Indian Journal of History of Science (2023) 58:1–12
https://doi.org/10.1007/s43539-023-00075-3

ARTICLE

Calculation for 'chain-reduction' in the *Triśatībhāṣya*

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Received: 9 November 2022 / Accepted: 25 January 2023 / Published online: 29 March 2023

Abstract
The *Triśatībhāṣya* is an anonymous commentary on Śrīdhara's *Triśatī*. 'Chain-reduction' (*vallīsavarṇana*) is a rule for unifying quantities expressed in several units into the highest one, but the usage of the rule in the *Triśatībhāṣya* is slightly different. The present paper tries to explain, by comparison with the procedures illustrated in other arithmetic texts, why the commentator applies the 'chain-reduction' in an irregular way.

Keywords Chain-reduction (*vallīsavarṇana*) · *Triśatī* · *Triśatībhāṣya* · *Pāṭīgaṇitaṭīkā* · Siṃhatilaka

Abbreviations

A_1	LD Institute, Ahmedabad, 1559.
BM	*Bakhshālī Manuscript*
GSK	*Gaṇitasārakaumudī* of Ṭhakkura Pherū
GT	*Gaṇitatilaka* of Śrīpati
K_{ED}	Kāśī edition of the *Triśatī*
MS	*Mahāsiddhānta* of Āryabhaṭa II
PG	*Pāṭīgaṇita* of Śrīdhara
PGṬ	*Pāṭīgaṇitaṭīkā* (anonymous comm.) on the PG
SGT	Siṃhatilaka's comm. on the GT
SŚ	*Siddhāntaśekhara* of Śrīpati
Tr	*Triśatī* (alias *Triśatikā* and *Gaṇitasāra*) of Śrīdhara
TrBh	*Triśatībhāṣya* (anonymous comm.) on the Tr

1 Introduction

The *Triśatībhāṣya* (hereafter TrBh) is an anonymous commentary on the Sanskrit arithmetic text *Triśatī* (hereafter Tr) by Śrīdhara (ca. 800 CE). The TrBh is available only in a single complete manuscript (LD Institute, Ahmedabad, 1559: hereafter A_1) and is not contained in the edition published at Kāśī (hereafer K_{ED}).[1] In my recent study, I investigated the date and the place of the author of the TrBh through an analysis of the linguistic features, and concluded that he flourished in Western India some time between the twelfth and fifteenth centuries CE.[2] The Tr presents arithmetic rules and examples briefly. On the other hand, the TrBh explains the computational procedures in detail.

'Chain-reduction' (*vallīsavarṇana*) is a rule for unifying quantities expressed in several units into the highest one, but the usage of the rule in the TrBh is slightly different. The present paper, by comparison with the procedures illustrated in other arithmetic texts, attempts to explain why the commentator applies the rule for 'chain-reduction' in an irregular way.

For that purpose, first, I will give a brief explanation of the rule for the 'chain-reduction.' Then, I will present the computational procedures in modern notation on the basis of the descriptions of the TrBh, the prose parts of the Tr, the *Pāṭīgaṇitaṭīkā* (hereafter PGṬ), and Siṃhatilaka's commentary (hereafter SGT) on the *Gaṇitatilaka* (hereafter GT) successively.

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[1] Kāśī is a historical name of the present-day Varanasi. In the present paper, verse numbers of the Tr follow K_{ED}, and I utilize the numbering system as follows: 1) Pbh*n* is assigned to definitions [of number words and weights and measures] (*paribhāṣā*), 2) *n* to rules, 3) E*n* to examples, and 4) *n*p to the prose commentary that occurs immediately after the *n*-th verse.

[2] See Tokutake (2022b)

2 Rule for 'chain-reduction'

Tr 26cd—27ab prescribes the rule for the 'chain-reduction' as follows:[3]

> Tr 26cd—27ab:
>
> प्राक्छेदांशौ गुणयेच्छेदेनाधःस्थितेन पूर्वांशे।
> धनमृणमधःस्थितांशं कुर्वीत सवर्णने वल्ल्याः॥
>
> *prākchedāṃśau guṇayec*
> *chedenādhaḥsthitena pūrvāṃśe/*[4]
> *dhanam ṛṇam adhaḥsthitāṃśaṃ*
> *kurvīta savarṇane vallyāḥ//*[5]
>
> In the 'chain-reduction,' one should multiply the former (the upper) denominator and numerator by the denominator located below. On the former (the upper) numerator, one should make the numerator located below positive [or] negative.[6]

When conversion ratios of four units U_1, U_2, U_3, and U_4 are p_1, p_2, and p_3 respectively, that is, $1\ U_1 = p_1\ U_2$, $1\ U_2 = p_2\ U_3$, $1\ U_3 = p_3\ U_4$, a quantity expressed in the four units can be converted to the highest unit by the following operation:

$$
\begin{aligned}
& a_1U_1 + a_2U_2 + a_3U_3 + a_4U_4 \\
&= \left(a_1 + \frac{a_2}{p_1} + \frac{a_3}{p_1p_2} + \frac{a_4}{p_1p_2p_3}\right)U_1 \\
&= \left(\frac{a_1p_1 + a_2}{p_1} + \frac{a_3}{p_1p_2} + \frac{a_4}{p_1p_2p_3}\right)U_1 \\
&= \left(\frac{(a_1p_1 + a_2)p_2 + a_3}{p_1p_2} + \frac{a_4}{p_1p_2p_3}\right)U_1 \\
&= \frac{\{(a_1p_1 + a_2)p_2 + a_3\}p_3 + a_4}{p_1p_2p_3}U_1,
\end{aligned}
$$

where a_1 is positive and a_i $(i > 1)$ is positive or negative.

This calculation is carried out on calculating board in the following way. First, one arranges a_i and p_i vertically and places 1 under a_1. This is called 'chain' (*vallī*). For four terms from the top of the chain, "one should multiply the former (the upper) denominator and numerator by the denominator located below" and "on the former (the upper) numerator, one should make the numerator located below positive [or] negative," that is, one should perform addition or subtraction. The lower two of the four terms are then erased, although this step is not explicitly stated in the text. The same operation is repeated until a single fraction is obtained.

$$
\begin{array}{|c|}
a_1 \\ 1 \\ a_2 \\ p_1 \\ a_3 \\ p_2 \\ a_4 \\ p_3
\end{array}
\longrightarrow
\begin{array}{|c|}
a_1p_1 + a_2 \\ p_1 \\ a_3 \\ p_2 \\ a_4 \\ p_3
\end{array}
\longrightarrow
\begin{array}{|c|}
(a_1p_1 + a_2)p_2 + a_3 \\ p_1p_2 \\ a_4 \\ p_3
\end{array}
$$

$$
\longrightarrow
\begin{array}{|c|}
\{(a_1p_1 + a_2)p_2 + a_3\}p_3 + a_4 \\ p_1p_2p_3
\end{array}
$$

3 TrBh

The TrBh applies the rule for 'chain-reduction' at three places: on Tr E22, E33 and E71–72. I will represent the procedure intended at each place. For English translations of the text, see Appendix 1.

3.1 TrBh on Tr E22

> Tr E22:
>
> पञ्च पुराणास्त्रिपणाः काकिण्येका वराटकेनोना।
> तत्पञ्चमभागोना सवर्णिते किं फलं भवति॥
>
> *pañca purāṇās tripaṇāḥ*
> *kākiṇy ekā varāṭakenonā/*[7]
> *tatpañcamabhāgonā*
> *savarṇite kiṃ phalaṃ bhavati//*[8]
>
> There are five *purāṇa*-s, three *paṇa*-s, one *kākinī*, decreased by one *varāṭaka* and one-fifth of that (one *varāṭaka*). When they are reduced to the same color,[9] what is the result?

[3] Hereafter, a brief explanation of a word in translation is marked with parentheses ( ), and additions to the translation with square brackets [ ]. As for notation in apparatuses, see Appendix 1.

[4] prākche°] K_{ED}, prākache° A_1; °yec chedenādhaḥsthi°] K_{ED}, °yet/chedenādhasthi° A_1

[5] ṛṇam adhaḥsthi°] K_{ED}, ūṇam adhasthi° A_1; kurvīta] K_{ED}, kuvvāta A_1; vallyāḥ] K_{ED}, valyaḥ A_1

[6] Cf. BM Q6; PG 41; MS 15.18; GT 62; GSK 2.12.

[7] tripaṇāḥ kā°] K_{ED}, tripuṇakā° A_1; °ṭakenonā] K_{ED}, °ṭakenyenā A_1

[8] savarṇite] A_1, samāsataḥ K_{ED}

[9] 'Reduction to the same colour' (*savarṇana*) means the reduction of a 'composite' fraction to a 'simple' fraction.

That is, 5 *purāṇa*-s + 3 *paṇa*-s + 1 *kākiṇī* − 1 *varāṭaka* − $\frac{1}{5}$ *varāṭaka* are to be unified into the unit of *purāṇa*.[10] The commentator first constructs a 'chain' from the given quantities and their conversion ratios:[11]

5	1	3	16	1	4	·1	20	·1	5

Then, he performs the calculation in the following four steps:

$$\frac{5}{1}+\frac{3}{16}=\frac{5\cdot 16+3\cdot 1}{1\cdot 16}=\frac{83}{16},$$
$$\frac{83}{16}+\frac{1}{16\cdot 4}=\frac{83\cdot 4+1}{16\cdot 4}=\frac{333}{64},$$
$$\frac{333}{64}-\frac{1}{64\cdot 20}=\frac{333\cdot 20-1}{64\cdot 20}=\frac{6659}{1280},$$
$$\frac{6659}{1280}-\frac{1}{5}=\frac{6659\cdot 5-1}{1280\cdot 5}=\frac{33294}{6400}=\frac{16,647}{3200}\ \textit{purāṇa}\text{-s}$$

In the first step, the commentator cites part of the stanza for the 'part-class' (*bhāgajāti*) prescribed in PG 37.[12] It is noteworthy that in A_1 (fol. 7b) this stanza is regarded as the rule of the Tr, although it is not contained in K_{ED};[13] and that the stanza is cited in SGT on GT 54.[14] At the moment there is too little evidence to determine whether Siṃhatilaka (second half of the 13th century CE), the author of the SGT, referred to the text of the Tr including PG 37 or that of the PG; and whether the verse of PG 37 was contained in the original text of the Tr. The calculation in the first step is carried out on calculating board in the following manner.

(i) The two fractions are placed each below the other:

5
1
3
16

(ii) The upper numerator (5) is multiplied by the lower denominator (16), and the lower denominator (16) is then multiplied by the upper denominator (1):

$5\cdot 16$
1
3
$16\cdot 1$

(iii) The product of the numerator (3) and the denominator (1) in the middle is added to the upper numerator (80):

$80+1\cdot 3$
1
3
16

(iv) The numerator (3) and the denominator (1) in the middle are erased:[15]

83
16

After obtaining the answer $\frac{16647}{3200}$ *purāṇa*-s, the commentator inversely calculates the original numbers expressed in the lower units:[16]

$$\frac{16647}{3200}=5\frac{647}{3200}\ \textit{purāṇa}\text{-s},$$
$$\frac{647\cdot 16}{3200}=\frac{10352}{3200}=3\frac{752}{3200}\ \textit{paṇa}\text{-s},$$
$$\frac{752\cdot 4}{3200}\left[=\frac{3008}{3200}\ \textit{kākiṇī}\right],$$
$$\frac{3008\cdot 20}{3200}=\frac{60160}{3200}=18\frac{2560}{3200}=18\frac{4}{5}\ \textit{varāṭaka}\text{-s}.$$

The calculation ends here, but of course,

[10] Tr Pbh4: षोडशपणः पुराणः पणो भवेत्काकिणीचतुष्केण। पञ्चाहतैश्चतुर्भिर्वराटकैः काकिणी चैका॥ *ṣoḍaśapaṇaḥ purāṇaḥ paṇo bhavet kākiṇīcatuṣkeṇa/ pañcāhataiś caturbhir varāṭakaiḥ kākiṇī caikā//** (* °kaiḥ] K_{ED}, °kai A_1; caikā] A_1, hy ekā K_{ED}) "One *purāṇa* is made up of sixteen *paṇa*-s, one *paṇa* of four *kākiṇī*-s, and one *kākiṇī* of four *varāṭaka*-s multiplied by five."

	va	*kā*	*pa*	*pu*
varāṭaka	1			
kākiṇī	**4·5**	1		
paṇa	80	**4**	1	
purāṇa	1280	64	**16**	1

[11] The negative sign, a dot (·), is attached to subtractive/negative numbers in the 'chain.' Here and hereafter, I rotated the tall boxes through 90° to save space.

[12] PG 37: अधरहरोर्ध्वांशवधश्चोर्ध्वहरेणाधरं <हरं> हन्यात्। मध्यांशहराभ्यासं <विनिक्षिपेदुपरिमांशेषु>॥ *adharaharordhvāṃśavadhaś cordhvahareṇādharaṃ <haraṃ> hanyāt/ madhyāṃśaharābhyāsaṃ <vinikṣiped uparimāṃśeṣu>//* "By the lower denominator multiply the upper numerator, (then) by the upper denominator multiply the lower denominator, and (then) add the product of the numerator and the denominator in the middle to the upper numerator." Translation by Shukla (1959, transl. p. 17). This rule is given not as 'addition of fractions,' but as 'part-class' in SŚ 13.12 and A_1. See Hayashi (2019, p. 339) and Tokutake (2021, pp. 155–158).

[13] See Tokutake (2021, pp. 77, 155–156).

[14] See Petrocchi (2019, pp. 129, 335–336) and Hayashi (2019, p. 196).

[15] This step is not mentioned in PG 37.

[16] In the following text, I mark procedures that is not mentioned in the original texts with square brackets [ ].

$$\left[18\frac{4}{5} = \left(20 - 1 - \frac{1}{5}\right) \textit{varāṭaka-s} = 1\ \textit{kākiṇī} - 1\ \textit{varāṭaka} - \frac{1}{5}\ \textit{varāṭaka}.\right]$$

The four steps after obtaining the answer, $\frac{16647}{3200}$ *purāṇa*-s, are probably meant to be a verification. Siṃhatilaka carries out a similar calculation in SGT on GT 63. I will discuss it in more details in Sect. 6.1.

3.2 TrBh on Tr E33

The following is an example for Rule of Three (*trairāśika*):[17]

Tr E33:

धान्याद्रोणः सारर्धः कुडवत्रितयं च लभ्यते ऽष्टाभिः।
तद्रोणयुक्तखार्याः किं मूल्यं कथ्यतामाशु॥

dhānyadroṇaḥ sārdhaḥ
kuḍavatritayaṃ ca labhyate 'ṣṭābhiḥ/[18]
tad droṇayuktakhāryāḥ
kiṃ mūlyaṃ kathyatām āśu//[19]

[If] one and a half *droṇa*-s and three *kuḍava*-s of grain are obtained for eight [units of money], then, how much is the price of one *khārī* increased by one *droṇa*? Say quickly.

That is,

$$\left(1\frac{1}{2}\ \textit{droṇa-s} + 3\ \textit{kuḍava-s}\right) : 8 = (1\ \textit{khārī} + 1\ \textit{droṇa}) : x.$$

The commentator converts the first term, $1\frac{1}{2}$ *droṇa*-s + 3 *kuḍava*-s, into *kuḍava* in the following two ways. First, he simply calculates the numbers of *kuḍava*-s in the *prastha*, *āḍhaka*, and *droṇa* successively.[20]

$$1\ \textit{prastha} = 4\ \textit{kuḍava-s},$$
$$1\ \textit{āḍhaka} = 16\ \textit{kuḍava-s},$$
$$1\ \textit{droṇa} = 64\ \textit{kuḍava-s};$$
$$64 + 64 \cdot \frac{1}{2} = 64 + 32 = 96\ [\textit{kuḍava-s}];$$
$$96 + 3 = 99\ [\textit{kuḍava-s}]$$

Secondly, he expresses $1\frac{1}{2}$ *droṇa*-s successively in smaller and smaller units, *āḍhaka*, *prastha*, and *kuḍava*.

$$1\frac{1}{2} = \frac{3}{2}\ [\textit{droṇa-s}],$$
$$\left[\frac{3}{2} \cdot 4 =\right] 3 \cdot 2 = 6\ [\textit{āḍhaka-s}],$$
$$6 \cdot 4 = 24\ [\textit{prastha-s}],$$
$$24 \cdot 4 = 96\ [\textit{kuḍava-s}];$$
$$96 + 3 = 99\ [\textit{kuḍava-s}].$$

That this calculation was carried out on calculating board by the 'chain-reduction' is suggested by the words 'increased by three located below' (*adhasthitatrikasahite*). This step occurs at the end of the following 'chain-reduction.'

$$\begin{array}{|c|}\hline 3/2\\ 1\\ 0\\ 4\\ 0\\ 4\\ 3\\ 4\\ \hline\end{array} \longrightarrow \begin{array}{|c|}\hline (3/2)\cdot 4+0\\ 1\cdot 4\\ 0\\ 4\\ 3\\ 4\\ \hline\end{array} \longrightarrow \begin{array}{|c|}\hline 6\cdot 4+0\\ 4\cdot 4\\ 3\\ 4\\ \hline\end{array}$$

$$\longrightarrow \begin{array}{|c|}\hline 24\cdot 4+3\\ 16\cdot 4\\ \hline\end{array} \longrightarrow \begin{array}{|c|}\hline 99\\ 64\\ \hline\end{array}$$

Hence, it follows that:

[17] Tr 29: आद्यन्तयोस्त्रिराशावभिन्नजाती प्रमाणमिच्छा च। फलमन्यजाति मध्ये तदन्त्यगुणमादिना विभजेत्॥ *ādyantayos trirāśāv abhinnajātī pramāṇam icchā ca/* phalam anyajāti madhye tadantyaguṇam ādinā vibhajet//*** (* ādyanta°] K_{ED}, āvyaṃta° A_1; °jātī] A_1, °jāti K_{ED}; ca] K_{ED}, vā A_1 ** ādinā] em., ādimena K_{ED}, mārdinā A_1; vibhajet] A_1^{pc}, bhajet K_{ED}, vibhajetaṃ A_1^{ac}) "Among the three quantities, the 'standard' and the 'requisite' in the first and the last [positions respectively] are of the same denomination, [and] the 'fruit' [of the 'standard'] in the middle [position] is of a different denomination. By the first (the 'standard'), one should divide that (the 'fruit' of the 'standard') multiplied by the last (the 'requisite')." That is, if $a : b = c : x$, then $x = b \cdot c \div a$. The three quantities—the 'standard' (a), the 'fruit' of the 'standard' (b), and the 'requisite' (c)—are arranged horizontally: $\begin{array}{|c|c|c|}\hline a & b & c\\ \hline\end{array}$,

where a and c should be of the same denomination.

[18] sārdhaḥ] K_{ED}, sorddhaḥ A_1; °tritayaṃ ca] K_{ED}, °ttita/ yaṃ va A_1; 'ṣṭā°] K_{ED}, ṣṭā° A_1

[19] °khāryāḥ] K_{ED}, °ṣāṃryāḥ A_1

[20] Tr Pbh6: खार्येका षोडशभिर्द्रोणैश्चतुराढको भवेद्द्रोणः। प्रस्थैश्चतुर्भिराढक एकप्रस्थश्चतुःकुडवः॥ *khāry ekā ṣoḍaśabhir droṇaiś caturāḍhako bhaved droṇaḥ/* prasthaiś caturbhir āḍhaka ekaprasthaś catuḥkuḍavaḥ//*** (* bhaved] K_{ED}, bhave A_1 ** ekapra°] A_1, ekaḥ pra° K_{ED}; catuḥku°] K_{ED}, catuku° A_1) "One *khārī* should be made up of sixteen *droṇa*-s, one *droṇa* of four *āḍhaka*-s, one *āḍhaka* of four *prastha*-s, and one *prastha* of four *kuḍava*-s."

	ku	*pra*	*ā*	*dro*	*khā*
kuḍava	1				
prastha	**4**	1			
āḍhaka	16	**4**	1		
droṇa	64	16	**4**	1	
khārī	1024	256	64	**16**	1

$1\frac{1}{2}$ *droṇa*-s + 3 *kuḍava*-s = 99 *kuḍava*-s.

Then, the commentator converts the third term of the Rule of Three, 1 *khārī* + 1 *droṇa*, into *kuḍava* by means of the 'chain-reduction.'

$$\begin{array}{|c|}1\\1\\0\\4\\0\\4\\0\\4\end{array} \longrightarrow \begin{array}{|c|}1\cdot4+0\\1\cdot4\\0\\4\\0\\4\end{array} \longrightarrow \begin{array}{|c|}4\cdot4+0\\4\cdot4\\0\\4\end{array}$$

$$\longrightarrow \begin{array}{|c|}16\cdot4+0\\16\cdot4\end{array} \longrightarrow \begin{array}{|c|}64\\64\end{array}$$

Thus, 1 *droṇa* = 64 *kuḍava*-s. Only the first and last setting-downs are given in the text, and the first one is expressed as $\begin{array}{|c|c|}1&0\\1&4\end{array}$.

Similarly, 1 *khārī* = 1024 *kuḍava-s*. Hence, it follows that:

1 *khārī* + 1 *droṇa* = 1024 + 64 = 1088 *kuḍava*-s

The 'chain-reduction' is originally an operation for unifying quantities expressed in multiple units into the highest one, but here, the rule is applied to unify the quantities into the lowest one. Finally, the commentator carries out the Rule of Three as follows:

$99 : 8 = 1088 : x,$

$x = 8 \cdot 1088 \div 99 = 8704 \cdot \frac{1}{99} = \frac{8704}{99} = 87\frac{91}{99}$ *rūpa*-s.

The reason why the commentator unifies the values into the lowest unit is to simplify the calculation of the Rule of Three by reducing all the three quantities into integers without fractions.

3.3 TrBh on Tr E71–72

The author of the TrBh again applies the 'chain-reduction' to the problem of Tr E71–72, where several monetary quantities are unified into the highest unit. Since it is a regular usage just as prescribed in Tr 26cd–27ab, I do not discuss it in this paper.

4 Prose parts of the Tr

This section deals with prose parts of the Tr which is contained in K_{ED}, but not in A_1. Each of those parts follows a stanza of the Tr.

4.1 Tr E22p

Tr E22p:

न्यासः॥ [५/१ ३/१६ १/४ १/२० १/५] सवर्णिते जातं [१६६४७/३२००] लब्धं पुराणाः ५। पणाः ३। काकिणी ०। वराटकाः १८। वराटकभागाश्च४/५ । इति वल्लीसवर्णनम्॥

nyāsaḥ// $\left[\frac{5}{1}\ \frac{3}{16}\ \frac{1}{4}\ \frac{1}{20}\ \frac{1}{5}\right]$ *savarṇite jātaṃ* $\left[\frac{16647}{3200}\right]$ *labdhaṃ purāṇāḥ 5/ paṇāḥ 3/ kākiṇī 0/ varāṭakāḥ 18/ varā-ṭakabhāgāś ca*$\frac{4}{5}$ */ iti vallīsavarṇanam//*

Setting-down: $\left[\frac{5}{1}\ \frac{3}{16}\ \frac{1}{4}\ \frac{1}{20}\ \frac{1}{5}\right]$

Reduced to the same color, the result is $\left[\frac{16647}{3200}\right]$.

What is obtained is 5 *purāṇas*, 3 *paṇa*-s, 0 *kākiṇī*, 18 *varāṭaka*-s, and $\frac{4}{5}$ parts of *varāṭaka*. Thus is the 'chain-reduction.'

After the answer $\frac{16640}{3200}$, the original values which are given in Tr E22 are enumerated here (see Sect. 3.1). It is most probable that the author of the TrBh, referring to this prose part without citing the text, conducted a verification.

4.2 Tr E33p

Tr E33p:

आद्यन्तयोः कुडवैर्न्यासः॥ [९९/१ | ८/१ | १०८८/१] लब्धानि रूपाणि ८७ भागाश्च ९१/९९ ॥

ādyantayoḥ kuḍavair nyāsaḥ// $\begin{array}{|c|c|c|}\frac{99}{1}&\frac{8}{1}&\frac{1088}{1}\end{array}$ *labdhāni rūpāṇi 87 bhāgāś ca* $\frac{91}{99}$*//*

Setting-down with *kuḍava*-s of the first and last [terms]: $\begin{array}{|c|c|c|}\frac{99}{1}&\frac{8}{1}&\frac{1088}{1}\end{array}$. What is obtained is 87 *rūpa*-s and $\frac{91}{99}$ parts.

The above values, especially the three quantities in the unit of *kuḍava*, could also have influenced the author of the TrBh into carrying out the 'chain-reduction' for the sake of unifying the quantities into the lowest unit, as we have seen in TrBh on Tr E33 (see Sect. 3.2), although he does not mention this prose part of the Tr.

5 PGṬ

In 1959, an incomplete treatise of Śrīdhara was published under the title of *Pāṭīgaṇita* (hereafter PG). This title was given by Shukla, the editor of the text, according to the catalogue listing the extant and unique manuscript (Raghunātha Temple Library, Jammu, 3074). The manuscript ends with the twenty-third verse of the 'procedure of plane figure' (*kṣetravyavahāra*) and contains an anonymous commentary, the PGṬ. Shukla edited the PG with the PGṬ and translated only the PG into English. In his edition one can find many verses common to the Tr.

In this section, I will present the computational procedures of the 'chain-reduction' given in the PGṬ. For English translations of the text, see Appendix 2.

5.1 PGṬ on PG E22

PG E22:

पञ्च पुराणास्त्रिपणं काकिण्येका वराटकैकोना।
तत्पञ्चमभागोना समासतः किं धनं भवति॥

pañca purāṇās tripaṇaṃ
kākiṇy ekā varāṭakaikonā/
tatpañcamabhāgonā
samāsataḥ kiṃ dhanaṃ bhavati//

There are five *purāṇa*-s, three *paṇa*-s, one *kākinī*, decreased by one *varāṭaka* and one-fifth of that (one *varāṭaka*). What is [their] value in total?

The above stanza is almost the same as Tr E22. The commentator first mentions the conversion ratios of the monetary units defined in PG 9 (=Tr Pbh4), and then, gives only the 'chain' and the answer $5\frac{647}{3200}$ *purāṇa*-s. The calculation could be the same as that of TrBh on Tr E22, though no procedure is described in the PGṬ. The commentator does not perform the verification and seems not to be influenced by the values listed in Tr E22p. This means that the author of the PGṬ either did not possess or ignored manuscript(s) of the Tr including Tr E22p.

5.2 PGṬ on PG E27

PG E27:

धान्यद्रोणः सार्द्धः कुडवत्रितयं च लभ्यते ऽष्टाभिः।
खार्येका द्रोणयुता तत्कियता कथय यदि वेत्सि॥

dhānyadroṇaḥ sārddhaḥ
kuḍavatritayaṃ ca labhyate 'ṣṭābhiḥ/
khāry ekā droṇayutā
tat kiyatā kathaya yadi vetsi//

One and a half *droṇa*-s and three *kuḍava*-s of grain are obtained for eight [units of money]. In that case, say for how much one *khārī* increased by one *droṇa* [will be obtained], if you know?

The above stanza is almost the same as Tr E33. The commentator unifies the first and third terms for Rule of Three, ($1\frac{1}{2}$ *droṇa*-s + 3 *kuḍava*-s) and (1 *khārī* + 1 *droṇa*), into *droṇa*. As for the third term, because 1 *khārī* = 16 *droṇa-s*,

1 *khārī* + 1 *droṇa* = 16 + 1 = 17 *droṇa*-s.

After that, the commentator illustrates two types of 'chains' for the first term:

1 1 8 16 3 4	or	1 1 2 3 64

The left-hand 'chain' is constructed with $\frac{1}{2}$ *droṇa* = 8 *prastha*-s and the right-hand one with 1 *droṇa* = 64 *kuḍava*-s by keeping $\frac{1}{2}$ *droṇa*. Applying the rule for the 'chain-reduction' to the left, the procedure can be reconstructed as follows:

$$\left[\frac{1}{1}+\frac{8}{16}=\frac{1\cdot 16+8}{1\cdot 16}=\frac{24}{16};\right]$$
$$\left[\frac{24\cdot 4+3}{16\cdot 4}=\right]\frac{99}{64}[\textit{droṇa}\text{-s}].$$

With regard to the right-hand 'chain,' the commentator first carries out 'other's part addition' (*parabhāgānubandha*),[21] and then, applies the rule for the 'chain-reduction':

$$\left[1\frac{1}{2}=\frac{1\cdot 2+1}{2}=\frac{3}{2};\right]$$
$$\left[\frac{3/2}{1}+\frac{3}{64}=\frac{(3/2)\cdot 64+3}{1\cdot 64}=\right]\frac{99}{64}\ [\textit{droṇa}\text{-s}].$$

Hence, it follows that:

$$1\frac{1}{2}\ \textit{droṇa}\text{-s}+3\ \textit{kuḍava}\text{-s}=\frac{99}{64}\ \textit{droṇa}\text{-s}.$$

[21] The rule for 'other's part addition' is: $n+\frac{b}{a}=\frac{na+b}{a}$. This is in contrast to 'one's own part addition' (*svabhāgānubandha*), that is, $\frac{b_1}{a_1}(1+\frac{b_2}{a_2})=\frac{b_1(a_2+b_2)}{a_1a_2}$.

The calculations of the ‘other’s part addition’ and the ‘chain-reduction’ are here collectively called ‘calculation in the two principles’ (*tantradvayakriyā*). Finally, the commentator carries out the Rule of Three as follows:

$$\frac{99}{64} : 8 = 17 : x,$$
$$x = 8 \cdot 17 \div \frac{99}{64} = 136 \cdot \frac{64}{99}\left[= \frac{8704}{99}\right] = 87\frac{91}{99}.$$

It is clear from the above that the usage of the ‘chain-reduction’ in the PGṬ is in accordance with the rule prescribed in PG 41 (= Tr 26cd–27ab).[22]

6 SGT

The GT is a Sanskrit arithmetic text written by Śrīpati (11th century CE). The text has been handed down to us together with the SGT in an incomplete manuscript. Siṃhatilaka was a Śvetāmbara Mūrtipūjaka monk affiliated to the Kharataragaccha, one of the Jaina monastic orders in early medieval Gujarat and Rajasthan.[23] It is notable that he was quite familiar with the Tr (and also the PG) because Tr Pbh4, 11, 15, 24, 31, and PG 37 are cited or mentioned in the SGT.[24] The GT and SGT have been fully studied and translated into English by Petrocchi (2019) and into Japanese by Hayashi (2019). This section is based on both of these studies.

6.1 SGT on GT 63

GT 63:

द्रम्मद्वयं पञ्च पणास्तथैका
काकिण्यहो मित्र कपर्दिकोना।
तदंह्रिणा चापि सवर्णयित्वा
व्यावर्ण्यतां द्राग्यदि बोबुधीषि॥

drammadvayaṃ pañca paṇās tathaikā
kākiṇy aho mitra kapardikonā/
tadaṃhriṇā cāpi savarṇayitvā
vyāvarṇyatāṃ drāg yadi bobudhīṣi//

O friend, if you wish to learn [mathematics], having performed their simplification, calculate quickly two *dramma*s, also five *paṇa*s and one *kakiṇī* minus one *kapardikā*, and [minus] its one-fourth too.[25]

That is, 2 *dramma*-s + 5 *paṇa*-s + (1 *kākiṇī* $-1\frac{1}{4}$ *kapardikā*-s) are to be unified to the unit of *dramma*.[26] This example is of the same type as Tr E22. Siṃhatilaka first constructs a ‘chain’ from the given quantities and their conversion rates:[27]

2	1	5	16	1	4	∘1	20	∘1	4

Then, he performs the calculation in the following steps:[28]

$$\frac{2}{1} + \frac{5}{16} = \frac{2 \cdot 16 + 5}{1 \cdot 16} = \frac{37}{16},$$
$$\frac{37}{16} + \frac{1}{16 \cdot 4} = \frac{37 \cdot 4 + 1}{16 \cdot 4} = \frac{149}{64},$$
$$\frac{149}{64} - \frac{1}{64 \cdot 20} = \frac{149 \cdot 20 - 1}{64 \cdot 20} = \frac{2979}{1280},$$
$$\frac{2979}{1280} - \frac{1}{1280 \cdot 4} = \frac{2979 \cdot 4 - 1}{1280 \cdot 4}$$
$$= \frac{11915}{5120} = \frac{2383}{1024} \textit{ dramma-s.}$$

This is the answer, but Siṃhatilaka continues to calculate and obtains the original values expressed in the lower units:

$$\frac{2383}{1024} = 2\frac{335}{1024} \textit{ dramma-s},$$
$$\frac{335 \cdot 16}{1024} = \frac{5360}{1024} = 5\frac{240}{1024} \textit{ paṇa-s},$$
$$\frac{240 \cdot 4}{1024} = \frac{960}{1024} = 0\frac{960}{1024} \textit{ kākiṇī},$$
$$\frac{960 \cdot 20}{1024} = \frac{19200}{1024} = 18\frac{768}{1024} \textit{ kapardaka-s},$$
$$\frac{768}{1024} \div \frac{1}{4} = \frac{768 \cdot 4}{1024 \cdot 1} = \frac{3072}{1024} = 3, \quad \frac{768}{1024} = \frac{3}{4}.$$

The calculation ends here, but of course,

[22] In PGṬ on PG E55–56 (p. 58, lines 1–11 in Shukla’s edition), the rule of the ‘chain-reduction’ is applied to unify two time units, *māsa* (month) and *dina* (day), into the higher one, that is, *māsa*.

[23] See Petrocchi (2019, pp. 12–16) and Hayashi (2019, pp. 16–19).

[24] See Petrocchi (2019, p. 417) and Hayashi (2019, pp. 24–26).

[25] Translation by Petrocchi (2019, p. 153).

[26] GT 4: स्यात्काकिणी पञ्चगुणैश्चतुर्भिर्वराटकैः २० काकिणिकाचतुष्कम्। पणं भणन्ति व्यवहारतज्ज्ञा द्रम्मश्च तैः षोडशभिः प्रसिद्धः॥ *syāt kākiṇī pañcaguṇaiś caturbhir varāṭakaiḥ 20 kākiṇikācatuṣkam/ paṇaṃ bhaṇanti vyavahāratajjñā drammaś ca taiḥ ṣoḍaśabhiḥ prasiddhaḥ//* “There is one *kākiṇī* in five times four cowry-shells, 20. Those who are familiar with this practice say that one *paṇa* is [equal to] four *kākiṇīs*, and one *dramma* is known to be in sixteen of these [*paṇas*].” Translation by Petrocchi (2019, p. 49).

	va	kā	pa	dra
varāṭaka	1			
kākiṇī	**5·4**	1		
paṇa	80	**4**	1	
dramma	1280	64	**16**	1

[27] The negative sign, a circle (∘), is attached to subtractive/negative numbers in the following ‘chain.’

[28] The following procedures are based on Hayashi (2019, p. 218).

$\left[18\frac{3}{4} = \left(20 - 1\frac{1}{4}\right) \textit{kapardaka}\text{-s} = 1 k\bar{a}ki\d{n}\bar{\imath} - 1\frac{1}{4}\ \textit{kapardaka}\text{-s.}\right]$

The procedure after the answer is a verification just as in TrBh on Tr E22 (see Sect. 3.1). It is most likely that manuscript(s) of the Tr containing Tr E22p was available to Siṃhatilaka and that he conducted the verification in accordance with the description of it.

7 Concluding remarks

The reason why the author of the TrBh applies the 'chain-reduction' to Tr E22 and E33 in an irregular manner can be explained in the following ways.

As for the Tr E22, the commentator first performs the calculation in a regular way, but after that, he inversely finds the original values given in the example. From the mathematical perspective, the latter half of the procedure is clearly a verification. The same type of verification can be found in SGT on GT 63 but not in PGṬ on PG E22 (=Tr E22). Moreover, the values obtained through the verification in the TrBh are set out in Tr E22p too. Therefore, it is most probable that the author of the TrBh and also Siṃhatilaka were able to access the manuscript(s) of the Tr including Tr E22p.

As for the Tr E33, the author of the TrBh applies the 'chain-reduction' in order to unify the quantities into the lowest unit. The mathematical reason for it is that he attempts to reduce all the three quantities into integers without fractions for the sake of easy calculation with Rule of Three. On the other hand, in PGṬ on PG E27 (=Tr E33) the commentator unifies the given values into the highest unit by means of the 'chain-reduction.' It is likely that the prose part of Tr E33 was available to the author of the TrBh, but unavailable to that of the PGṬ.

The relationship between the texts can be illustrated in Fig. 1. The vertical line at left-hand side denotes the common era (CE), and each text is arranged accordingly. The PGṬ and PG 37 are surrounded by a dashed line, because the date of the commentator is unknown and it is not certain whether the stanza of PG 37 was contained in the original text of the Tr. The box of the TrBh corresponds to the estimated date of its author, that is, some time between the 12th and 15th centuries CE . The arrows → indicate the text(s) to which the author of each commentary explicitly referred, and the dashed arrows ⇢ text(s) to which each author may or may not have referred. At present there is too little evidence to determine whether the text of the Tr including PG 37 or that of the PG was accessible to Siṃhatilaka.

The author of the TrBh neither mentions nor quotes the prose parts of the Tr, but they are repeated in the TrBh in his own words. Otherwise, it might be that the prose parts were originally cited in the TrBh, but the scribe of the A_1 or someone else omitted them.

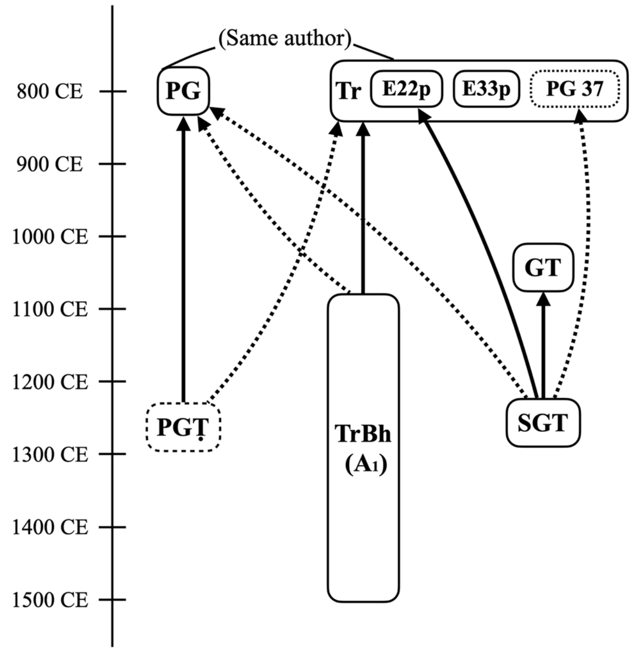


Fig. 1 Relationship between the texts

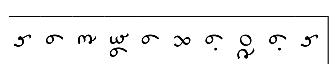
Appendix 1: Text and translation of the TrBh

The text edited here of the TrBh is based on A_1. The word(s) cited in the TrBh from the Tr is printed in bold. I remove or add *daṇḍa*-s (/) in A_1 for the ease of reading. Phonological irregularities have been left in this edition just as they are in the manuscript.

Notations

- *ac* — *Ante correcturam*, i.e., the reading before the correction by the scribe
- em. — Emendation (I do not distinguish it from 'correction' and 'conjecture')
- *pc* — *Post correcturam*, i.e., the reading after the correction by the scribe
- ⟨A⟩ — A is supplied by the editor
- 'A' — A is quotation
- (A) — A is reference of quoted passage
- ° — Truncation (of letters) in long Sanskrit words

TrBh on Tr E22 (A_1 fols. 9b–10a)

न्यासः।  '**<विनि > क्षिपेत्**' (PG 37d) इति कृते जातं ८३ / १६ एव। चतुष्केण **छेदांशौ गुणयेत्**। **पूर्वांशे**

एकं प्रक्षिपेत्। प्रक्षिप्ते जातं ३३३/६४ । ततो विंशत्या **छेदांशौ गुणयेत्**। अंशेभ्यः एकं पातयेत्। ऊर्द्धस्थानात्पातिते जातं ६६५९/१२८० । ततः पंचकेन **छेदांशौ गुणयेत्**। अंशेभ्यः एककं च पातयेत्। पातिते जातं ३३२९४/६४०० । ततः छेदांशौ दलयेत्। दलिते जातं १६६४७/३२०० । छेदेन हृते लब्धं पुराणाः पंच ५। ततः शेषे षोडशभिर्हते लभ्यंते पणाः ३। शेषे चतुष्केण हते ततो विंशत्या हते लभ्यंते वराटकाः १८। शेषे षड्भिः शतैश्चत्वारिंशदधिकैरपवर्त्तिते भागाः ४/५ । एवं वल्लीसवर्णनं समाप्तं॥ छ॥

nyāsaḥ/ 5 1 3 16 1 4 ·1 20 ·1 5 ‘ ***<vini> kṣipet***’ *(PG 37d)*[29] *iti kṛte jātaṃ* 83/16 *eva/*[30] *catuṣkena* ***chedāṃśau guṇayet/***[31] ***pūrvāṃśe*** *ekaṃ prakṣipet*[32] *prakṣipte jātaṃ* 333/64 */ tato viṃśatyā* ***chedāṃśau guṇayet/***[33] *aṃśebhyaḥ ekaṃ pātayet/*[34] *ūrdvasthānāt pātite jātaṃ* 6659/1280 */* [35] *tataḥ paṃcakena* ***chedāṃśau guṇayet/*** *aṃśebhyaḥ ekakaṃ ca pātayet/*[36] *pātite jātaṃ* 33294/6400 */ tataḥ chedāṃśau dalayet/*[37] dalite jātaṃ 16647/3200 */ chedena hṛte labdhaṃ purāṇāḥ paṃca 5/*[38] *tataḥ śeṣe ṣoḍaśabhir hate labhyaṃte paṇāḥ 3/ śeṣe catuṣkeṇa hate tato viṃśatyā hate labhyaṃte varāṭakāḥ 18/*[39] *śeṣe ṣaḍbhiḥ śataiś catvāriṃśad adhikair apavarttite bhāgāḥ* 4/5 */*[40] *evaṃ vallīsavarṇanaṃ samāptaṃ// cha//*

Setting-down: 5 1 3 16 1 4 ·1 20 ·1 5.

When [the operation] “**one should add**” (PG 37d) is carried out, the result is exactly 83/16. **One should multiply [the former (the upper)] denominator and numerator** by four. One should add one to **the former (the upper) numerator**. When [one] is added [to it], the result is 333/64. Then, **one should multiply [the former (the upper)] denominator and numerator** by twenty. One should subtract one from the [upper] numerator. When [one] is subtracted from the above place, the result is 6659/1280. Next, **one should multiply [the former (the upper)] denominator and numerator** by five and subtract one from the [upper] numerator. When [one is] subtracted [from them], the result is 33294/6400. After that, one should halve the denominator and the numerator. When [they are] halved, the result is 16647/3200.

When [the numerator is] divided by the denominator, the quotient is five, i.e., 5 *purāṇa*-s. Then, when the remainder is multiplied by sixteen, 3 *paṇa*-s are obtained. When the remainder is multiplied by four and further multiplied by twenty, 18 *varāṭaka*-s are obtained. When the remainder is reduced by six hundred increased by forty, the parts are 4/5. Thus, [the topic of] the ‘chain-reduction’ is completed.

TrBh on Tr E33 (A_1 fols. 11a–11b)

द्रोणैः कुडवे कृते न्यासः। द्रोणस्य कुडवकरणार्थं युक्तिः प्रस्थे कुडव ४ आढके कुडव १६ द्रोणे कुडव ६४। तदर्द्धभागे ३२। उभयं ९६ त्रिभिर्योगे ९९। अथवा अन्या युक्तिः सवर्ण्णने जातं ३/२ त्रिकद्विकयोरन्योन्यघाते जातं ६ चतुर्गुणे २४ पुनश्चतुर्गुणे ९६ अधस्थितत्रिकसहिते ९९। तद्वा वल्लीसवण्णने कुडवानयनं। द्रोण १। आढकस्थाने आढकचतुष्टये<न> द्रोणः स्यादिति चतुष्कदर्शनं ४। प्रस्थस्थाने ० प्रस्थचतुष्टयेनाढकः स्यादिति चतुष्कदर्शनं ४। कुडवचतुष्टयेन प्रस्थं स्यात् ४। एकत्रस्थापनं १/१ ०/४ गुणने ६४/६४ एते द्रोणकुडवाः ६४। खार्या १०२४। उभयं जातं १०८८। अंतिमानयनं। अथ स्थापनं। ९९। ८। १०८८। **तदंत्यगुणं** फलं ८ गुणिते ८७०४ आदिमेन छेदांशविपर्यासेन गुणिता ८७०४/९९ ॥ भागे हृते लब्धानि रूपाणि ८७ रूपभागाः ९१/९९ ॥

droṇaiḥ kuḍave kṛte nyāsaḥ/[41] *droṇasya kuḍavakaraṇārthaṃ yuktiḥ prasthe kuḍava 4 ādhake kuḍava 16 droṇe kuḍava 64/*[42] *tadarddhabhāge 32/ ubhayaṃ 96 tribhir yoge 99/*

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[29] A_1 contains the full version of this stanza, though K_{ED} does not have it.

[30] 5 1 3 16 1 4 ·1 20 ·1 5] em., 5 1 3 16 ·1 4 1 20 ·1 5 A_1

[31] °ṇayet] em., °ṇayat A_1

[32] ekaṃ] em., evaṃ$_1$

[33] viṃśa°] em., viśa° A_1

[34] aṃśe°] em., aśe° A_1

[35] ūrdvasthānāt] em., ūratvātā A_1

[36] ca] em., va A_1

[37] °dāṃśau] em., °dāṃśo A_1

[38] °rāṇāḥ] em., °rāṇā A_1

[39] śeṣe catu°] em., śoṣe vatu° A_1; viṃśa°] em., viśa° A_1

[40] ṣaḍbhiḥ] em., ṣaḍabhiḥ A_1; °riṃśadadhikair] em., °riśadadhiker A_1

[41] ° ḍave kṛte] em., °ḍavaḥ kṛteḥ A_1

[42] prasthe] em., prasthi A_1

athavā anyā yuktiḥ savarṇṇane jātaṃ $\begin{array}{|c|}3\\2\end{array}$ *trikadvikayor anyonyaghāte jātaṃ 6 caturguṇe 24 punaś caturguṇe 96 adhasthitatrikasahite 99/*[43] *tadvā vallīsavarṇṇane kuḍavānayanaṃ/*[44] *droṇa 1/ āḍhakasthāne āḍhakacatuṣṭaye<na> droṇaḥ syād iti catuṣkadarśanaṃ 4/*[45] *prasthasthāne 0 prasthacatuṣṭayenāḍhakaḥ syād iti catuṣkadarśanaṃ 4/*[46] *kuḍavacatuṣṭayena prasthaṃ syāt 4/ ekatrasthāpanaṃ* $\begin{array}{|c|c|}1&0\\1&4\end{array}$ *guṇane* $\begin{array}{|c|}64\\64\end{array}$ *ete droṇakuḍavāḥ 64/*[47] *khāryā 1024/*[48] *ubhayaṃ jātaṃ 1088/ aṃtimānayanaṃ/*[49] *atha sthāpanaṃ/ 99/ 8/ 1088/* ***tadaṃtyaguṇaṃ*** *phalaṃ 8 guṇite 8704 ādimena chedāṃśaviparyāsena guṇitā* $\begin{array}{|c|}8704\\99\end{array}$ */*[50] *bhāge hṛte labdhāni rūpāṇi 87 rūpabhāgāḥ* $\begin{array}{|c|}91\\99\end{array}$ */.*

When the *kuḍava* is produced by the *droṇa*-s, setting-down is [as follows]. The principle (*yukti*) for the sake of converting the *droṇa* into the *kuḍava* is: 4 *kuḍava*-s for one *prastha*, 16 *kuḍava*-s for one *āḍhaka*, 64 *kuḍava*-s for one *droṇa*. For a half part of them, there is 32. Both [of 64 and 32 added together] are 96. Increased by three, [the result is] 99. Alternatively, there is the other principle. Reduced to the same color, the result is $\begin{array}{|c|}3\\2\end{array}$. When three and two are multiplied mutually, the result is 6. Multiplied by four, [the result is] 24. Further multiplied by four, [the result is] 96. Increased by three located below (*adhasthita*),[51] [the result is] 99. Similarly (*tadvat*),[52] in the 'chain-reduction,' [one] calculates the *kuḍava*. 1 *droṇa* is [at the top of the 'chain']. Since, in the place of *āḍhaka*, one *droṇa* should be made up of four *āḍhaka*-s, four, i.e., 4 is shown. [The numerator is] 0 in the place of *prastha*. Because one *āḍhaka* should be made up of four *prastha*-s, four, i.e., 4 is shown. One *prastha* should be made up of four *kuḍava*-s. 4 [is shown]. Put [the digits] in one place, $\begin{array}{|c|c|}1&0\\1&4\end{array}$. [Performing] the multiplication, [the result is] $\begin{array}{|c|}64\\64\end{array}$. These are 64 *kuḍava*-s [contained] in one *droṇa*. [The *kuḍava*-s contained] in one *khārī* are 1024. Both [of 64 and 1024 added together] are 1088. [Thus is] the calculation of the last [term of a Rule of Three]. Now, setting-down is 99, 8, 1088. **That (the 'fruit' of the 'standard') multiplied by the last [quantity]**. The 'fruit' [of the 'standard'] is 8. Multiplied [by the last quantity, the result is] 8704. Multiplied by the first [quantity] through interchanging the denominator and the numerator, [the result is] $\begin{array}{|c|}8704\\99\end{array}$. When [the numerator is] divided [by the denominator], what is obtained is 87 units [and] $\begin{array}{|c|}91\\99\end{array}$ parts of unity.

Appendix 2: Text and translation of the PGṬ

The text of the PGṬ is based on the following edition and manuscript:

L_{ED}: Lucknow edition by K. S. Shukla, 1959

J_1: Raghunātha Temple Library, Jammu, 3074

PGṬ on PG E22 (L_{ED} p. 35, line 28—p. 36, line 11)

अत्र पुराणानां रूपच्छेदनतास्वातन्त्र्यात्पणादीनां चावयवविशेषत्वे संज्ञाविशेषत्वाच्छेदलाभः। यतः पुराणषोडशभागः पणः, पणचतुर्भागः काकिणी, काकिणीविंशतिभागो वराटकः, अतस्तैरेव रूपच्छेदस्थापनम्— $\begin{array}{|cccccccc|cc|}५&१&३&१६&१&४&१&२०&१&५\end{array}$ लब्धं पुराणाः ५, पुराणभागाः $\frac{६४०}{३२००}$ ।भागापवाहजातिः समाप्ता वल्ली च।

atra purāṇānāṃ rūpacchedanatāsvātantryāt paṇādīnāṃ cāvayavaviśeṣatve saṃjñāviśeṣatvāc chedalābhaḥ/ yataḥ purāṇaṣoḍaśabhāgaḥ paṇaḥ, paṇacaturbhāgaḥ kākiṇī, kākiṇīviṃśatibhāgo varāṭakaḥ, atas tair eva rūpacchedasthāpanam– $\begin{array}{|cccccccc|cc|}5&1&3&16&1&4&1&20&1&5\end{array}$ *labdhaṃ purāṇāḥ 5, purāṇabhāgāḥ* $\frac{640}{3200}$ */ bhāgāpavāhajātiḥ samāptā vallī ca/*

In this [example], the *purāṇa*-s are independent because they possess unity as the denominator, and the *paṇa*-s and so on have particular designations as particular parts [of the preceding units]. For that reason, they acquire the denominators. This is because one *paṇa* is made up of one-sixteenth of one *purāṇa*, one *kākiṇī* one-fourth of one *paṇa*, and one *varāṭaka* one-twentieth of one *kākiṇī*. Therefore, by means of those [conversion ratios], the setting-down (*sthāpana*) of the unity and the denominators is [as follows]: $\begin{array}{|cccccccc|cc|}5&1&3&16&1&4&1&20&1&5\end{array}$.

[43] $\begin{array}{|c|}3\\2\end{array}$ trikadvikayor anyonya°] em., trikadvikayo $\begin{array}{|c|}3\\2\end{array}$ anyonya° A_1; caturguṇe] em., caturguṇai A_1

[44] °nayanaṃ] em., °nayana A_1

[45] droṇaḥ syāditicatuṣka°] em., droṇasya ditivatuṣka° A_1

[46] °darśanaṃ] em., °daśaṃnaṃ A_1

[47] $\begin{array}{|c|}64\\64\end{array}$] em., $\begin{array}{|c|}64\\64\\3\end{array}$ A_1

[48] 1024] em., 10244 A_1

[49] °timāna°] em., °timona° A_1

[50] tadaṃtya°] em., taṃdaṃtya° A_1; chedāṃśa°] em., chedāṃ $\begin{array}{|c|c|}1&3\\4&4\end{array}$/ śa° A_1 ^{pc}, chedāṃ $\begin{array}{|c|c|}1&3\\4&4\end{array}$ guṇane/ śa° A_1 ^{ac}

[51] *adhasthita*- for *adhaḥsthita*-.

[52] According to the dictionary of Monier Williams, *tadvā* = *tadvat*.

What is obtained is 5 *purāṇa*-s [and] $\frac{647}{3200}$ parts of one *purāṇa*. [The topics of] 'part-subtraction class' and 'chain' are completed.

PGṬ on PG E27 (L_{ED} p. 39, lines 4–21)

धान्यस्य द्रोणः अर्द्धेन प्रस्थाष्टकेन सहितस्तथा कुडवैस्त्रिभिर्युक्तः **अष्टाभिर्यैः** कैश्चिद्देशनियतैर्व्यावहारिकै रूपैश्चे**ल्लभ्यते** तदा **एका खारी द्रोणयुता कियता** लभ्यते।

dhānyasya droṇaḥ arddhena prasthāṣṭakena sahitas tathā kuḍavais tribhir yuktaḥ ***aṣṭābhir*** yaiḥ kaiścid deśaniyatair vyāvahārikai rūpaiś cel ***labhyate*** tadā ***ekā khārī droṇayutā kiyatā*** labhyate/

If one *droṇa* of grain, increased by a half, that is, by eight *prastha*-s, and increased by three *kuḍava*-s, **are obtained for eight [units of money]**, that is, for [eight of] certain practical (*vyāvahārika*) units limited to the region(s), then, **for how much one *khārī* increased by one *droṇa*** [will] be obtained?

अत्र **धान्यद्रोणो** यथोक्तप्रस्थकुडवान्वितो ज्ञातमूल्यत्वात्प्रमाणराशिः। तत्र प्रमाणेच्छाराश्योः सवर्णमुपादीयते। द्रोणानां खारी कार्या, तत्खार्य्यश्च द्रोणाः <वा> कार्याः। द्रोणस्य प्रस्थादिसानुबन्धत्वात्प्रतिपत्तिगौरवं स्यात्। खारी तु षोडशगुणा द्रोणाः तावन्त एव, एकद्रोणाधिकाः सप्तदश, द्रोणस्य यदि रूपार्द्धेन योगं कृत्वा कुडवयोगः क्रियते तदा कुडवा चतुःषष्टिच्छेदाः कार्याः, तावत्कुडवैर्द्रोण इति। अथ द्रोणस्यार्द्धेन परभागानुबन्धः कुडवैः वल्ली इति तन्त्रद्वयक्रियायामायासस्तदा द्रोणो रूपच्छिन्न उपरि, तदधः प्रस्थाष्टकं षोडशच्छिन्नं तदधस्त्रयः कुडवा प्रस्थव्यवस्थया चतुर्भक्ताः स्थाप्याः।

१		१
१		
८	यद्वा	१
१६		२
३		३
४		६४

उभयत्रापि प्रमाणराशिः सवर्णि<ते> इदं भवति $\frac{९९}{६४}$ मध्यमराशिः स्वरूपस्थ एव ८, अन्त्यराशिः १७। प्रमाणराशेर्हरत्वाच्छेदांशविपर्यासेऽनन्तरं प्रभागकर्मणि लब्धं ८७ भागाः $\frac{९१}{९९}$।

atra ***dhānyadroṇo*** *yathoktaprasthakuḍavānvito jñātamūlyatvāt pramāṇarāśiḥ/ tatra pramāṇecchārāśyoḥ savarṇam upādīyate/ droṇānāṃ khārī kāryā, tatkhāryyaś ca droṇāḥ <vā> kāryāḥ/ droṇasya prasthādisānubandhatvāt pratipattigauravaṃ syāt/ khārī tu ṣoḍaśaguṇā droṇāḥ tāvanta eva, ekadroṇādhikāḥ saptadaśa, droṇasya yadi rūpārddhena yogaṃ kṛtvā kuḍavayogaḥ kriyate tadā kuḍavā catuḥṣaṣṭicchedāḥ kāryāḥ, tāvatkuḍavair droṇa iti/*[53] *atha droṇasyārddhena parabhāgānubandhaḥ kuḍavaiḥ vallī iti tantradvayakriyāyām āyāsas tadā droṇo rūpacchinna upari, tadadhaḥ prasthāṣṭakaṃ ṣoḍaśacchinnaṃ tadadhas trayaḥ kuḍavā prasthavyavasthayā caturbhaktāḥ sthāpyāḥ/*[54]

1		*1*
1		
8	*yadvā*	*1*
16		*2*
3		*3*
4		*64*

ubhayatrāpi pramāṇarāśiḥ savarṇi<te> idaṃ bhavati $\frac{99}{64}$ *madhyamarāśiḥ svarūpastha eva 8, antyarāśiḥ 17/ pramāṇarāśer haratvāc chedāṃśaviparyāse 'nantaraṃ prabhāgakarmaṇi labdhaṃ 87 bhāgāḥ* $\frac{91}{99}$ */*

In this [example], **one *droṇa* of grain** increased by the aforementioned *prastha*-s and *kuḍava*-s is the 'standard' quantity, because its price is already known. In that case, the same color for the 'standard' and the 'requisite' quantities is taken.[55] The *droṇa*-s are to be [converted into] *khārī*, or the *khārī*-s are to be [converted into] *droṇa*-s. [However, since] the *droṇa* [of the 'standard' quantity] is accompanied by the *prastha* and so on, it would be difficult to carry out [the conversion of them into *khārī*]. On the other hand, one *khārī* [of the 'requisite' quantity] is just as much as one *droṇa* multiplied by sixteen. Adding one *droṇa* to it, seventeen [*droṇa*-s are obtained, and so, let the units of the 'standard' quantity be unified into *droṇa*]. If one *droṇa* is increased by half of the unit and [three] *kuḍava*-s are added [to it], then, *kuḍava*-s possessing sixty-four as the denominator are to be produced. [To be precise], one *droṇa* is made up of such number of *kuḍava*-s. Next, 'other's part addition' is by means of a half *droṇa*, [and] 'chain' is by means of *kuḍava*-s. Thus, [one makes] an effort at calculation in the two principles (*tantradvayakriyā*). Then, one *droṇa* divided by unity is at the top. Eight *prastha*-s divided by sixteen are below that. Three *kuḍava*-s divided by four are to be placed below them in accordance with the settled rule of *prastha* (*prasthavyavasthā*):

1		1
1		
8	or	1
16		2
3		3
4		64

---

53 droṇasya] J_1, doṇasya L_{ED}; catuḥṣaṣṭicche°] em., <dviguṇa>dvātriṃśacche° L_{ED}, dvātriṃśacche° J_1

54 tadadhaḥ pra°] em., tadaṃśaḥ pra° $L_{ED}J_1$

55 That is to say, "units of the 'standard' and the 'requisite' quantities are unified into the same one."

There is the 'standard' quantity on both sides. Reduced to the same color, this is produced $\frac{99}{64}$. The middle quantity is indeed in its own unit (*svarūpastha*), i.e., 8. The last quantity is 17. Since the 'standard' quantity is the divisor, the denominator and the numerator are interchanged, and after that, the calculation for 'multi-part' [is carried out]. What is obtained is 87 [and] $\frac{91}{99}$ parts.

Acknowledgements I wish to express my sincere gratitude to Prof. Takao Hayashi (Doshisha University) for his helpful comments and suggestions, though any mistakes are my own.

Funding This work was supported by JSPS KAKENHI Grant Number JP21J23080.

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