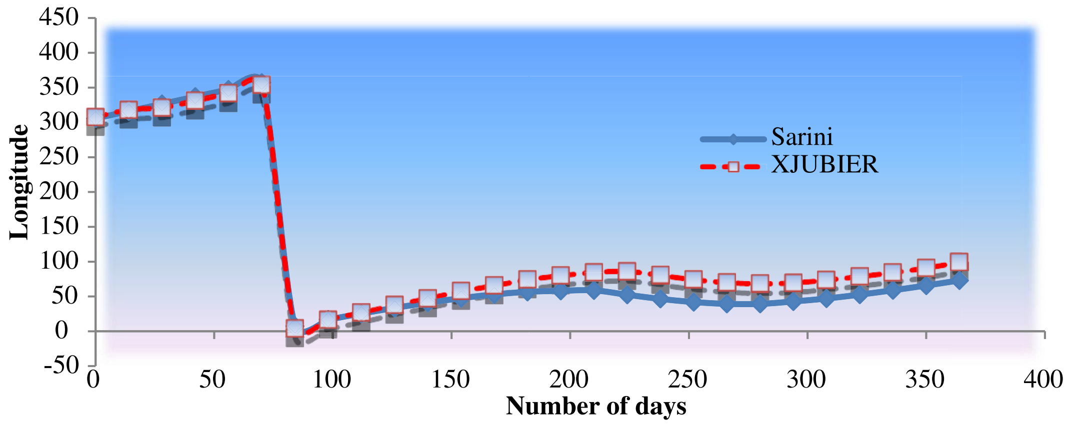

Fig. 3 The longitude of *Kuja* (Mars) for the year 1612 CE values from *sāriṇī* compared to those from Xjubier

The first entry of Table M1 from the manuscript provides the true value for the mean value of sun = 0 and the mean value for the planet = 0. The next column is for the sun having moved in 14 days to 13°| 47′| 27″. The mean and true values for *kuja* would have changed accordingly. The consecutive columns increase the longitude in steps of 14 days. Thus, the true positions are available for the whole year.

The next one, Table M2, from the manuscript, repeats this pattern for the mean value of the planet as 6°; the last column is for the year-end. The next ones are for 12°, 18°, 24° and so on, to cover all the possible values from 0 to 360° of the mean longitude.

Therefore, the procedure would be:

a. To get the mean values of the sun and the planets for the beginning of the year
b. To calculate the planet's mean value (p) for the date corresponding to the mean value of the sun = 0.
c. To reach the table corresponding to the mean value of the planet as p.
d. Move to the column corresponding to the desired date in the table.
e. Interpolate between this column and the next column using the instantaneous *gati* (speed) provided beneath.
f. Interpolate between the tables if the mean value is not a multiple of 6.

We have tabulated the result for the specific example of the previous section in the second column in Table 2. The mean value of the sun for the beginning of *Śaka* year was 0^R|0°|30′|10″. This correction was added to the extrapolated values from the *sāriṇī*. The second and third are from different texts *Brahmatulya Udāharaṇam* and *Makaranda-sāriṇī* of the seventeenth century. The last column is the result of the procedure described in the previous section.

4 Discussion on the tables of *Mahādevī-sāriṇī*

The tabulated results from the manuals of different periods show that there has always been a constant effort on achieving accuracy. The differences in the results presented in the last row can be attributed to the extrapolations for small angles in using sine tables. Moreover, the *manda* corrections have been arrived at by using a variable epicycle radius (Uma et al., 2022; personal communication, Balachandra Rao et al., 2008, S3–S12).

The software Stellarium provides 352°66′; Xjubier gives 352°36′.[1] These are the *sāyana* (precession corrected) values obtainable with *ayanāmśa* of 18°|9′. We have also calculated the longitudes for the *Budha* (Mercury). We find that the differences are more pronounced. Between *Brahmatulya Udāharaṇam* (1^R|16°) and *Makaranda-sāriṇī* (1^R 19°), there is a difference of 3 degrees. *Mahadevi-sāriṇī* value (1^R|22°) does not concur with any of these. We observed a similar trend for *Budha* in the context of stationary points in retrograde motion calculations for *Budha* (Shubha & Shylaja, 2020, pp. 40–48).

Neugebauer and Pingree, (1967, pp. 69–92) have discussed the tables for the Sun, Moon, and *Rāhu* (the node). The copy that we procured is confined to the 5 planets. They also commented on the 'countless scribal errors' in the tables. The copy that we studied is relatively free of such errors as we observed in this verification process. Further, they had access to yet another set of tables for the mean values.

The *sāriṇī*, as explained earlier, provides true values for the given mean values. We have compared the mean values in Table 2, which demonstrates the completeness of the procedure. Starting with the mean value of 9^R|27°, we looked up the corresponding tables and interpolated them as per the difference in the fraction of the degree.

[1] http://xjubier.free.fr/en/site_pages/astronomy/ephemerides.html.

Fig. 4 Saturn, top, and Jupiter, below, are seen after sunset from Shenandoah National Park, Sunday, Dec. 13, 2020, in Luray, Virginia. The two planets are drawing closer to each other in the sky as they head towards a “great conjunction” on December 21, where the two giant planets will appear a tenth of a degree apart. *Credits: NASA/ Bill Ingalls*

We extended the application and verified the longitudes for one whole year. Fig. 3 shows the longitude of Mars for 1612 CE as derived from the *sāriṇī*. We have included in the graph the longitudes with the computations provided by Xjubier for comparison.[2]

Planetary conjunctions can be used for verifying the accuracies of the tables and methods of calculations. We are providing an example of the use of these *sāriṇī* tables corresponding to the conjunction of Jupiter and Saturn which was witnessed all over the world on 21st December 2020. It had a separation of 6′ and the two dots were resolved for naked eye visibility. (Fig. 4).

We calculated the mean values of both planets for the year 2020 and reached the corresponding tables. (Fig. 5).

The conjunction of the planets is quite clear from this exercise. The x-axis represents the number of blocks of 14 days as given in the *sāriṇī*. Dotted curves Jupiter (S) and Saturn (S) are from the *sāriṇī*. The ones with the suffix X are from the ephemeris shown as continuous lines. There are two obvious inferences from the figure–the vertical shift by about 100 degrees and the horizontal shift by 70 days. These may be explained as follows:

a. The tables are not corrected for precession. Hence *ayanāṃśa* of 24° needs to be added.
b. The tables of the *sāriṇī* are prepared for the mean sun = 0. However, for the year 2020 on March 21st, the mean was not 0 but 287°|45′|58″. This corresponds to a difference of about 74°.

The total shift would be 98°. This explains the vertical shift. #2 is also the reason for the horizontal shift of 74 days for the day of conjunction.

Thus, these tables are applicable for any year and therefore, offer a great opportunity to study the development of computational practices.

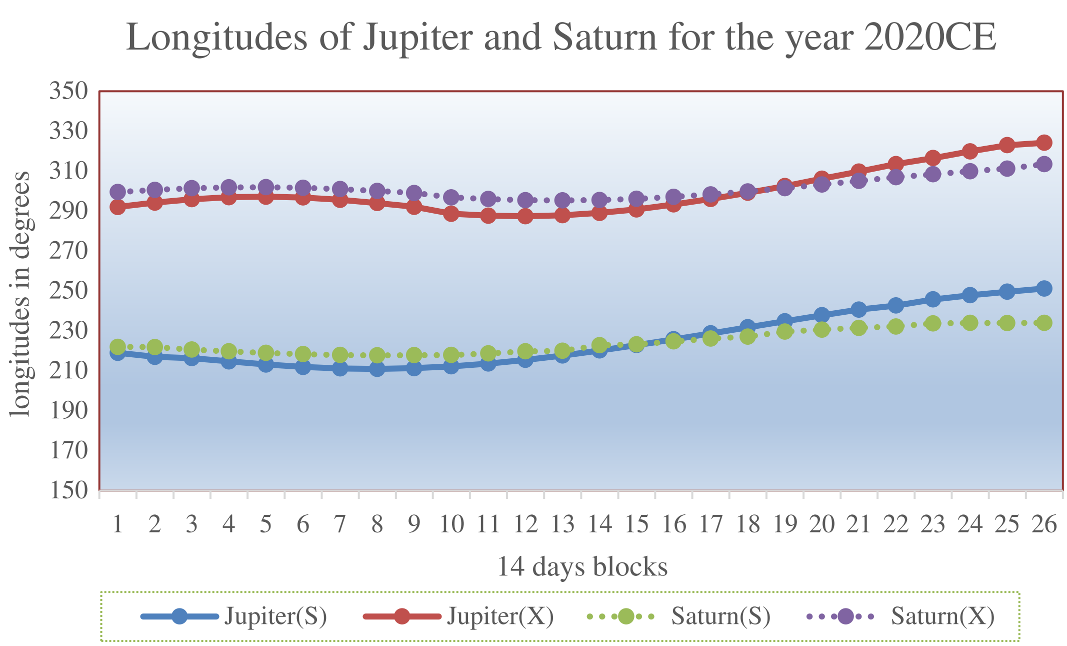

Fig. 5 Comparison of the longitudes of Jupiter and Saturn from the *sāriṇī* (suffix S) to check the conjunction compared with the values from Indian Astronomical Ephemeris (with suffix X)

[2] http://xjubier.free.fr/en/site_pages/astronomy/ephemerides.html.

The study is confined to the five planets available in this manuscript. Any discovery of a similar work of the thirteenth century (among thousands waiting for their turn) would offer great scope for a comparative study. As shown above, *Mahādevī-sāriṇī* is relevant for any year and can be a good starting point for understanding computations.

We plan to provide these tables as an interactive online application for all the planets for future users.

5 Conclusion

We have worked out the procedures for the preparation of the tables *Mahādevī-sāriṇī*, which is unique in providing true longitudes readable directly from the mean values. In comparison with other methods of calculation for Mars, we find small differences which can be explained by various factors such as the difference in the sine tables and the precession. This study also hints at the evolution of the methods of computation. We have verified the application of the tables to the planetary conjunction of December 21, 2020, CE, and discussed the relevance of the tables for any year of interest.

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Declarations

Conflict of interest Not applicable, Both the authors do not have any such intention.

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