Indian Journal of History of Science (2021) 56:85–92
https://doi.org/10.1007/s43539-021-00012-2

ORIGINAL ARTICLE

Computing the number of perfumes that constitute the *gandhārṇava* and *kacchapuṭa* of Varāhamihira

Shriram Chauthaiwale[1] · Jayant Deopujari[2] · Atharva Chauthaiwale[3]

Received: 25 March 2020 / Accepted: 22 April 2021 / Published online: 29 October 2021

Abstract
The sixth-century Indian scholar Varāhamihira elaborated on the processes for the preparation of perfumes in the '*gandhayukti*' chapter of the *Bṛhatsaṁhitā*. He adapted the combinatorial analysis and magic square structure for arriving at the possible number of perfumes from the set of specified substances. This attempt is an excellent example of the multidisciplinary thought process of the Indian scholars in general and Varāhamihira in particular. This paper focuses on the verses under the captions '*gandhārṇava*' and '*kacchapuṭa*'. In the first case, four substances are selected from sixteen, and the chosen four are permuted with the pre-defined proportions. In the second case, selected substances are placed in a 4×4 magic square. The claims by Varāhamihira and the same amended by the commentator Bhaṭṭotpala for the number of perfumes are verified here. The constraints imposed by Varāhamihira in the first case on the proportions of the two substances are considered. The correct number for the possible perfumes is determined mathematically in the first case and by the specially designed computer program in the second case. Valid lists of perfumes with ingredients obtained as the output are attached in the form of web links in the first case, and the same is placed in the Appendix for the second case.

Keywords Bhaṭṭotpala · *Gandhārṇava* · *Kacchapuṭa* · Permutations and combinations · Python programming language · Varāhamihira

1 Introduction

Varāhamihira (480–587CE), a polymath with exceptional brilliance, elaborated on the preparations of perfumes, scents, oils, and incenses in the chapter entitled *gandhayukti* (the blending of fragrant substances, preparation of perfumes) (Williams, 2002, p. 345) of his voluminous work the *Bṛhatsaṁhitā* (*BS*). Later, Bhaṭṭotpala (950 CE) critically analyzed the content of the chapter in his "Sanskrit Commentary on *BS* (BS-Com., 1968, pp. 834–851)".

This paper focuses only on verses 13 to 26, excluding verses 16 and 22 of this chapter (BS-Com., 1968, pp. 841–848). In the first set of verses from 13 to 21, Varāhamihira suggested selecting four substances in a definite proportion out of the specified 16 and then permute each combination. He claimed that one could prepare 1,74,720 perfumes from the mixtures after following the various mechanical processes. Bhaṭṭotpala amended this number to 43,680 perfumes, i.e., one-fourth of the previous estimate. These numbers sensibly justify the term *gandhārṇava* (the ocean of the perfumes).

Krishnamurthy has extensively discussed these aromatic materials with a short account of *gandhārṇava* in the chapter "Cosmetics and Varāhamihira" (Krishnamurthy, 2001, pp. 51–65). Similarly, Pandey has explained these verses in the chapter "Development of Algebra at Varaḥmihira Gurukula" (Pandey, 2010, pp. 127–131). However, the constraints imposed by Varāhamihira on the proportions of the two substances, that one should not select coriander and camphor in more than one part, remained unnoticed for a long time. Iyer addressed this issue in the chapter entitled "On Perfume Mixture" (Iyer, 1884, Ed., 2021, pp. 139–154).

Varāhamihira, in the second set of verses from 23 to 26, placed sixteen substances with pre-defined proportions, in *kacchapuṭa* (a box with compartments) (Williams, 2002, p. 241) or more precisely 'a square figure having the same number (four in the present case) of cells in each row and in

✉ Shriram Chauthaiwale
vedicganit.shriram@gmail.com

[1] Retired Lecturer in Mathematics, Amolakchand College, Yavatmal, Maharashtra, India

[2] National Commission for Indian System of Medicine, Ministry of AYUSH, Govt. of India, New Delhi, India

[3] SLACK Technologies, Pune, India

each column' (Hayashi, 1987, p. 161). Varāhamihira then explored the possible combinations. Only those combinations are said to be fit for the preparations of perfumes where the sum of the proportions of the four distinct substances is 18. He termed these perfumes as *gandhāḥ sarvatobhadraḥ* (the perfumes for all good purposes) but did not mention any number for the same. Bhaṭṭotpala noted 18 such combinations with a remark that many more can be guessed (BS-Com., 1968, p. 847). Later, Hayashi observed that *kacchapuṭa* is a pan-diagonal magic square with its sum 18 (Hayashi, 1987, p 161). The mere observation of this magic square reveals some additional combinations with 18 as the sum. Hence, such tetrads need precise counting and listing with the four ingredients in each of the combinations.

Here, the authors first examined the relevant calculations mathematically. Two separate computer programs, specially designed for evaluating or confirming and then listing the valid combinations with ingredients, were executed. Considering the premise of the Varāhamihira and Bhaṭṭotpala in *BS-Com.*, we interchanged the order of the verses in the original text for a better appreciation of the subject content. The simple translation of the verses is attested to, followed by the comments from the authors. The pseudo-code of the programs, executed in the "Python" programming language, is provided. Output is interpreted, and generated lists are attached in the links or placed in the Appendix.

Verse 16 suggests some mechanical processes (BS-Com., 1968, p. 842) and verse 22 describe the *loṣṭakaprastāra*) for the evaluation of the number of combinations (BS-Com., 1968, pp. 844–846). Hence, we do not consider these two verses here.

2 *Gandhārṇava*

2.1 The claim

Varāhamihira says (BS-Com., 1968, p. 841, p. 843):

घनवालकशैलेयककर्पूरोशीरनागपुष्पाणि ।
व्याघ्रनखस्पृक्कागुरुदमनकनखतगरधान्यानि ॥ 13 ॥
कर्चूरचोलमलयैः स्वेच्छापरिवर्तितैश्चतुर्भिरतः ।
एकद्वित्रिचतुर्भिर्भागैर्गन्धार्णवो भवति ॥ 14 ॥
अत्र सहस्रचतुष्टयमन्यानि च सप्ततिसहस्राणि ।
लक्षं शतानि सप्त विंशतियुक्तानि गन्धानाम् ॥ 17 ॥

ghanavālakaśaileyaka–
karpūrośīranāgapuṣpāṇi ।
vyāghranakhaspṛkkāguru–
damanakanakhatagaradhānyāni ॥13 ॥
karcūracolamalyaiḥ
swvecchāparivartitaiścaturbhirataḥ ।
ekadvitricaturbhir–
bhāgairgandhārṇavo bhavati ॥14 ॥
atra sahasracatuṣṭaya–
manyāni ca saptatisahasrāṇi ।
lakṣaṁ śatāni sapta–
viṁśatiyuktāni gandhānām ॥17 ॥

(From the sixteen substances, namely) *ghana*, *vālaka*, *śaileyaka*, *karpūra*, *uśīra*, *nāgapuṣpa*, *vyāghranakha*, *spṛkkā*, *aguru*, *damanaka*, *nakha*, *tagara*, *dhānya*, *karcūra*, *cola* and *malaya*, select any four substances at will. Fix the proportions (for selected four) as 1, 2, 3, and 4 parts and permute. This act generates number of perfumes. Here, (permutations of the obtained combinations give) 4000 + 70000 + 100000 + 720 (= 1,74,720) perfumes. For botanical names and the properties of the selected substances, along with the details of the process of preparation of the perfumes, one can refer to (Krishnamurthy, 2001, pp. 41–65) and (Deopujari, 2009, pp. 275–277).

2.2 Justification

Varāhamihira says (BS-Com., 1968, pp. 843–844):

षोडशके द्रव्यगणे चतुर्विकल्पेन भिद्यमानानाम् ।
अष्टादश जायन्ते शतानि सहितानि विंशत्या ॥ 20 ॥

ṣoḍaśake dravyagaṇe caturvikalpena bhidyamānānāṁ ।
aṣṭādaśa jāyante śatāni sahitāni viṁśatyā ॥ 20 ॥

The selection of the four substances from the set of sixteen gives 1800 + 20 (=1820 combinations).

एकैकमेकभागं द्वित्रिचतुर्भागिकैर्युतं द्रव्यैः ।
षड्गन्धकरं तद्वद् द्वित्रिचतुर्भागिकं कुरुते ॥ 18 ॥

ekaikabhāgaṁ dvitricatur–
bhāgikairyutaṁ dravyaiḥ ।
ṣaḍgandhakaraṁ tadvad–
dvitricaturbhāgikaṁ kurute ॥ 18 ॥

When combined with three others in two, three, and four parts, one substance selected as the one part produces six perfumes. Similar is the case when the first substance is specified as two, three, or four parts.

द्रव्यचतुष्टययोगाद् गन्धचतुर्विंशतिर्यथैकस्य ।
एवं शेषाणामपि षण्णवतिः सर्वपिण्डोऽत्र ॥ 19 ॥

dravyacatuṣṭayayogād
gandhacaturviṁśatiryathaikasya ।

evaṁ śeṣāṇāmapi
ṣaṇṇavatiḥ sarvapiṇḍo'tra ॥ 19 ॥

As one group of four substances (in four different proportions) sum up to 24 perfumes and consider the remaining (three tetrads), this sums up to 96.

षण्णवतिभेदभिन्नश्चतुर्विकल्पो गणो यतस्तस्मात्।
षण्णवतिगुणः कार्यः सा सङ्ख्याभवति गन्धानाम्॥ 21 ॥

ṣaṇṇavatibhedabhinna–
ścaturvikalpo gaṇo yatastasmāt ।
ṣaṇṇavatiguṇaḥ kāryaḥ
sā saṅkhyābhavati gandhānāṁ ॥

Thus because, the group has ninety-six different variants, (one) should multiply by 96. Then, that number (96 × 1820 = 1,74,720) is the number of (possible) perfumes.

2.3 Bhaṭṭotpala's amendment

Bhaṭṭotpala, while commenting on verse 21 says (BS-Com., 1968, p. 844):

… एतत् गौणवृत्या न मुख्यया। मुख्यवृत्त्या त्रिचत्वारिं–शत्सहस्राणि षड्शतान्यशीत्यधिकानि भवन्ति । यतश्चतुर्विकल्पो गणश्चतुर्विंशतिभेदभिन्नाः॥

… etat gauṇavṛtyā na mukhyayā । *mukhyavṛtyā tricatvāriṁśatsahasrāṇi ṣaṭśatānyaśītyadhikāni bhavanti* । *yataścaturvikalpo gaṇaścaturviṁśatibhedabhinnāḥ* ॥

This way of multiplication (96 × 1820 = 174,720) is (to be) rejected. By the better method, it (the number of perfumes) is 43,000 + 600 + 80 (= 43,680). Because twenty-four variants (are there) when the group of four is permuted.

The footnote on the same page says (BS-Com., 1968, p. 844):

अष्टादशशतानि विंशत्यधिकानि चतुर्विंशत्या गुणानि त्रिचत्वारिंशत्सहस्राणि षड्शतान्यशीत्यधिकानि जायन्ते।

aṣṭādaśaśatāni viṁśatyadhikāni caturviṁśatyā guṇāni tricatvāriśatsahasrāṇi ṣaṭśatānyaśītyadhikāni jāyante ।

(When) 1820 is multiplied by 24, it becomes 43,680.

2.4 Comments

(a) Here, Varāhamihira is applying the rules of the combinations and permutations to arrive at the number of possible perfumes. However, the applications of these concepts are traceable in the Piṅgala's *Chandaśāstra*, Bharata's *Nātyaśastra*, *Caraka Saṁhitā* and in the Jain literature (Pandey, 2010, p. 123). Varāhamihira used the same concepts in the *Bṛhatjātaka* to determine the verities of what he termed as *anaphā*, *sunphā* and *durdharā yogas* (Pandey, 2010, pp. 124–126). (b) Mahāvīra (850 CE), in the *Gaṇitasārasaṅgraha* (*GSS*), introduced the formula for the combinations (Padmavatamma, 2000, verse 218, p. 348) and Bhāskara II (1150 CE) in the *Līlāvatī* stated the formula for the number of permutations (*Līlāvatī*, 2007, verse 262, p. 275]. (c) Thus, number of perfumes is ${}^{16}C_4 \times {}^{4}P_4 = 43{,}680$. This number expectedly matches with Bhaṭṭotpala's amended count.

2.5 The constraints

Varāhamihira says (BS-Com., 1968, p. 842):

अत्युल्बणगन्धत्वादेकांशोनित्यमेव धान्यानाम् ।
कर्पूरस्य तदूनोनैतो द्वित्र्यादिभिर्देयौ॥ 15 ॥

atyulbaṇagandhatvādekāṁśonityameva dhānyānāṁ ।
karpūrasya tadūnonaito dvitryādibhirdeyau ॥15 ॥

Due to strong smells, coriander (*dhānya*) should always be selected as one part, and camphor (*karpūra*) should be chosen in lesser proportions. (Therefore), these (two substances) should not be considered in two, three, or other (more) parts (in the preparation of the perfumes).

Bhaṭṭotpala explains this as (BS-Com., 1968, p. 842):

..... एतौ द्वौ द्वित्र्यादिभिर्भागै: प्राप्तावपि न देयौ न दातव्यौ । यतस्तदुत्कटत्वादन्यद्रव्याणां गन्धहानिर्भवति ।

..... etau dvau dvitryādibhirbhāgaiḥ prāptāvapi na deyau na dātavyau। *yatastadutkaṭatvādanyadravyāṇāṁ gandhahānirbhavati* ।

Even if obtained in two or more parts (in the process of permutations), these two substances should not be considered. The strong fragrance of these two components overpowers the aroma of the other ingredients.

3 Number and the list of perfumes with constraints

The constraints on the proportions of the two ingredients will reduce the count (43,680) of the possible perfumes. Iyer noted this fact and claimed that the valid count is 28,392 after a long discussion. (Iyer, 1884, Ed. 2021, pp. 143–147). Here, the following combinatorial arguments justify this count.

3.1 Justification

There is no constraint when *dhānya* and *karpūra* are not involved in the selection. Hence the number of perfumes is—

$$ {}^{14}C_4 \times {}^{4}P_4 = 1001 \times 24 = 24{,}024. $$

Again, both *dhānya* and *karpūra* cannot be used in any perfume as both can only be used in 1 proportion/part. When either of them is used in 1 proportion/part, we can choose three of the other 13 basic substances in $^{14}C_3 = 364$ ways and mix them in proportions 2:3:4 in 6 ways. Hence, the number of perfumes when *dhānya* or *karpūra* is included is—

$$2 \times 364 \times 6 = 4368.$$

Hence, total number of perfumes taking into consideration the constraints mentioned in verse 15 is 24,024 + 4368 = 28,392.

3.2 The list

The authors executed the following Python language program to enlist the ingredients in each of the 28,392 perfumes in *gandhārṇava*. This list serves as a reckoner for the preparation of the perfumes and eventually confirms the count.

Deopujari and his team prepared 24 perfume samples using eight raw materials with constraints and tested them on human skin and clothes. All the perfumes with a six-month shelf-life gave a satisfactory response (Deopujari, 2009, pp. 276–277).

3.3 Pseudo code of the program

```
function gandhashastra_method_1 ()
{
    ingredients = ['ghana', 'valaka', 'shaileyaka',
  'karpura', 'ushira', 'nagapuspa', 'vyaghranakha',
  'spruka', 'aguru', 'madanaka', 'nakha',
  'tagara', 'dhanya', 'karchura', 'cola', 'malaya']
    total_combinations = combinations (ingredients,16, 4)
    final_valid_list = []
    for each c in total_combinations:
        parts = [1,2,3,4]
        ingredients_with_parts = permutation (c, parts)
        for each p in ingredients_with_parts:
            If p.element is 'dhanya' or 'karpura' and p.part = 1:
                add p to final_valid_list
print 'Total number of possible gandhas:' + count (final_valid_list)
```

3.4 The Output (1)

The list of 28,392 perfumes, each having four different proportions with constraints. [Link 1].

3.5 Conclusion (I)

The total number of perfumes is 28,392. This number is considerable enough, even after a 35% reduction in Bhaṭṭotpala's count, to justify the name *gandhārṇava*.

4 *Kacchapuṭa*

4.1 The claims

Varāhamihira says (BS-Com., 1968, p. 846).Varāhamihira says (*BS-Com.,* 1968, p. 846).

द्वित्रीन्द्रियाष्टभागै अगुरुः पत्रं तुरुष्कशैलयौ ।
विषयाष्टपक्षदहनाः प्रियङ्गुमुस्तारसाः केशः ॥ 23 ॥
स्पृक्कात्वक्तगराणां मांस्याश्च कृतैकसप्तषड्भागाः ।
सप्तर्तुवेदचन्द्रैः मलयनखश्रीककुन्दुरुकाः ॥ 24 ॥

dvitrīndriyāṣṭabhāgaiḥ
aguruḥ patraṁ turuṣkaśailayau ।
viṣayāṣṭapakṣadahanāḥ
priyaṅgumustārasāḥ keśaḥ ॥ 23 ॥
spṛukkātvaktagarāṇāṁ
māṁsyāśca kṛtaikasaptaṣaḍbhāgāḥ ।
saptartuvedacandraiḥ
malayanakhaśrīkakundurukāḥ ॥ 24 ॥

The word-numerals used to denote the proportions are:

चन्द्र = 1, पक्ष = 2, दहन = 3, वेद, कृति = 4, इन्द्रिय, विषय = 5, ऋतु = 6.

(First select) two, three, five and eight parts of *aguru* (S1), *patra* (S2), *turuṣka* (S3), *śaileyaka* (S4) (respectively); (then) five, eight, two and three (parts of) *priyaṅgu* (S5), *mustā* (S6), *rasa* (S7), *keśa* (S8); (then) four, one, seven and six parts of *spṛkkā* (S9), *tvak* (S10), *tagara* (S11), *māṁsī* (S12) (and finally select) seven, six, four, and one part of *malaya* (S13), *nakha* (S14), *śrīka* (S15), *kunduruka* (S16) (respectively).

Varāhamihira further says (BS-Com., 1968, p. 847):

षोडशके कच्छपुटे यथा तथा मिश्रिते चतुर्द्रव्ये ।
येऽत्राष्टादशभागास्तेऽस्मिन् गन्धादयो योगाः ॥ 25 ॥

ṣoḍaśake kacchapuṭe yathā tathā miśrite caturdravye ।
ye'trāṣṭādaśabhāgāste'smin gandhādayo yogāḥ ॥ 25 ॥

When any four substances, out of sixteen arranged in a *kacchapuṭa*, are randomly mixed (the mixtures), where the sum of the proportions (of the selected

substances) is 18, are worthy of producing the perfumes.

He also said that (from the mixture so formed) the perfumes which are suitable for all purposes should be prepared. {गन्धाः कर्तव्याः सर्वतोभद्राः} [*BS-Com*., v 26, p. 847).

The *kacchapuṭa* is as shown in the figure below.

aguru **2**	*patra* **3**	*turuṣka* **5**	*śaileyaka* **8**
priyaṅgu **5**	*mustā* **8**	*rasa* **2**	*keśa* **3**
spṛkkā **4**	*tvak* **1**	*tagara* **7**	*māṁsī* **6**
malaya **7**	*nakha* **6**	*śrīka* **4**	*kunduruka* **1**

Bhaṭṭotpala says (BS-Com., 1968, p. 847):

अस्मिन् षोडशके षोडशककोष्ठके कच्छपुटे यथा तथा येन तेन प्रकारेण चतुर्द्रव्ये मिश्रिते एकीकृते । चतुर्भिर्द्रव्यैः यथाभागपरिकल्पितैः मिश्रीकृतैरत्र येऽष्टादश भागा भवन्ति तेऽस्मिन् कच्छपुटे गन्धादय ऊर्ध्वाधःक्रमेण तिर्यग्वा चतुर्षु कोणेषु वा मध्यमचतुष्कोणे वाकोणकोष्ठचतुष्टये वा प्राक्पङ्क्तौ वा मध्यमकोष्ठद्वये वा अन्त्यपङ्क्तौ मध्यमकोष्ठद्वये वा द्वितीयतृतीयपङ्क्तौ वा आद्यन्त: कोष्ठके वा येन केन प्रकारेण चतुर्षु मिश्रितेषु अष्टादशभागा भवन्ति ।

asmin ṣoḍaśake ṣoḍaśakakoṣṭake kacchapuṭe yathā tathā yena tena prakāreṇa caturdravye miśrite ekīkṛte । caturbhirdravyaiḥ yathābhāgaparikalpitaiḥ miśrīkṛtairatra ye'ṣṭādaśa bhāgā bhavanti te'smin kacchapuṭe gandhadaya ūrdhvārdhaḥ krameṇa tiryagvā caturṣu koṇeṣu vā madhyamacatuṣkoṇe vā koṇakoṣṭhacatuṣṭaye vā prākpaṅktau vā madhyamakoṣṭhadvaye vā antyapaṅktau madhyamakoṣṭhadvaye vā dvitītyatṛtīyapaṅktau vā ādyantakoṣṭake vā yena kena prakāreṇa caturṣu miśriteṣu aṣṭādaśabhāgā bhavanti ।

In this *kacchapuṭa* with sixteen cells, four substances with specified proportions are mixed at will. Here, when the sum of the proportions is 18, a perfume is generated out of the mixture. This also happens when substances are mixed from top to bottom, horizontally, or along the diagonals. Further, perfume can also be prepared when substances are mixed in the following ways:

- Substances at each of the four corners or
- at the central quadrangle or
- at four corner cells or
- each at middle two cells of the first row with those at the last row or
- each at the first and last cell of the second and third-row or in any other manner

4.2 Comments

(a) Indian tradition assigns the origin of the 'Magic Squares' to the lord Śiva who taught this to magician Maṇibhadra. However, the undisputable study of magic squares is found in the work of famous alchemist and philosopher Nāgārjuna (f. First century CE) under the caption *kakṣapuṭa*. The Magic Squares discussed by him are all (4 × 4) types, and some of them claimed to be known before him. He mentioned these squares and elaborated the rules for the construction of magic squares with even and odd totals (Datta, 1992, pp. 51–58]. The detailed mathematical study of the magic squares is available in the "*Gaṇitakaumudī* (*GK*)" of Nārāyaṇa Paṇḍita (1356 CE) under the caption *bhadragaṇitaṁ*. (Dvivedi, 1942, pp. 353–411) (Singh, 2002, pp. 34–98) (b) Usually, Nāgārjunīya squares are constructed by filling distinct sixteen numbers, but in *kacchapuṭa*, only eight digits from 1 to 8 are found used, and each digit is repeated twice. (c) *Kacchapuṭa* is itself a particular case of Nāgārjunīya square with a total 2N = 18 as shown below (Datta, 1992, p. 57).

Nāgārjunīya magic square				Varāhamihira's magic square			
2	N – 6	5	N – 1	2	3	5	8
N – 4	8	N – 7	3	5	8	2	3
N – 5	1	N – 2	6	4	1	7	6
7	N – 3	4	N – 8	7	6	4	1

Thus, Varāhamihira followed Nāgārjuna and skillfully employed profound mathematical techniques to express his ideas. This act is, no doubt, an excellent example of a multidisciplinary activity of our ancestors. Hayashi remarked that 'Being pan-diagonal, this magic square has many other tetrads whose sums are 18 (Hayashi, 1987, p. 161). The sum of proportions in any combinations will be between 10 and 26, both inclusive. However, Varāhamihira directed that only those combinations are suitable for the perfume, in which the sum is 18. Bhaṭṭotpala mentioned eighteen such combinations. Now, add six along the pan diagonals raising the total to 24. Careful observation reveals many such tetrads. The *aguru*, *śaileyaka*, *spṛkā*, *tagara*, *nakha* and *malaya* are the six common substances for *gandhārṇava* and *kacchapuṭa*. However, the second list does not contain *dhānya* and *karpūra* for which the constraints are applicable.

5 Second program

The authors executed the following Python-language-based program to determine how many of the $^{16}C_4 = 1820$ quadruples of entries add to the magic constant and enlist such entries pattern-wise for a specified magic square (A) and those for Varāhamihira's Magic Square "*kacchapuṭa*."

Magic square (A)				Varāhamihira's magic square			
2	11	5	16	2	3	5	8
13	8	10	3	5	8	2	3
12	1	15	6	4	1	7	6
7	14	4	9	7	6	4	1

5.1 Pseudocode

function gandhashastra_method_2 ()

```
{
        ingredients = {'aguru': 2, 'patra': 3, 'turushka':
5, 'saileyaka': 8, 'priyangu': 5, 'musta': 8, 'rasa': 2, 'kesha':
3, 'spruka': 4, 'tvak': 1, ' tagara': 7, 'mamsi': 6, 'malaya': 7,'nakha':
6, 'shrika': 4, 'kunduruka': 1}
  comb = combinations (ingredients, 4)
count = 0
for c in comb:
if c[0].quantity + c[1].quantity + c[2].quantity + c[3].quantity == 18:
count += 1
print c
print "Total number of valid combinations are" + count
}
```

5.2 Output (II)

For magic square (A), 86 of 1820 quadruples add to magic constant 34. For *kacchapuṭa,* 172 of 1820 quadruples add to the magic constant 18. The list of the ingredients in these 172 quadruples is mentioned in (Appendix).

5.2.1 Notes

- One can execute the program (II) and compare the output for (4 × 4) magic squares discussed by Nārāyaṇa Paṇḍita (Datta, 1992, p. 66), for (4 × 4) Jaina Magic Squares (Vijayraghavan, 1941, pp. 97–102] and those discussed in by Sridharan and Srinivas (2011, pp. 383–391).
- The output (86 quadruples) for (4 × 4) pan-diagonal magic squares where the entries are all different and the result (172 quadruples) for *kacchapuṭa* where each entry is repeated twice shows the level of symmetry.

5.3 Conclusions (II)

When 16 specified substances with pre-defined proportions are wrapped up in the *kacchapuṭa* then, the total number of perfumes is 172. As per Euler's criterion [Link 2], this *kacchapuṭa* is a "Most Perfect Magic Square" in which entries in 36 quadruples [List, S. No. 1 to 36] sum up to 18. Also, 16 knight tour patterns add up to 18. [List, S. No. 37 to 52]. This *kacchapuṭa* is a 'Super Magic Square' as another 120 quadruples sum up to 18 [List, S. No. 53 to 172]. Authors attributed the adjective 'Super' to this magic square. This attempt exhibits the profound character and the formidable thought process of Varāhamihira and the Indian scholars in general in multidisciplinary activities like *gandhayukti*.

Appendix

List of the 172 perfumes, where proportions of four ingredients add to 18

No.	Substances				No.	Substances				No.	Substances				No.	Substances			
1	S1	S2	S3	S4	**46**	S5	S9	S15	S3	**91**	S2	S9	S13	S15	**136**	S4	S8	S14	S16
2	S5	S6	S7	S8	**47**	S9	S13	S3	S7	**92**	S2	S3	S8	S11	**137**	S4	S7	S13	S16
3	S9	S10	S11	S12	**48**	S13	S1	S7	S11	**93**	S2	S9	S11	S15	**138**	S4	S7	S9	S15
4	S13	S14	S15	S16	**49**	S2	S6	S12	S16	**94**	S2	S4	S8	S9	**139**	S4	S5	S15	S16
5	S1	S5	S9	S13	**50**	S6	S10	S16	S4	**95**	S2	S4	S8	S15	**140**	S5	S7	S9	S11
6	S2	S6	S10	S14	**51**	S10	S14	S4	S8	**96**	S2	S6	S8	S9	**141**	S5	S8	S12	S15
7	S3	S7	S11	S15	**52**	S14	S2	S8	S12	**97**	S2	S6	S8	S15	**142**	S5	S8	S9	S14
8	S4	S8	S12	S16	**53**	S1	S3	S13	S15	**98**	S2	S5	S9	S12	**143**	S5	S6	S9	S16
9	S1	S6	S11	S16	**54**	S1	S6	S7	S12	**99**	S2	S5	S14	S15	**144**	S5	S6	S10	S15
10	S4	S7	S10	S13	**55**	S1	S3	S6	S8	**100**	S2	S7	S11	S12	**145**	S5	S7	S11	S15
11	S2	S5	S12	S15	**56**	S1	S8	S12	S13	**101**	S2	S5	S6	S7	**146**	S5	S7	S9	S13
12	S3	S8	S9	S14	**57**	S1	S6	S10	S13	**102**	S2	S4	S12	S16	**147**	S5	S10	S12	S14
13	S1	S8	S11	S14	**58**	S1	S8	S13	S14	**103**	S2	S3	S4	S7	**148**	S5	S12	S14	S16
14	S4	S5	S10	S15	**59**	S1	S2	S12	S13	**104**	S2	S4	S10	S14	**149**	S6	S8	S10	S12
15	S2	S7	S12	S13	**60**	S1	S8	S11	S12	**105**	S2	S6	S10	S12	**150**	S6	S7	S9	S15
16	S3	S6	S9	S16	**61**	S1	S2	S11	S14	**106**	S2	S6	S14	S16	**151**	S6	S7	S9	S13
17	S1	S2	S5	S6	**62**	S1	S9	S12	S14	**107**	S2	S11	S13	S16	**152**	S6	S8	S12	S16
18	S2	S3	S6	S7	**63**	S1	S12	S14	S15	**108**	S2	S10	S11	S13	**153**	S6	S8	S10	S14
19	S3	S4	S7	S8	**64**	S1	S6	S9	S15	**109**	S2	S3	S12	S15	**154**	S6	S7	S11	S16
20	S5	S6	S9	S10	**65**	S1	S4	S10	S13	**110**	S2	S3	S9	S14	**155**	S7	S9	S12	S14
21	S6	S7	S10	S11	**66**	S1	S3	S11	S15	**111**	S3	S8	S12	S15	**156**	S7	S8	S12	S13
22	S7	S8	S11	S12	**67**	S1	S5	S6	S8	**112**	S3	S6	S10	S15	**157**	S7	S12	S14	S15
23	S9	S10	S13	S14	**68**	S1	S2	S3	S6	**113**	S3	S5	S7	S14	**158**	S7	S8	S11	S14
24	S10	S11	S14	S15	**69**	S1	S3	S4	S8	**114**	S3	S5	S13	S16	**159**	S8	S10	S11	S13
25	S11	S12	S15	S16	**70**	S1	S4	S11	S16	**115**	S3	S5	S10	S13	**160**	S8	S9	S13	S15
26	S1	S3	S9	S11	**71**	S1	S5	S13	S15	**116**	S3	S5	S10	S11	**161**	S8	S11	S13	S16
27	S2	S4	S10	S12	**72**	S1	S5	S9	S11	**117**	S3	S8	S9	S12	**162**	S8	S9	S11	S15
28	S5	S7	S13	S15	**73**	S1	S2	S4	S5	**118**	S3	S8	S14	S15	**163**	S9	S11	S14	S16
29	S6	S8	S14	S16	**74**	S1	S3	S9	S13	**119**	S3	S4	S9	S16	**164**	S9	S13	S14	S16
30	S1	S4	S13	S16	**75**	S1	S6	S10	S11	**120**	S3	S4	S10	S15	**165**	S9	S10	S11	S14
31	S1	S2	S13	S14	**76**	S1	S6	S13	S16	**121**	S3	S6	S9	S10	**166**	S9	S11	S12	S16
32	S2	S3	S14	S15	**77**	S1	S4	S7	S12	**122**	S3	S6	S7	S8	**167**	S9	S10	S12	S13
33	S3	S4	S15	S16	**78**	S1	S4	S7	S14	**123**	S3	S7	S9	S11	**168**	S10	S12	S13	S15
34	S1	S4	S5	S8	**79**	S1	S6	S7	S14	**124**	S3	S7	S13	S15	**169**	S10	S13	S14	S15
35	S5	S8	S9	S12	**80**	S1	S3	S5	S12	**125**	S3	S5	S7	S12	**170**	S10	S11	S12	S15
36	S9	S12	S13	S16	**81**	S1	S3	S5	S14	**126**	S3	S5	S11	S16	**171**	S11	S14	S15	S16
37	S1	S2	S11	S12	**82**	S1	S4	S9	S15	**127**	S3	S12	S14	S16	**172**	S12	S13	S15	S16
38	S2	S3	S12	S9	**83**	S2	S4	S14	S16	**128**	S3	S10	S12	S14					
39	S3	S4	S9	S10	**84**	S2	S4	S5	S7	**129**	S3	S6	S15	S16					
40	S4	S1	S10	S11	**85**	S2	S5	S8	S11	**130**	S4	S7	S11	S16					
41	S5	S6	S15	S16	**86**	S2	S7	S11	S14	**131**	S4	S5	S9	S16					
42	S6	S7	S16	S13	**87**	S2	S5	S9	S14	**132**	S4	S5	S7	S8					
43	S7	S8	S13	S14	**88**	S2	S7	S13	S14	**133**	S4	S5	S9	S10					
44	S8	S5	S14	S15	**89**	S2	S3	S8	S13	**134**	S4	S8	S10	S12					
45	S1	S5	S11	S15	**90**	S2	S5	S8	S13	**135**	S4	S7	S10	S11					

Acknowledgements We are grateful to the reviewers for their comments, suggestions, and for providing valuable references. This guidance will remain cardinal in our minds.

References

BS-Com. (1968). *Śrī Varāhamihirācāryaviracitā Bhaṭṭotpalavivṛtīsahitā Bṛhatsaṁhitā*, Varanasi: *Sarasvatī Granthamālā* (97).

Datta, B., & Singh, A. N. (1992). Magic squares in India, *Indian Journal of. History of Science, 27*(1), 51–120.

Deopujari, J. Y., Mandavgane, S. A., & Holey, P. P. (2009). Preparation and testing of perfumes as described in *Bṛhatsaṁhitā*. *Indian Journal of Traditional Knowledge, 8*(2), 275.

Dvivedi Padmakara (Ed.). (1942). The *Gaṇitakaumudī* (*GK*) by Nārāyaṇa Paṇḍita (Part II), Chapter 15, Benares.

Hayashi, T. (1987). Varāhamihira's Pan-diagonal magic square of the order four. *Historia Mathematica, 14*, 159–166.

Iyer Chidambaram, N. (Tr). (1884). *The Brihat Samhita of Varahmihura*, Vol II, New Delhi: (Ed 2021), Jyoti Enterprises

Krishnamurthy, K. H. (2001). *Ayurvedic Technical Studies and Herbal Cosmetics of Ancient India*. B.R. Publishing House.

Līlāvatī, Srīmad Bhāskarācāryaviracita (Reprint 2007). Pune: Ānadāśrama Sanskrit *Granthāvaliḥ* (107).

[Link 1] The list of 28,392 perfumes. https://drive.google.com/file/d/1ZcVtQPGtMrtFJ20yj1m3N3XEoZiosEa/view?usp=sharing.

[Link 2] Euler's criterion.

https://en.m.wikipedia.org > Magic Square ^ variations of the magic square.

[List] List of 172 perfumes placed as an appendix.

Padmavatamma. (Ed.) (2000). *Mahāvīra's Gaṇitasārasaṅgraha* (*GSS*), Hombuja, Shimoga: India, *Siddhānta Kīrthi Granthamāla*.

Pandey, G. S. (2010). *A Study in the Mathematical Contributions of Varahamihira and his Heritage*. Shimla: Indian Institute of Advanced Studies

Singh, P. (2002) The *Gaṇitakaumudī* of Nārāyaṇa Paṇḍita, chapter XIV, (Eng. Tr. with Notes), *Gaṇita Bhāratī*, *24*.

Sridharan, Raja and Srinivas, M. D. (2011). Study of Magic Squares in India, In: Sujatha, R et al. (ed.) *Math Unlimited: Essays in Mathematics*, CRC Press.

Monier, W. (2002). *A Sanskrit English Dictionary*. Motilal Banarsidass Publishers.

Vijayaraghavan, T. (1941). On Jaina magic squares. *The Mathematics Student, 9*(3), 97–102.